

This book explores recent research underlining the remarkable connections between the algebraic and arithmetic world of Galois theory and the topological and geometric world of fundamental groups. Arising from an MSRI program held in the fall of 1999, it contains ten articles, all of which aim to present new results in a context of expository introductions to theories that are ramifications and extensions of classical Galois theory.

B. H. Matzat and M. van der Put introduce differential Galois theory and solve the differential inverse Galois problem over global fields in positive characteristic; D. Harbater gives a comparative exposition of formal and rigid patching starting from the familiar complex case. S. Mochizuki discusses aspects of Grothendieck's famous anabelian geometry, while the articles by R. Guralnick, A. Tamagawa, and F. Pop and M. Saïdi investigate the structure of the fundamental groups of curves over different kinds of characteristic p fields. M. Imbert and L. Schneps study the structure of the Hurwitz spaces and moduli spaces of curves, which are of great importance to Galois theory because of the Galois action on their fundamental groups. The first interesting such group is $SL_2(\mathbb{Z})$, a family of special subgroups of which is studied by F. Bogomolov and Y. Tschinkel. Finally, R. Hain and M. Matsumoto present their result proving part of a conjecture by Deligne on the structure of the Lie algebra associated to the Galois action on the fundamental group of the thrice-punctured projective plane.

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41

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