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Go Thermography: The 4/21/98 Jiang–Rui Endgame

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Go thermography is more complex than thermography for classical combinatorial games because of loopy games called kos. In most situations, go rules forbid the loopiness of a ko by banning the play that repeats a position. Because of the ko ban one player may be able to force her opponent to play elsewhere while she makes more than one play in the ko, and that fact gives new slopes to the lines of ko thermographs. Each line of a thermograph is associated with at least one line of orthodox play [Berlekamp 2000, 2001]. Multiple kos require a new definition of thermograph, one based on orthodox play in an enriched environment, rather than on taxes or on composing thermographs from the thermographs of the followers [Spight 1998]. Orthodox play is optimal in such an environment.

Reading a Thermograph

Many go terms have associations with thermographs. They are not defined in terms of thermographs, but I will be using them in talking about the game, and it will be helpful to be able to visualize the thermographs when I do. The inverse of the slope of a thermographic line indicates the net number of local plays. If one player makes 2 local plays while her opponent makes only plays in the environment, the slope will be plus or minus $\frac{1}{2}$. The color of a line indicates which player can afford to play locally at a certain temperature. The player does not necessarily wish to play at that temperature, but she can do so without loss.

In the simple thermograph in Figure 1, the black mast at the top extends upward to infinity. It indicates a region of temperature in which neither player can afford to play locally without taking a loss. The mast starts at temperature, $t = 2$, which we call the temperature of this game, at the top of the hill. It also indicates the local count (or mast value), which is -5. Just below the top of the hill, the blue line of the Left wall indicates that Black (or Left) will not be unhappy to make a local play in this region of temperature. And it shows what

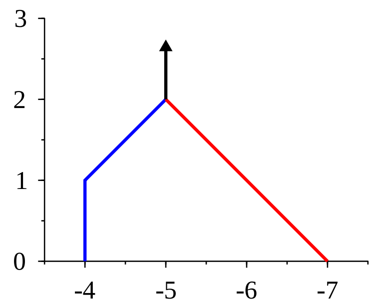


Figure 1. Simple Thermograph.

the local score at each temperature would be when Black plays first. Similarly, the red line of the Right wall indicates an initial play by White (or Right). A vertical slope, whether of a mast or a wall, indicates that each player has made the same number of local plays. The vertical section of the Left wall indicates that Black initiated play and that the whole sequence of play contained an equal number of local plays.

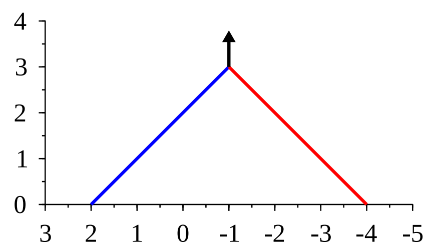


Figure 2. Gote.

Figure 2 shows the prototypical gote thermograph. Two inclined slopes meet at the top of the hill, which means that each player is inclined to play below the temperature of the game, which is 3. (The mast value is -1.) She initiates a sequence of play in which she makes one more play locally than her opponent. Above the temperature of the game the mast is black, which means that neither player is inclined to play locally.

Figure 3 shows the prototypical sente thermograph. Its mast value is 2 and its temperature is 1. It has one vertical wall at the top of the hill, which indicates that, just below the temperature of the game, one player will be able to play locally and force her opponent to make an equal number of local plays. This is White's sente because the Right wall is vertical. Also, the mast is colored red, which means that White will not be unhappy to initiate local play in the region between the temperature of the game and the temperature above which the mast

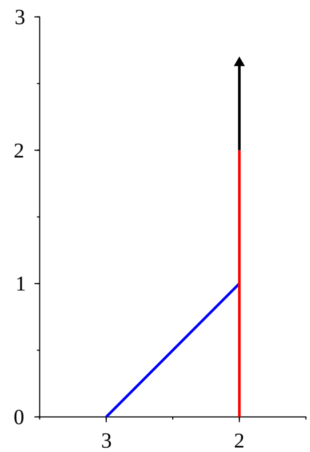


Figure 3. Sente.

is black. So when $1 < t < 2$, White will be able to play in the game with sente, as Black does best to reply locally.

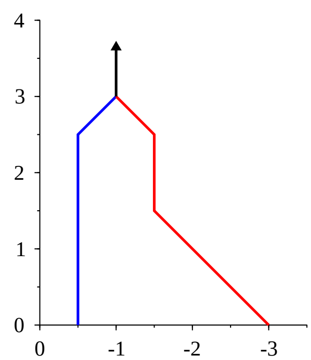


Figure 4. "Double sente".

A common go term is double sente. At first it does not seem to make much sense in terms of thermography, because it suggests two distinct vertical lines, which can never meet to form a mast. However, it does make sense as a temperature relative term. In the thermograph in Figure 4, both walls are vertical when $1\frac{1}{2} < t < 2\frac{1}{2}$. At those temperatures, each player will be able to play in the game with sente, and will be eager to do so, as the gain from playing costs nothing (versus allowing the opponent to play first locally). Hence the go proverb, "Play double sente early." Of course, earlier, when $t > 2\frac{1}{2}$, a local play is gote, but double sente tend to arise when the ambient temperature is lower than the local temperature.

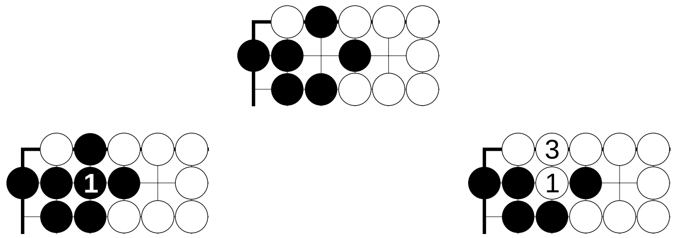


Figure 5. Ko.

The top central position in Figure 5 is a ko. The single Black stone on the upper side has only one adjacent free point (liberty) and may be captured by a play on that point (W 1). Then the stone at 1 has only one liberty, and could be captured except for the ko ban.

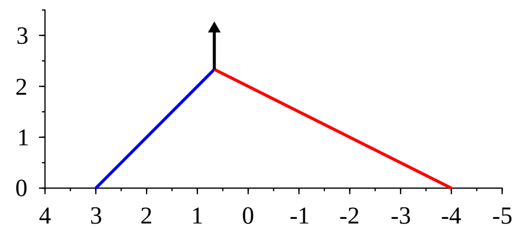


Figure 6. Ko Thermograph (White komaster).

Figure 6 shows the thermograph for the ko in Figure 5 when White is komaster, which means that he can take and win the ko if he plays first [Berlekamp 1996]. If Black wins the ko by filling with B 1, she gets 3 points locally, 2 points of territory plus 1 point for the dead White stone on the 2-1 point. The Left wall indicates the single Black play to a local score of 3. If White captures the ko with W 1 and then, after Black plays elsewhere, wins it with W 3, he gets 2 points of territory plus 1 point for the dead Black stone on the 2-4 point plus 1 point for the stone captured by W 1, for a total of 4 points. The Right wall indicates the 2 White moves to a local score of -4. The mast value of the ko is $\frac{2}{3}$, and its temperature is $2\frac{1}{3}$.

Figure 7 shows the thermograph of the same ko when Black is komaster. Even if White plays first, Black can win the ko. We do not see a separate Right wall, because it coincides with the Left wall. Since the two walls coincide, they form a mast. The inclined section of the mast when $t < 2\frac{1}{3}$ is purple, as it combines blue and red lines. When White takes the ko, Black makes a play that carries a threat which is too large for White to ignore. White replies to this ko threat, and then Black takes the ko back and wins it on the following move. The local result is the same as when Black simply wins the ko. The purple mast indicates that either player can play without a local loss at each temperature in

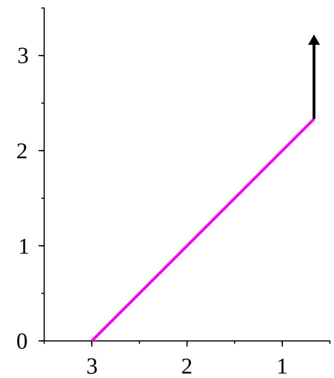


Figure 7. (Black komaster).

that range, but that fact does not take into account the dynamic aspects of the ko. If Black can afford to wait, as the temperature drops the value of the game increases, and that favors Black. So Black will be inclined not to win the ko yet. Therefore the koloser, White, should take the ko and force Black to win it before the temperature drops. It costs Black one move to win the ko, and that allows White to make a play in the environment. The hotter the ambient temperature, the more White gains from that play. This idea is counterintuitive, and most amateur go players have no conception of starting a ko when they cannot win it. They shy away from it.

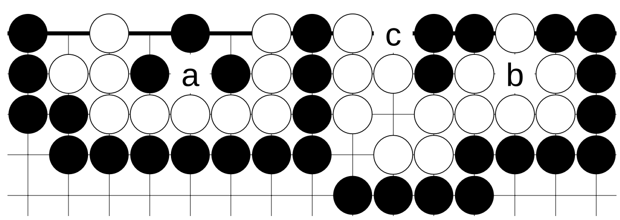


Figure 8. Ko and Gote.

Figure 8 shows a ko on the right and a gote on the left. In the gote, if Black plays at a she kills the White group for 23 points; if White plays at a he saves the group for a local score of -8. The gote has a mast value of $7\frac{1}{2}$ and a temperature of $15\frac{1}{2}$. If Black takes the ko at b and then connects 1 point above b, she kills the White group for 28 points; if White plays at c to take the 3 Black stones, he gets 8 points. The ko has a mast value of 4 and a temperature of 12.

Figure 9 illustrates the unusual thermographs that can occur with sums including kos. This is the thermograph of the sum of the gote and ko in Figure 8, when neither player has a ko threat The gote is hotter than the ko, so that when $t > 12$, each player will prefer the gote to the ko, and the thermograph looks like the standard gote thermograph. But when $t < 12$, both plays are

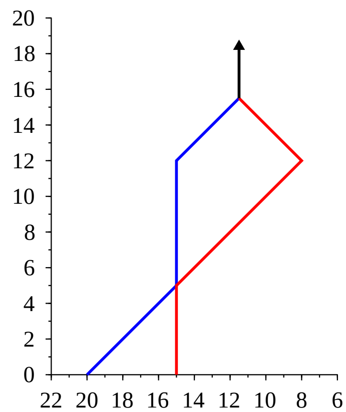


Figure 9. Sum of Ko plus Gote.

viable, and their interaction produces a curious loopback and osculation. When $5 < t < 12$, Black to move will play in the gote, allowing White to win the ko. That exchange produces the vertical segment of the Left wall. White to move will also play in the gote, allowing Black to take and win the ko while White makes a play elsewhere. That exchange produces the backward loop in the Right wall. Below the osculation point, when $t < 5$, each player will prefer to play in the cooler component, the ko.

The Jiang–Rui Environmental Go Game

On April 21, 1998, Jiang Zhujiu and Rui Nai Wei, 9-dan professional players, played the first environmental go game. Jiang played Black and Rui played White. Jiang won by $\frac{1}{2}$ point. The environment for environmental go is a stack of coupons. Instead of making a play on the board, a player may take the top coupon. This coupon is worth t points, the ambient temperature. For this game the values of the coupons ranged from 20 to $\frac{1}{2}$ in $\frac{1}{2}$ point decrements. To offset the advantage of playing first, White received compensation (komi) of $9\frac{1}{2}$ points. (In theory the komi should be 10 points, half the ambient temperature, but a non-integral komi is customary to avoid ties.) For go regions of even moderate size, exhaustive search is not possible, even with computer assistance, and we cannot absolutely guarantee all of our results. However, with the collaboration of two of the best go players in the world, we feel confident that we know the main lines of orthodox play from this point on.

Figure 10 shows a position from the game indicating marked territories and the temperatures of some regions. (The temperatures assume orthodox play with White to play in this position.) The hottest region is the Southwest corner, extending into the central South region. The top coupon is now worth 4 points, and this is a double sente at that temperature. We do not need to know the local temperature precisely, only that it is greater than 4. It is approximately 5.

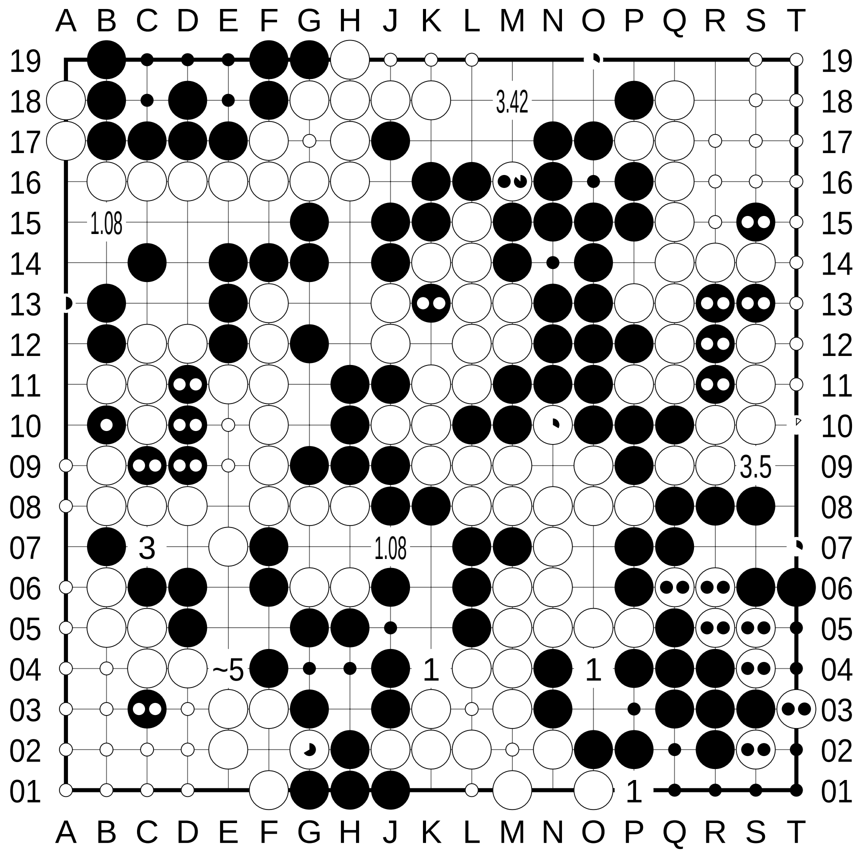
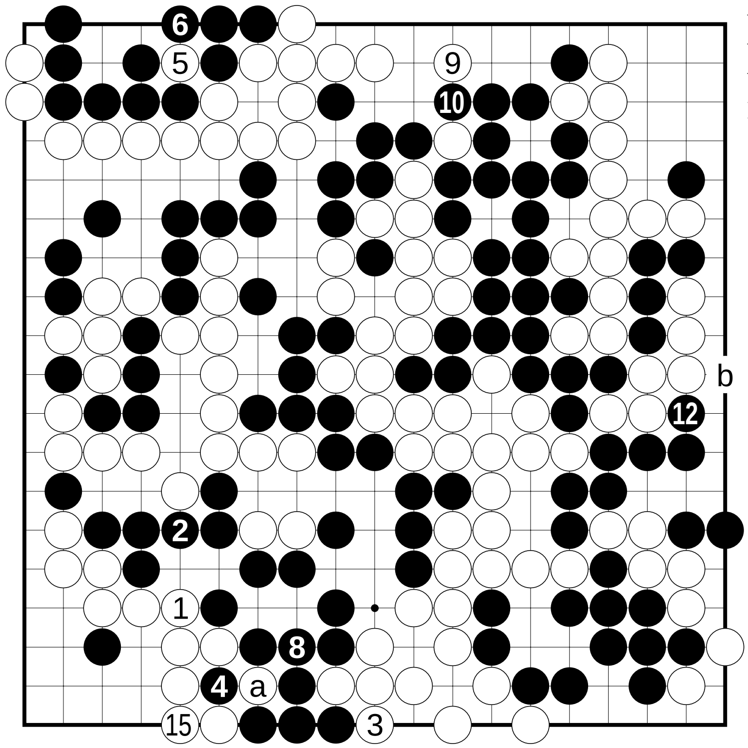


Figure 10. Territories and Temperatures.

If White plays at E04, she threatens to cut off the area just above that west of the E coordinate, which is fairly large. That threat is worth 6 points or so. If Black plays there, he threatens to cut at D03. This cut does not win the cut off stones, but it allows a sizable incursion into the corner. The cut is also hotter than 4 points. The next hottest play, worth $3\frac{1}{2}$ points, is in the East. The next hottest play, a White sente in the North, is worth $3\frac{5}{12}$. Then Black has a 3 point sente at C07 in the Southwest. Next there are 2 plays worth $1\frac{1}{12}$ points in the Northwest and Center, which are related. Finally there are some 1 point plays in the South.

Orthodox Play

Now let's take a look at what we believe to be orthodox play. There are several orthodox variations which all produce the same result: White wins by $2\frac{1}{2}$ points. White actually lost the game by $\frac{1}{2}$ point. Considering that the ambient temperature was only 4, White took a sizable loss of 3 points versus orthodox play.



W 7 = ko (a)
W 11 = 4 pt cpn
W 13 = $3\frac{1}{2}$ pt cpn
B 14 = ko (4)

Figure 11. Orthodox play (1–15).

Figure 11: First White plays the double sente at 1. After B 2 White should play W 3 in the South. This is like a double sente, because the mast is purple at the ambient temperature. W 3 threatens to take 4 Black stones by a second play at 8. (Stones which may be taken at the opponent's next play are said to be in atari.) Black does not submit meekly, but takes the ko with B 4, threatening to invade White's corner. This threat is also hotter than the ambient temperature. However, W 5 threatens to kill Black's group in the Northwest corner. Since White would gain a sizable territory, Black replies. Then W 7 takes the ko and forces Black to play B 8, saving his stones by connecting them to living neighbors. Black has a ko threat but nothing that would be bigger than White's taking these 4 stones, because that capture would put Black's whole group in the Southwest and Center in jeopardy. If White waits to play at 3, Black can take the ko first. Then, even though White can still force Black to connect at 8, it costs an extra ko threat to do so.

Next, White takes her sente in the North with W 9. This play has a nominal temperature of less than 4, but it raises the local temperature to around $4\frac{1}{3}$, so Black replies. If White omits this play Black has the possibility of making a gain through unorthodox play. White plays sente while she still can. In general, with combinatorial games, it is a good idea to play sente early. In go, it is usually advisable to wait, because a sente may be used as a ko threat.

After B 10, the mast value of the game is $\frac{1}{4}$. Since the temperature is 4, with a perfectly dense environment the final result with orthodox play would be 2 points better for White, $-1\frac{3}{4}$. The sparseness of the environment happens to favor White in this case, by $\frac{3}{4}$ point. W 11 takes the 4 point coupon.

Now the mast value is $-3\frac{3}{4}$. Black can make a $3\frac{1}{2}$ point play in the East or take the $3\frac{1}{2}$ point coupon. These plays have the same temperature.[1] In go terms, they are miai. Whichever one Black chooses, White gets the other. Usually it is a matter of indifference which one is chosen, and that is the case here. In this variation, Black plays the board play at 12 and White takes the coupon. (Black has an alternative local play at *b*, also worth $3\frac{1}{2}$ points. The final result is the same if Black plays there, as well.)

Next, B 14 takes the ko in the Southwest at 4. This ko has a low temperature, only $1\frac{2}{3}$. Despite the fact that White is komaster, she meekly replies, because Black's threat to invade the corner is so large, over 4 points, that the mast is blue up to that temperature. Even if White fights the ko, its temperature is so low that she cannot afford to win it yet. Black does not need to play a ko threat, but can simply continue with his normal play, and take the ko back on his next turn. Black can continue to force White to waste ko threats in this fashion. Rather than do so, she connects at 15.

This kind of ko, in which one player takes the ko, forcing the opponent to connect, and then later the other player takes the ko back, forcing the first player to connect, is not uncommon. At first the mast is purple, but after Black connects at 8, the resulting ko carries no further threat for White, only one for Black, and its mast is blue.

Figure 12: At this point the mast value is still $-3\frac{3}{4}$, and the ambient temperature has dropped to 3. In a perfect environment with orthodox play the result would be $-2\frac{1}{4}$. The sparseness of the environment between temperatures 3 and 4 has benefited White by $\frac{1}{2}$ point.

B 16 is a 3 point Black sente. If Black cuts off White's Southwest corner, White will need to protect it from an inside attack. After White's response, Black takes the 3 point coupon. If Black had taken the coupon without playing the sente, White could have made a reverse sente play at 16 and gotten the last 3 point move. In the sparse environment of a real game, it is normally best to get the last play before the temperature drops.

The mast value is now $-\frac{3}{4}$. The local temperature in the North is $2\frac{2}{3}$. After W 19 Black responds with B 20. W 19 is not a prototypical sente, because the mast is black. The temperature of B 20 is the same as that of W 19, so White cannot play sente early and raise the local temperature to force Black's reply.

[1]By the more common traditional form of go evaluation, these plays are 7 point gote. The less common form, called miai valuation, corresponds to temperature. Whenever I refer to the value of a play, I am using the miai value.

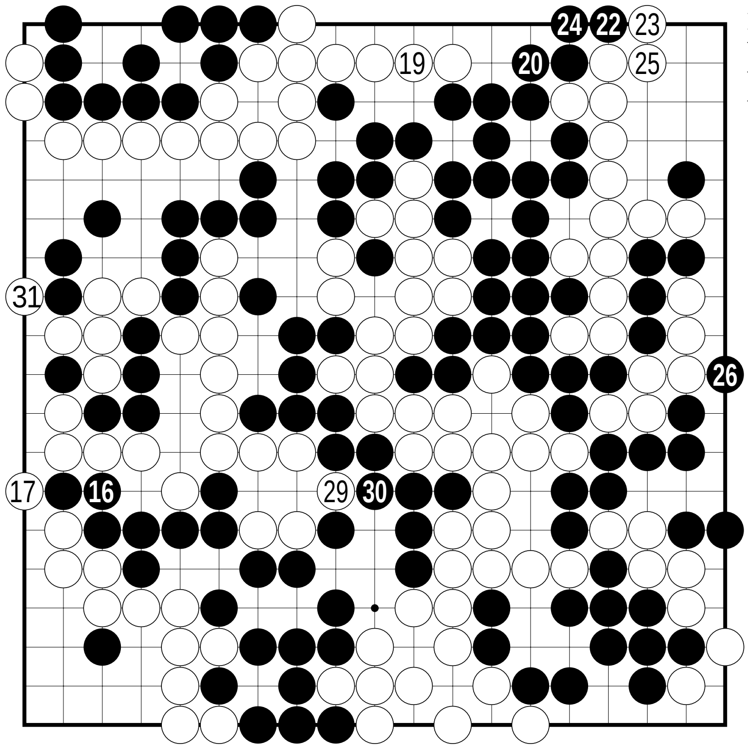


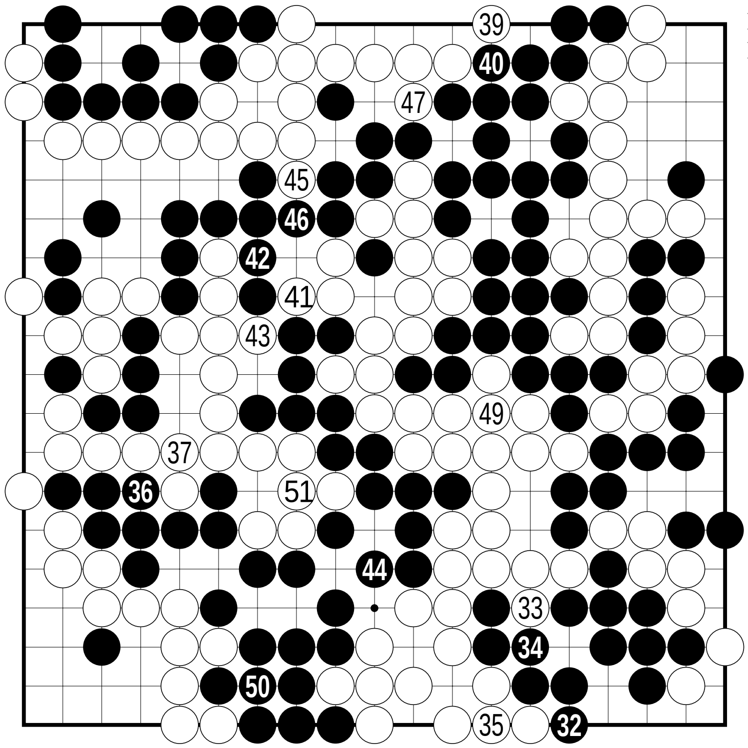
Figure 12. Orthodox play (16–31).

Next White takes the $2\frac{1}{2}$ point coupon. Now B 22 initiates a sente sequence through W 25, with temperature $2\frac{1}{3}$. Black threatens a large invasion of White's corner.

The mast value is $-3\frac{1}{4}$, and the ambient temperature is 2. Again we have a miai between the 2 point coupon and a play at 26 in the East. And again it does not seem to matter which alternative Black chooses. Here Black takes the board play. After White takes the 2 point coupon Black takes the $1\frac{1}{2}$ point coupon. The mast value is now $-1\frac{3}{4}$.

White has a clever play at W 29 in the center, with a temperature of $1\frac{1}{12}$. Black must respond at 30 or lose his group in the South. W 31 now completes the play. If White plays first at 31, the temperature in the center rises to $1\frac{1}{6}$ (!), and Black gets to make the hotter play at 29. These plays are related by White's threat to cut at 41 in Figure 13.

Figure 13: The mast value is now $-2\frac{5}{6}$, and the ambient temperature is 1. B 32 threatens an iterated ko, with 3 steps between winning and losing, raising the local temperature to about $1\frac{2}{5}$. Before replying at 35, White interjects a 1 point sente at W 33, which threatens Black's group in the corner. B 36 is another 1 point sente, and then B 38 takes the 1 point coupon. Now W 39 takes the $\frac{5}{6}$ point play in the North. The mast value is now $-2\frac{2}{3}$, and the ambient



B 38 = 1 pt cpn
B 48 = $\frac{1}{2}$ pt cpn

Figure 13. Orthodox play (32–51) — one point and below.

temperature is $\frac{1}{2}$. In addition to the $\frac{1}{2}$ point coupon there are 2 board plays worth $\frac{1}{2}$, one in the North and one in the North Central section. Again, it does not matter who gets these plays. B 40 takes the play in the North, and then White takes the play in the North Central. This play is a gote, but the mast is red up to a very high temperature, because White's cut with W 41–43 carries a huge threat (Figure 14).

If Black omits B 4, White can play W 5. W 7 puts Black's stones in the center in atari and threatens to kill Black's large group. B 8 takes White's stone at *a* and puts the 2 neighboring White stones in atari. W 9 saves them and threatens to cut off Black's stones on the left with a play at 10. B 10 saves them, but now W 11 takes the ko at *a*, renewing the threat against Black's whole group. If Black connects at *b*, White takes at *c*, killing the group. Black cannot afford to fight this ko, and avoids it with B 44.

Such a ko is called a hanami ko, or flower-viewing ko, in reference to the Cherry Blossom festivals held each spring in Japan, when people gather to watch cherry blossoms bloom and later fall in clouds.

Black's group is originally alive, and if Black loses the ko he loses a huge amount by comparison. In contrast, if White loses the ko she loses only a couple of points. Black needs huge ko threats to fight the ko, while White needs only

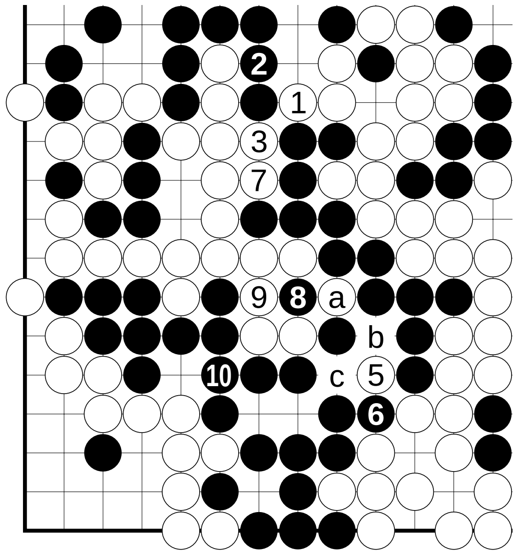


Figure 14. Hanami ko.

small ones. It is difficult for Black to be the komaster of White's hanami ko. We are just beginning the theoretical study of hanami kos, and of other kos whose structure affects the question of which player is komaster for that ko.

After B 48 in Figure 13 the mast value is $-2\frac{1}{6}$. There are 3 ko positions with temperature $\frac{1}{3}$ on the board. White gets two of them with W 49 and W 51 and Black gets one with B 50. (W 51 does not look like a ko, but if Black plays first he makes one with the same sequence as B 8, W 9, B 10 in Figure 14.) The result is a $2\frac{1}{2}$ point win for White.

Actual Play

Figure 15: White takes the 4 point coupon, but that is a costly mistake. After B 2 – W 3, White has lost 1 point on the South side by comparison with Figure 13. Then after B 4 – W 5, White has lost an additional $\frac{5}{8}$ point in the Southwest versus playing the double sente herself. Since Black plays B 2 and B 4 with sente, while White could have played with sente, these are differences between vertical thermographic lines, which means that Black's gains have come at no cost. "Play double sente early."

After W 5 the mast value is $-2\frac{1}{8}$, and the ambient temperature has dropped to $3\frac{1}{2}$. But the top coupon and the East side are miai, and Black should play his reverse sente sequence in the North (B 14 – W 11, B a), which is worth $3\frac{5}{12}$. Instead, B 6 is worth only $2\frac{5}{8}$ points. B 6 does aim at B 10, which gives Black several ko threats against White's Southwest corner, but with otherwise orthodox play, these threats do not seem to gain anything, nor did Black gain from them in the game.

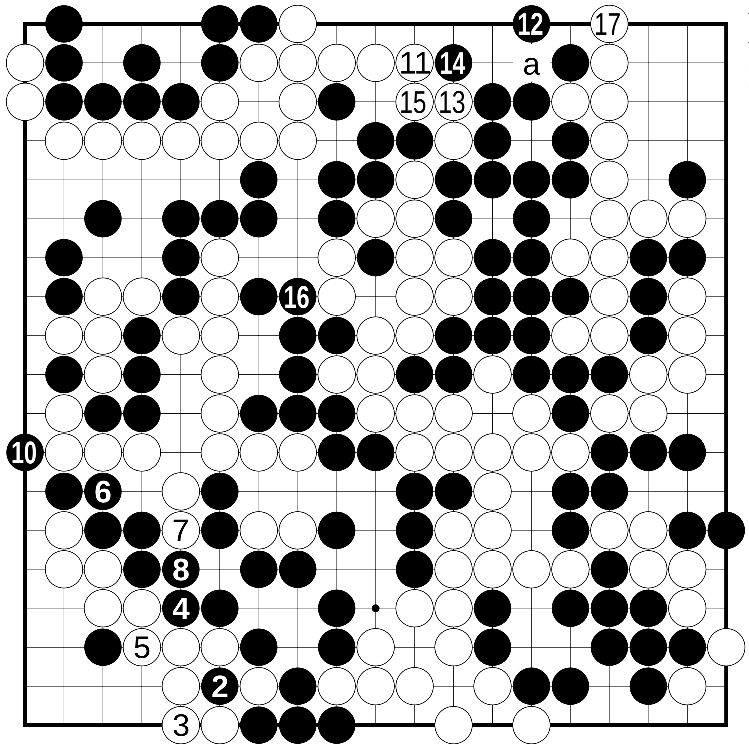


Figure 15. Actual play (1–18).

After W 9 takes the $3\frac{1}{2}$ point coupon, Black should take the $3\frac{1}{2}$ point play in the East. B 10 gives Black several ko threats in the Southwest corner to make Black komaster in general, but B 10 is worth only $2\frac{1}{4}$ points, plus $\frac{1}{2}$ the value of the threats. That value appears to be $\frac{1}{6}$ point, increasing the value of B 10 to $2\frac{1}{3}$ points.

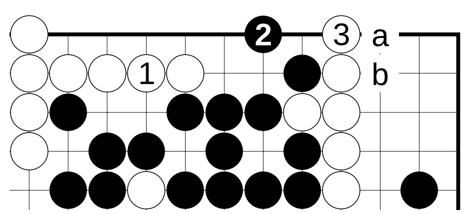
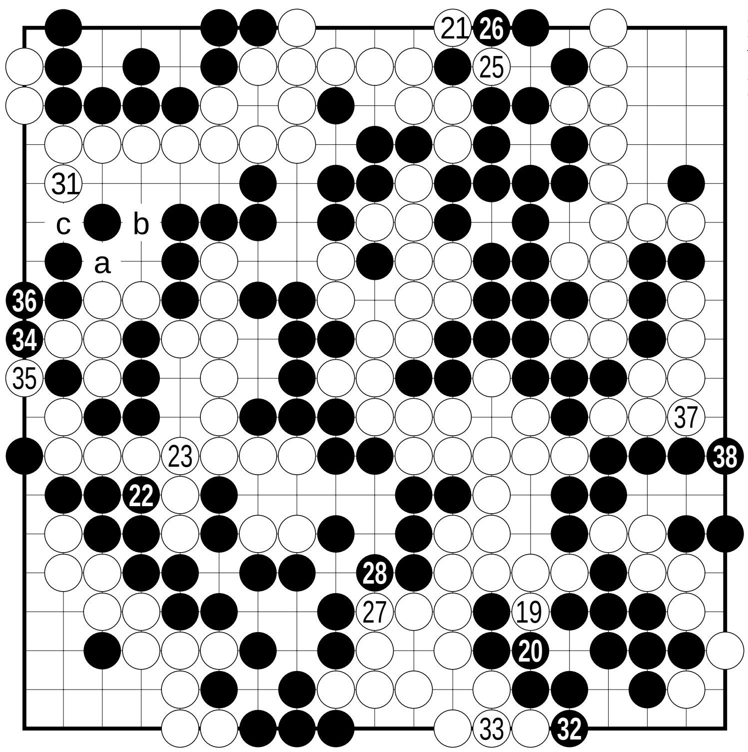


Figure 16.

After B 10 White should play her sente in the North, reaching the position in Figure 16. When White is komaster Black replies to W 1 at 1 point below 2, but since Black is komaster, B 2 is orthodox. If White omits W 3, Black can play there, and then if W a, B b makes a large ko. The mast value when Black is komaster is $\frac{1}{6}$ point greater than when White is komaster. There are some small kos and potential small kos on the board, but it appears that Black can realize no gain in their mast values from being komaster.

Returning to Figure 15, the fact that Black is now komaster after B 10 makes a play at 14 in the North double sente. White should play W 14 – B 13 in the North and then make the $3\frac{1}{2}$ point play in the East. Both players overlook the play in the East.

B 12 is sente, but may not be best. Black could try the aggressive B 17 – W a, B 12, making a large ko. Jiang 9-dan judged not to make that play in the game. The ko fight is difficult and has not yet been analyzed. Assuming that Black should not play at 17, his orthodox play is B a, a gote worth $3\frac{1}{6}$ points. In that case B 12 gives up $\frac{1}{3}$ point versus the mast value.



B 24 $= 2\frac{1}{2}$ pt cpn
W 29 $= 2$ pt cpn
B 30 $= 1\frac{1}{2}$ pt cpn

Figure 17. Actual play (19–38).

Figure 17: The mast value is now $-\frac{1}{3}$. Both players continue to ignore the East side, the hottest area on the board. W 21 is worth $2\frac{1}{2}$ points. W 25 is a $2\frac{1}{3}$ point sente. The exchange, W 27 – B 28, is unorthodox. It eliminates White options in some variations (See Figure 14 for an example.), and loses $\frac{1}{6}$ point versus the mast value, which is now $-\frac{1}{6}$.

After B 30 the mast value is $-\frac{2}{3}$. W 31 loses $\frac{1}{2}$ point versus the mast value. Before W 31, B 34 is worth $1\frac{1}{2}$ points, but afterwards it is worth 2 points.

If Black plays elsewhere after W 31, White can play at 35, threatening W a – B b, W c, which wins the 2 Black stones on the side. B 32 is a 1 point sente, but W 33 is worth less than a play in either the East or West. After B 36 the mast value is $-\frac{1}{6}$. White plays W 37 in the East, but loses 2 points by comparison with correct play.

Figure 18: In post-game discussion Rui Nai Wei spotted the correct play. After the skillful play of W 1, the local temperature drops to 3. But since the ambient temperature is less than that, Black replies. W 3 is another skillful play, threatening the Black stones on the side. Through B 8 the sequence is sente for White. White has the same territory as in the game, but Black has 2 points less. W 37 was White's last chance to win the game. If White had played as in Figure 18 the mast value would have been $-\frac{2}{3}$, but instead it is $1\frac{1}{3}$.

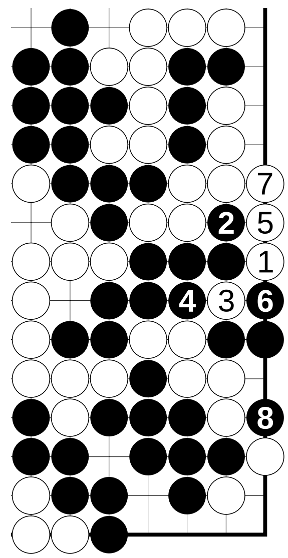
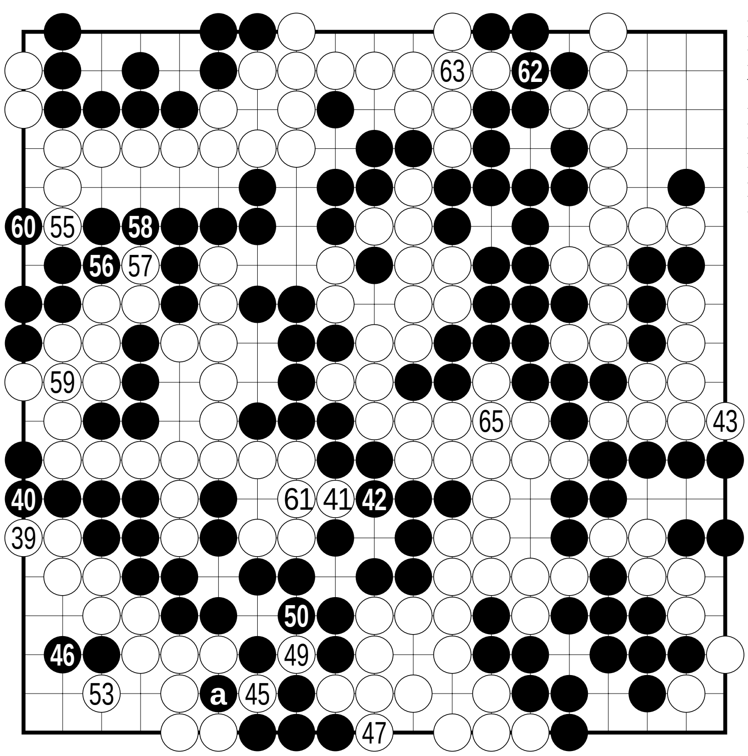


Figure 18.



B 44 = 1 pt cpn
B 48 = ko (a)
W 51 = ko (45)
B 52 = fills (49)
B 54 = $\frac{1}{2}$ pt cpn
B 64 = ko (a)
B 66 = fills (45)

Figure 19. Actual play (39–66).

Figure 19: There are some slight deviations from orthodox play at the end. W 39 should take the ko at 45 in the South. The ko has a temperature of $1\frac{1}{3}$. W 39 is a 1 point sente, and a possible threat for this ko. B 44 misses a chance when he takes the 1 point coupon instead of filling at 45. If Black fills at 45, White takes the 1 point coupon, and Black can play at 47 with sente. Then Black takes the $\frac{1}{2}$ point card. At that point there would be only 2 kos on the board. White would have to win the ko fight to get the same result as in the game. If Black won the ko fight, he would score 1 point more.

B 46 is an oversight. White does not have to answer this apparent ko threat right away. White plays atari at 47, and then when Black takes the ko back, White plays at 49. If Black fills the ko White can capture Black's 7 stones, so Black must take at 50. Then White takes the ko back and Black has to connect at 49. Since White is komaster, Black should simply play B 46 at 47. That gets the last 1 point play, but then White gets the $\frac{1}{2}$ point card instead of Black, so there is no difference in the score.

Summary

When White took the 4 point coupon, she overlooked the double sente in the South and Southwest, allowing Black to make the game close. Then Black overestimated the value of his plays in the West, allowing White to play first in the North. White chose the wrong spot in the North, ending up with gote. Then for a long time both players overlooked White's skillful option in the East and underestimated its temperature. White lost several chances to win, while Black failed to secure the game. Finally, White made the wrong play in the East, losing 2 points and the game.

Traditional evaluation of go plays produces the temperatures and mean values of classical thermography. Thermographs, however, yield additional information about orthodox play in different environments. The concept of komaster allows us to find the mast values and temperatures of all positions involving single kos. Defining thermographs in terms of play in a universal enriched environment extends thermography to positions with multiple kos. Research continues into positions in which neither player is komaster and into what conditions allow a player to be komaster.

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