

# Signal Processing in Optical Fibers

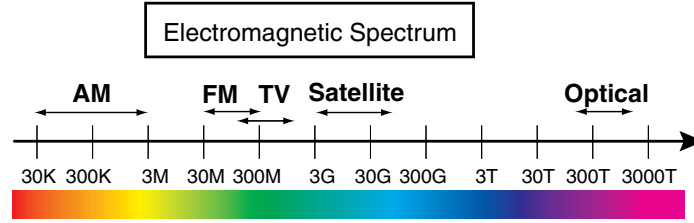
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**ABSTRACT.** This paper addresses some of the fundamental problems which have to be solved in order for optical networks to utilize the full bandwidth of optical fibers. It discusses some of the premises for signal processing in optical fibers. It gives a short historical comparison between the development of transmission techniques for radio and microwaves to that of optical fibers. There is also a discussion of bandwidth with a particular emphasis on what physical interactions limit the speed in optical fibers. Finally, there is a section on line codes and some recent developments in optical encoding of wavelets.

## 1. Introduction

When Claude Shannon developed the mathematical theory of communication [1] he knew nothing about lasers and optical fibers. What he was mostly concerned with were communication channels using radio- and microwaves. Inherently, these channels have a narrower bandwidth than do optical fibers because of the lower carrier frequency (longer wavelength). More serious than this theoretical limitation are the practical bandwidth limitations imposed by weather and other environmental hazards. In contrast, optical fibers are a marvellously stable and predictable medium for transporting information and the influence of noise from the fiber itself can to a large degree be neglected. So, until recently there was no real need for any advanced signal processing in optical fiber communications systems. This has all changed over the last few years with the development of the internet.

Optical fiber communication became an economic reality in the early 1970s when absorption of less than 20 dB/km was achieved in optical fibers and lifetimes of more than 1 million hours for semiconductor lasers were accomplished. Both of these breakthroughs in material science were related to minimizing the number of defects in the materials used. For optical fiber glass, it is absolutely necessary to have fewer than 1 parts per billion (ppb) of any defect or transition metal in the glass in order to obtain necessary performance.



**Figure 1.** Electromagnetic spectrum of importance for communication. Frequencies are given in Hertz.

For the last thirty years, optical fibers have in many ways been a system engineer's dream. They have had, literally, an infinite bandwidth and as mentioned above, a stable and reproducible noise floor. So no wonder it's been sufficient to use intensity pulse-code modulation, also known as *on-off keying* (OOK), for transmitting information in optical fibers.

The bit-rate distance product for optical fibers has grown exponentially over the last 30 years. (Using bandwidth times length as a measurement makes sense, since any medium can transport a huge bandwidth if the distance is short enough.) For this growth to occur, several fundamental and technical problems had to be overcome. In this paper we will limit ourselves to three fundamental processes; absorption, dispersion and nonlinear optical interactions. Historically, absorption and dispersion were the first physical limitations that had to be addressed. As the bit-rate increase shows, great progress has been made in reducing the effects of absorption and dispersion on the effective bandwidth. As a consequence, nonlinear effects have emerged as a significant obstacle for using the full bandwidth potential of optical fibers.

These three processes are undoubtedly the most researched physical processes in optical glass fibers, which is one reason for discussing them. Another reason, of great importance to mathematicians, is that recent developments in time/frequency and wavelet analysis have introduced novel line coding schemes which seem to be able to drastically reduce the impact from many of the deleterious physical processes occurring in optical fiber communications.

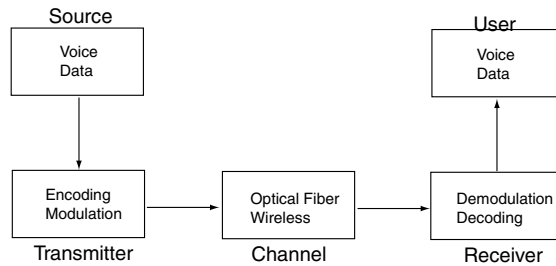
## 2. Signal Processing in Optical Fibers

The spectrum of electromagnetic waves of interest for different kinds of communication is shown in Figure 1.

A typical communications system for using these waves to convey information is shown in Figure 2. This system assumes digitized information but is otherwise completely transparent to any type of physical medium used for the channel.

Any electromagnetic wave is *completely* characterized by its *amplitude* and *phase*:

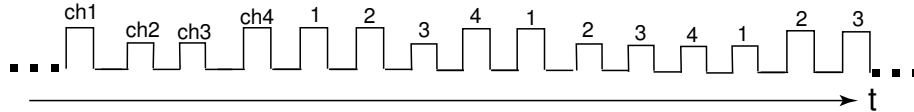
$$\mathbf{E}(\mathbf{r}, t) = \mathbf{A}(\mathbf{r}, t) \exp(\phi(\mathbf{r}, t))$$



**Figure 2.** Typical block diagram of a digital communications system.

where  $A$  is the amplitude and  $\phi(\mathbf{r}, t)$  is the phase. So, amplitude and phase are the two physical properties that we can vary in order to send information in the form of a wave. The variations can be in either analog or digital form. Note that even today, in our digitally swamped society, analog transmission is still used in some cases. One example is cable-TV (CATV), where the large S/N ratio (because of the short distances involved) provides a faithful transmission of the analog signal. The advantage in using analog transmission is that it takes up less bandwidth than a digital transmission with the same information content.

The first optical fiber systems in the 1970s used *time-division multiplexing* (TDM), each individual channel was multiplexed onto a trunk line using protocols called T1-T5, where T1-T5 refers to particular bit rates; see Figure 3.



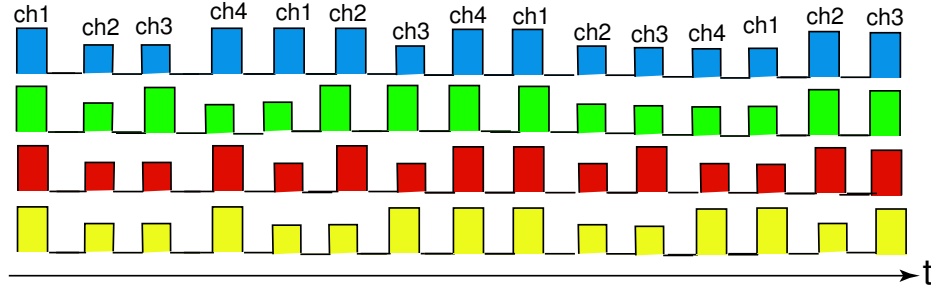
**Figure 3.** Time-division multiplexing.

Each individual channel was in turn encoded with the users' digital information.

TDM is still the most common scheme used for sending information down an optical fiber. Today, we are using a multiplexing protocol called *SONET* which uses the acronyms OC48, OC96, etc., where OC48 corresponds to a bit rate of 565 Mbits/sec and each doubling of the OC-number corresponds to a doubling of the bit rate. The increase in speed has been made possible by the dramatic improvement of electronic circuits and the shift from multi-mode fibers to dispersion-compensated single-mode fibers. Several large national labs are testing, in the laboratory, time-multiplexed systems up to 100 Gbits/sec, commercially most systems are still  $\lesssim 2.5$  Gbits/sec.

As industry is preparing for an ever growing demand of bandwidth it is clear that electronics cannot keep up with the optical bandwidth, which is estimated to be 30 Tbits/sec for optical fibers. Because of this *wavelength-division multiplexing* (WDM) has attracted a lot of attention. In a TDM system each bit is an

optical pulse, for WDM system each bit can either be a pulse or a continuous wave (CW). WDM systems rely on the fact that light of different wavelengths do not interfere with each other (in the linear regime); see Figure 4.



**Figure 4.** Wavelength-division multiplexing.

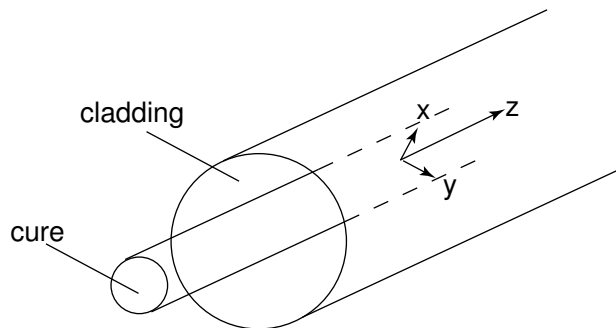
Signal processing in optical fibers has, historically, been separated into two distinct areas: pulse propagation and signal processing. To introduce these areas we will keep with tradition and describe them separately, however, please bear in mind that *the area in which mathematicians may play the most important role in future signal processing is to understand the physical limitations imposed by basic processes that are part of the pulse propagation and invent new signal processing schemes which oppose these deleterious effects.*

A pulse propagating in an optical fiber can be expressed by

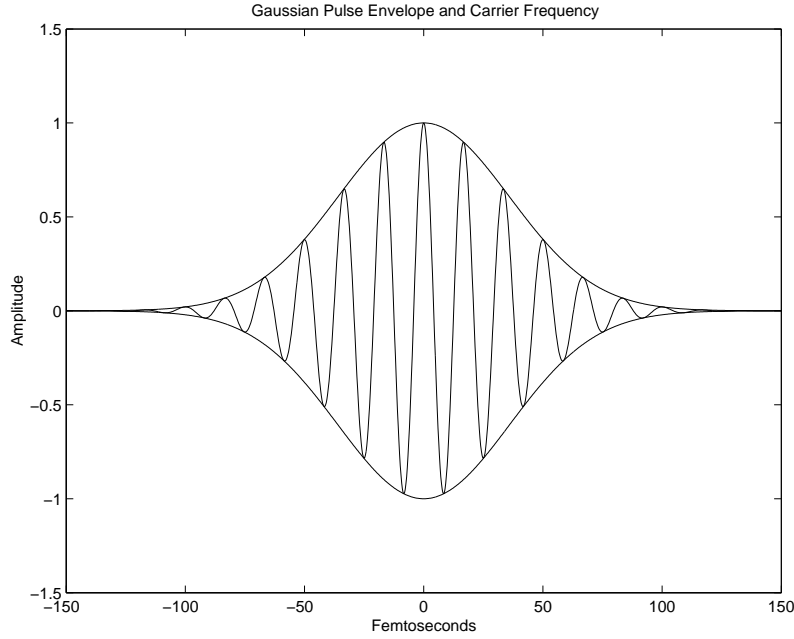
$$\mathbf{E}(x, y, z, t) = \hat{x}E_x(x, y, z, t) + \hat{y}E_y(x, y, z, t) + \hat{z}E_z(x, y, z, t),$$

where  $z$  is the direction of propagation and  $x, y$  are in the transversal plane; see Figure 5. The geometry shown in Figure 5 is for a single-mode fiber.

In such a fiber, the light has been confined to such a small region that only one type of spatial beam (mode) can propagate over a long distance. Even though this mode's spatial dependence is described by a Bessel function it is for most purposes sufficient to spatially model it as a *plane wave*. Therefore, the signal



**Figure 5.** Optical fiber geometry.



**Figure 6.** Gaussian pulse with the carrier frequency illustrated. The optical equivalent pulse has a  $10^{15}$  times higher carrier frequency than shown here.

pulse representing a *bit* can mathematically be written as

$$\mathbf{E}(z, t) = \hat{\mathbf{x}}E_x(z, t),$$

where the subscript  $x$  is often ignored, tacitly assuming that we only have to deal with one (arbitrary) scalar component of the full vectorial electromagnetic field.

In a glass optical fiber the signal has to obey the following wave equation

$$\nabla^2 E(z, t) = \frac{1}{c^2} \frac{\partial^2 E(z, t)}{\partial t^2},$$

where  $c$  is the speed of light.

A solution to this equation can be written as

$$E(z, t) = p(z, t)e^{i(kz - \omega_0 t)},$$

where  $p(z, t)$  is the temporal shape of the pulse (bit) representing a 1 or a 0. For a Gaussian pulse at  $z = 0$ ,

$$p(0, t) = Ae^{-t^2/(2T^2)},$$

and the electromagnetic field at  $z = 0$

$$E(0, t) = Ae^{-t^2/(2T^2)}e^{-i\omega_0 t}, \quad (2-1)$$

where  $\omega_0$  is the carrier frequency. This pulse is depicted in Figure 6.

To describe how this pulse changes as it propagates along the fiber we start by taking the Fourier transform (FT) of the field in equation (2-1):

$$\tilde{E}(0, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(0, t) e^{i\omega t} dt. \quad (2-2)$$

The reason for moving to the frequency domain is because in this domain the actual propagation step consists of “simply” *multiplying* the field with the phase factor  $e^{ikz}$ , where  $k$  is the *wavenumber*. To find out the temporal pulse shape after a distance  $z$  we then transform back to the time domain; that is,

$$E(z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{E}(0, \omega) e^{-i\omega t + ikz} d\omega.$$

So the principle is quite easy; nevertheless in reality it becomes more complicated because the phase factor,  $e^{ikz}$ , is different for different frequencies  $\omega$  since  $k = k(\omega)$ . The wavenumber  $k$  is related to the refractive index via

$$k(\omega) = \frac{\omega n(\omega)}{c}.$$

The refractive index can be described for most materials, at optical frequencies, using the Lorentz formula

$$n(\omega) = \sqrt{n_0^2 + \sum_j \frac{b_j^2}{\omega^2 - \omega_{0j}^2 + i2\delta_j\omega}}, \quad (2-3)$$

where the different  $j$ 's refer to different resonances in the media,  $b$  is the strength of the resonance and  $\delta$  is the damping term ( $\approx$  the width of the resonance).

For picosecond pulses ( $10^{-12}$  sec) or longer the pulse spectrum is concentrated around the carrier frequency  $\omega_0$  and we may therefore Taylor expand  $k(\omega)$  around  $k(\omega_0)$ :

$$k(\omega) = \sum_{n=0}^{\infty} \frac{1}{n!} k_n(\omega_0) (\omega - \omega_0)^n,$$

where  $k_n(\omega_0) = \frac{\partial^n k}{\partial \omega^n} |_{\omega=\omega_0}$ .

Typically, it is sufficient to carry this expansion to the  $\omega^2$ -term. Using this expansion we can now rewrite (2-2) as

$$E(z, t) = \frac{e^{i(k_0 z - \omega_0 t)}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{E}(0, \omega) e^{i[k(\omega_0) + k_1(\omega_0)(\omega - \omega_0) + k_2(\omega_0)(\omega - \omega_0)^2]} e^{-i\omega t} d\omega,$$

which can be further rewritten as

$$E(z, t) = p(z, t) e^{i(k(\omega_0)z - \omega_0 t)},$$

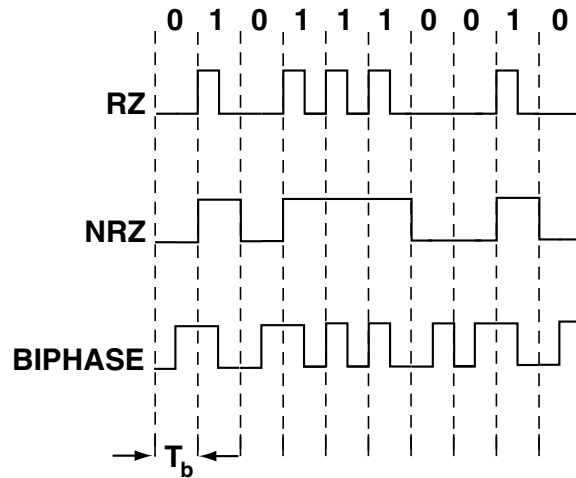
where, for a gaussian input pulse,  $p(z, t)$  is

$$p(z, t) = \frac{A}{(1 + k_2(\omega_0)z^2/T^4)^{1/4}} \exp\left(-\frac{(k_1(\omega_0)z - t)^2}{2T^2(1 + k_2(\omega_0)z^2/T^4)}\right).$$

Hence, the envelope remains Gaussian as the pulse is propagating along the optical fiber, however its width is increased and the amplitude is reduced (conservation of energy). From this type of analysis one may determine the optimum bit-rate (necessary temporal guard bands) for avoiding cross talk.

**Line coding.** In addition to using both time and wavelength multiplexing to increase the speed of optical fiber networks it is also necessary to use signal processing to maintain bit-error rates (BER) of  $\lesssim 10^{-9}$  for voice and  $\lesssim 10^{-12}$  for data. (BER is defined as the probability that the received bit differs from the transmitted bit, on average.) A ubiquitous signal processing method is *line coding* in which binary symbols are mapped onto specific waveforms; see Figure 7. In this way, pulses can be preconditioned to make them more robust to transmission impairments. Specific line codes are chosen which are adjusted differently for various physical communications media by arranging the mapping accordingly.

Line codes (three different types are shown in Figure 7) are all examples of *pulse-code modulation* or *on-off keying*. In this case it is only the amplitude which is varied; this is done by simply sending more or less light down the fiber.



**Figure 7.** Three types of line codes for optical fiber communications.

The choice of line codes depends on the specific features of the communication channel that needs to be opposed [5]. Common properties among all line codes include:

- (i) the coded spectrum goes to zero as the frequency approaches zero (DC energy cannot be transmitted).
- (ii) the clock can be recovered from the coded data stream (necessary for detection).
- (iii) they can detect errors (if not correct).

Another consideration in choosing a line code is that different coding formats will use more or less bandwidth. It is known that for a given bit-rate per bandwidth (bits/s/Hz), an ideal Nyquist channel uses the narrowest bandwidth [7]. Typically, adopting a line code will increase the needed transmission bandwidth, since redundancy is built into the system (table 1) where everything is normalized to the Nyquist bandwidth  $B$ .

Line codes	Transmission bandwidth	Bandwidth efficiency
RZ	$\pm 2B$	$\frac{1}{4}$ bit/s/Hz
NRZ	$\pm B$	$\frac{1}{2}$ bit/s/Hz
Duobinary	$\pm \frac{1}{2}B$	1 bit/s/Hz
Single Sideband	$\pm \frac{1}{2}B$	1 bit/s/Hz
M-ary ASK ( $M = 2^N$ )	$\pm B/N$	$\log_2 N$

**Table 1.** Bandwidth characteristics for different types of line codes.

Even though in the past, binary line codes were preferred to multilevel codes due to optical nonlinearities, it is now firmly established that multilevel line codes can be, spectrally, as efficient as a Nyquist channel. In particular, duobinary line coding (which uses three levels) have recently been shown to be very successful in reducing ISI due to dispersion [6].

Closely related to line coding is pulse or waveform generation. The waveform associated with a Nyquist channel is a sinc-pulse (giving rise to the “minimum” rect-shaped spectrum). The main problem with this waveform is that it requires perfect timing (no jitter) to avoid large ISI. The reason for this intolerance to timing jitter is found in the (infinitely) sharp fall-off of the spectrum. To address this problem, pulses are generated using a “raised-cosine” spectrum [1; 7] which removes the “sharp zeroes”. Unfortunately, it makes the transmission bandwidth twice as large as the Nyquist channel. Lately, it has been suggested that wavelet like pulses (local trigonometric bases) are a good choice for achieving efficient time/frequency localization [8] (see section on novel line coding schemes).



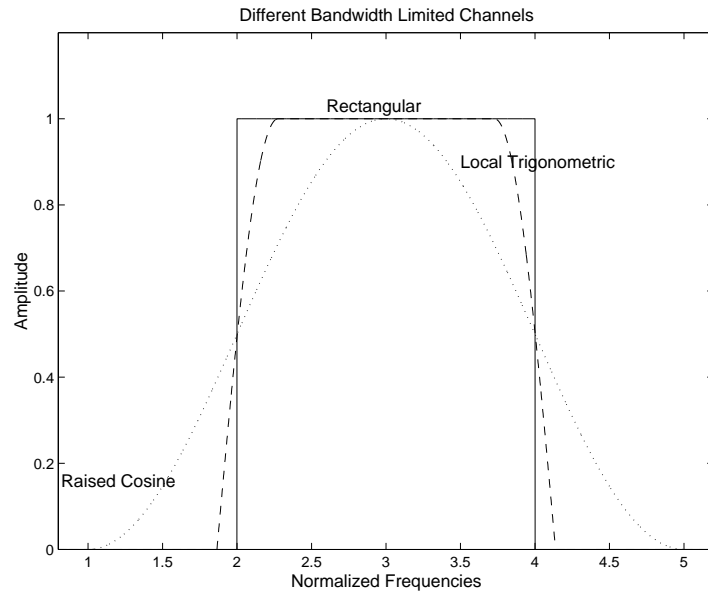


Figure 8. Examples of different bandwidth limited channels.

### 3. Physical Processes in Optical Fibers

**Absorption.** It may seem strange that the small absorption in optical fibers, which in the late 1960s was less than 20 dB/km (that is, over a distance of  $L$  km we have  $P_{in}/P_{out} \geq 10^{-20 L/10}$ ), still was not sufficient to make optical communications viable (in an economical sense).

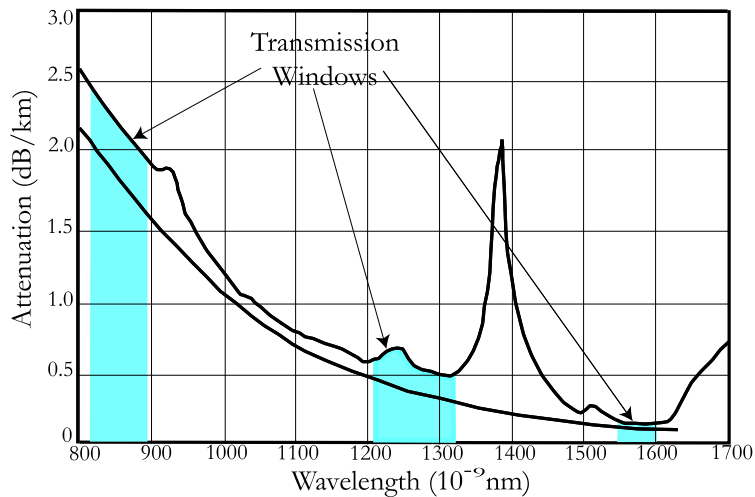
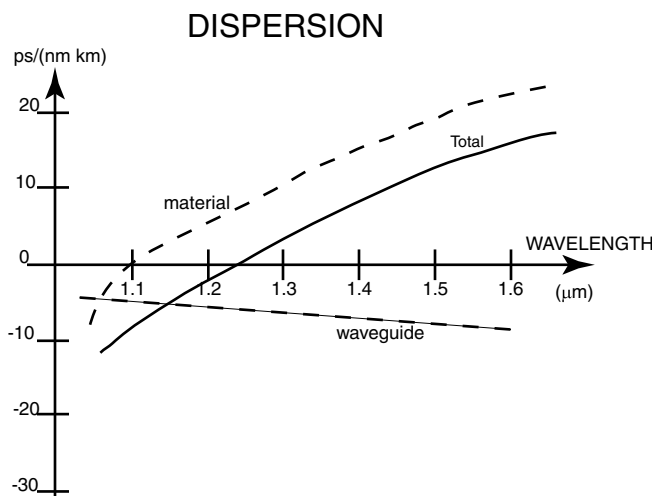


Figure 9. Absorption in optical fibers.

From 1970 to 1972 scientists managed to make fibers of even greater purity which reduced the absorption to no more than 3 dB/km at 800 nm (Figure 9). Using more or less the same type of fibers the absorption could be reduced to no more than 0.15 dB/km by going to longer wavelengths, such as 1.3  $\mu\text{m}$  and 1.55  $\mu\text{m}$ . This was possible through the invention of new semiconductor lasers using InGaAsP material. Despite this very low absorption, again, seen from an economical perspective, absorption was still the limiting factor. This changed with the invention of the erbium-doped fiber amplifier (EDFA). A short piece of fiber (only a few meters long) doped with Erbium and spliced to the system's fiber could now amplify the propagating pulses (bits) to "arbitrary" levels, thereby removing absorption as a system's physical limitation.

**Dispersion.** The next attribute which required attention was dispersion. Signal dispersion (mathematically described via the  $\omega^2$ -term in equation (2-3)) a source of *intersymbol interference* (ISI) in which consecutive pulses blend into each other. Again, it turns out that optical glass fibers have inherently outstanding dispersion properties. As a matter of fact, any particular fiber has a characteristic wavelength for which the dispersion is zero. This is typically between 1.27–1.39  $\mu\text{m}$ . However, as is the case for absorption, long distance transmission can cause dispersion.



**Figure 10.** Dispersion in optical fibers.

There are two major contributors to dispersion: material and waveguide structure. (A waveguide is a device, such as a duct, coaxial cable, or glass fiber, designed to confine and direct the propagation of electromagnetic waves. In optical fibers the confinement is achieved by having a region with a larger refractive index.)

Material dispersion, which comes from electronic transitions in the solid, is determined as soon as the chemical constituents of the glass have been fixed. Waveguide dispersion is a function of the geometry of the core or, more precisely, how the refractive index in the core and cladding vary in space. This is important because it means that fiber manufacturers have a fair amount of flexibility in modifying the total dispersion of the fiber. Today, there is a plethora of fibers with different dispersion characteristics. However, it is not yet possible to reliably manufacture fibers with zero dispersion for all wavelengths between, say, 1400–1550 nm. Thus, even though the dispersion can be made as small as 2–4 ps/nm·km over this wavelength region, we still need to worry about dispersion for long-distance networks. Two methods used to combat dispersion are *fiber Bragg gratings* and *line coding* and combinations of the two. We now describe each of these in turn.

Optical fiber Bragg gratings are short pieces of fiber ( $\lesssim 10$  cm) in which the refractive index in the core has been altered to modify the dispersion properties. Mathematically, the fiber Bragg grating is a *filter* whose properties can be described using a transfer function. Similarly, we can describe pulse propagation over a distance  $z$  in an optical fiber using a transfer function. If linear effects up to the quadratic frequency term (group-velocity dispersion) in the Taylor expansion of  $k$  in (2-3) are included, the transfer function is

$$H(\omega) = H_0 \underbrace{\exp(-\alpha z/2)}_{\text{amplitude}} \underbrace{\exp(-jknz) \exp(-jD\omega^2 z/(4\pi))}_{\text{phase}},$$

where  $k$  is the propagation constant,  $\omega$  is the angular frequency,  $n$  is the refractive index,  $\alpha$  is the absorption coefficient, and  $D$  is the dispersion coefficient. So for a known distance  $L$ , an EDFA can be used to amplify the amplitude and the Bragg grating (with a transfer function  $H^{-1}$ ) can mostly remove the influence of the dispersion (the dispersion is primarily modeled by the  $\exp(-jD\omega^2 z)$  term in the phase). The severest limitation to this scheme are *nonlinear effects* which can change both absorption and dispersion in a dramatic fashion.

**Nonlinear optics.** A description of electromagnetic waves interacting with matter ends up dealing with the electric and magnetic susceptibilities  $\chi_e$  and  $\chi_m$ , respectively. In this short exposé of nonlinear optics we will limit ourselves to non-magnetic materials, such as the glass that optical fibers are made of. The more common (in a linear description) dielectric constant,  $\varepsilon_r$ , is related to the susceptibility  $\chi_e^{(1)}$  via  $\varepsilon_r = 1 + \chi_e^{(1)}$ . The susceptibility, in turn, has complete information about how the material interacts with electromagnetic waves. The wave equation for an arbitrary dielectric medium can be written as

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) = \frac{\partial^2 \mathbf{P}(\mathbf{r}, t)}{\partial t^2},$$

where  $\mathbf{E}(\mathbf{r}, t)$  is the electric field and  $\mathbf{P}(\mathbf{r}, t)$  is the induced polarization field (an identical wave equation can be written for the magnetic field  $\mathbf{H}(\mathbf{r}, t)$ ). All

linear interactions can be described by assuming that the polarization field and the electric field are related via the constitutive relation,

$$\mathbf{P}(\mathbf{r}, \omega_s) = \varepsilon_0 \chi_e^{(1)}(\omega_s; -\omega_s) \mathbf{E}(\mathbf{r}, \omega_s).$$

Unfortunately, most real phenomena are not linear and this holds for electromagnetic interactions with matter. For waves whose wavelengths do not coincide with specific resonant transitions in the material, we can describe the polarization using a Taylor series expansion of the field amplitudes,

$$\begin{aligned} \mathbf{P}(\mathbf{r}, \omega_s) = \varepsilon_0 \cdot & (\chi_e^{(1)}(\omega_s; -\omega_s) \mathbf{E}(\mathbf{r}, \omega_s) + \chi_e^{(2)}(\omega_s; \omega_1, \omega_2) \mathbf{E}_1(\mathbf{r}, \omega_1) \cdot \mathbf{E}_2(\mathbf{r}, \omega_2) \\ & + \chi_e^{(3)}(\omega_s; \omega_1, \omega_2, \omega_3) \mathbf{E}_1(\mathbf{r}, \omega_1) \cdot \mathbf{E}_2(\mathbf{r}, \omega_2) \cdot \mathbf{E}_3(\mathbf{r}, \omega_3) + \dots), \end{aligned}$$

where  $\omega_s$  is the frequency of the generated polarization,  $\chi^{(n)}$  is the electric susceptibility of first, second and third order for  $n = 1, 2, 3$ , respectively,  $\mathbf{E}(\mathbf{r}, \omega_n)$  are the electric field amplitudes at different carrier frequencies,  $\omega_1, \omega_2, \omega_3$ , etc.

The susceptibilities have a general form given by

$$\chi_{i,j,k,\dots}^{(n)}(\omega; \omega_1, \omega_2, \dots) = \sum \frac{\langle g|\mathbf{r}|f\rangle}{(\omega_0^2 - \omega^2 - j2\omega\gamma)} = \frac{\text{spatial dispersion}}{\text{frequency dispersion}}. \quad (3-1)$$

The subscripts  $i, j, k, \dots$ , are connected with the structural symmetry of the material (spatial dispersion) and the particular polarization of the electromagnetic waves. The denominator describes the frequency dispersion with  $\omega$  being the frequency of an electromagnetic wave,  $\omega_0$  being a resonant frequency in the material and  $\gamma$  being the width of the resonance. The summation is over all the possible states that can occur in the material while it is interacting with the electromagnetic waves. As can be seen from (3-1), the electronic susceptibilities are complex quantities. It is common to separate the susceptibilities into a real and imaginary part. For the third-order nonlinear susceptibility this could look like

$$\chi_{ijkl}^{(3)}(\omega_s; \omega_1, \omega_2, \omega_3) = \chi_{\text{Real}}^{(3)} + i \cdot \chi_{\text{Imaginary}}^{(3)}.$$

In general, the real part describes light-matter interactions that leave the material in the original energy state, while the imaginary part describes interactions that transfer energy between the electromagnetic wave and the material in such a way as to leave the material in a different energy state than the original state. Processes described by the real part are commonly referred to as parametric processes and two examples of such a process are four-photon mixing and self-phase modulation. It is interesting to note that nonlinear processes controlled by the real part require phase matching while processes due to the imaginary part do not. Examples of processes described by the imaginary part are Raman and Brillouin scattering, and two-photon absorption.

For Raman and Brillouin scattering one also needs to distinguish between spontaneous and stimulated processes. In simple terms, spontaneous Raman and Brillouin scattering are due to fluctuations in one or more optical properties

caused by the internal energy of the material. Stimulated scattering is driven by the light field itself, actively increasing the internal fluctuations of the material.

Nonlinear susceptibilities of importance for tele- and data communication are all made up of electric-dipole transitions. When these transitions are between real energy levels of the material we talk about resonant processes. In general, resonant processes are strong and slow; strong because the susceptibility gets large at resonances and slow because the electrons have to be physically relocated. The nonlinear susceptibilities of importance for us are all due to non-resonant processes. These nonlinearities are distinguished by their small susceptibilities but very fast response. This is in part due to the electrons only making virtual transitions. A virtual energy level only exists for the combined system, matter and light.

In optical glass fibers, for symmetry reasons, the third-order nonlinearity,  $\chi^{(3)}$ , is the dominant nonlinear susceptibility. For pulse modulated systems the three most important nonlinearities are self-phase modulation, four-photon mixing and stimulated Raman scattering. The pros and cons of these nonlinearities can be summarized as follows (see [2; 3; 4]):

**Self-phase modulation.** Positive effects: solitons, temporal compression. Negative effects: spectral broadening, hence enhanced GVD.

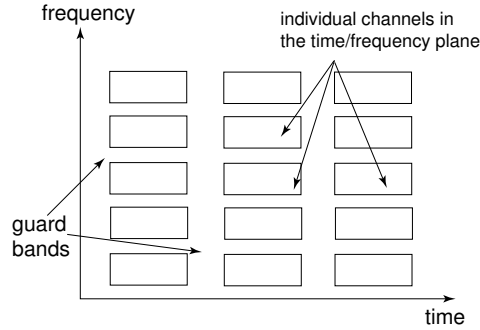
**Four-photon mixing.** Positive effects: generation of new wavelengths. Negative effects: crosstalk between different wavelength channels.

**Stimulated Raman Scattering.** Positive effects: amplification (broadband and wavelength independent). Negative effects: crosstalk between different wavelength channels.

#### 4. Novel Line Coding Schemes

With the introduction of communication channels in both time and wavelength (frequency) the challenge of fitting as much information as possible into a given time-frequency space, has become more similar to the problem that Shannon and, to some extent, Gabor were addressing in the 1940s. This is a fundamental problem—one which appears in many different fields such as; signal processing, image processing, quantum mechanics etc. Common to all of these different fields is the relation of *two* physical variables via a *Fourier transform*, which therefore, are subject to an “uncertainty relationship”, which ultimately determines the information capacity; see Figure 11.

To build robust pulse forms which have good time-frequency localization properties recent research in applied mathematics has shown that shaping optical pulses as wavelets can dramatically improve the spectral efficiency and robustness of an optical fiber network [8]. In table 2 we note that present systems (2.5 Gbs) only have a 5% spectral efficiency (that is, only 5% of the available bandwidth is used for sending information). It is hoped that in five to ten years we will have 40 Gbs systems utilizing 40% of the available spectral bandwidth.



**Figure 11.** Time/frequency representation of the available bandwidth for any communication channel.

Bit rate (Gbs)	Channel spacing(GHz)	Spectral efficiency(%)
2.5	100/50	2.5/5.0
10	200/100/50	5/10/20
40	100	40

**Table 2.** Spectral efficiency for present (2.5 Gbs) and future high-speed systems.

To achieve this spectral efficiency we can use an element of an orthonormal bases  $p(t)$  as our input pulse. Our total digital signal, with 1s and 0s can be described as a pulse train

$$s(t) = \sum_{j=1}^{2BT_b} a_j p(t - kT_b),$$

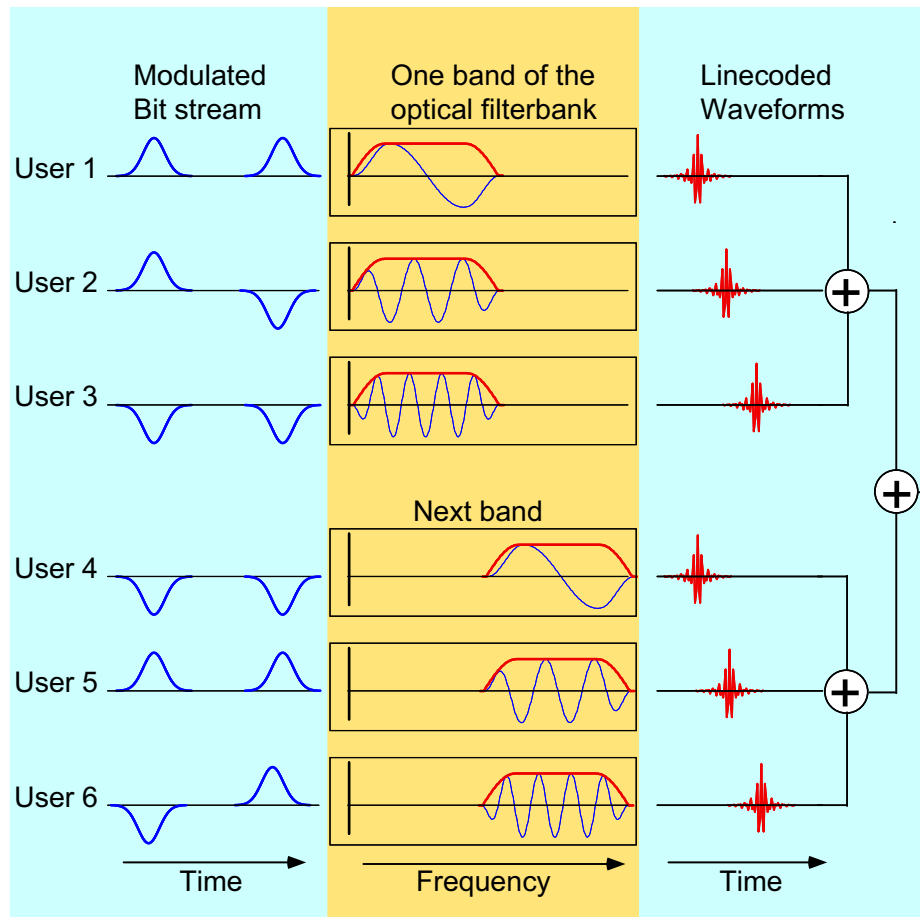
where  $B$  is the bandwidth of our channel,  $T_b$  is the time between pulses (Figure 7) and  $p(t)$  is the temporal shape of the bits. One possible choice for  $p(t)$  could be the local trigonometric bases,

$$p_{nk}(t) = w(t - n) \times \cos\left(\left(k + \frac{1}{2}\right)\pi(t - n)\right),$$

where  $w(t - n)$  is a window function; see Figures 8 and 12. The window function has very smooth edges, which partly explains the good time-frequency localization of these bases (Figure 12). Compared to other waveforms—sinc pulses, for instance—the local trigonometric bases have much better systems performance, they are particularly resistant to timing jitter. So, despite the fact that sinc pulses are theoretically the best pulses they are not the best choice for an imperfect communications system.

One possible way to use these special wavelets in a network could be to partition the fiber bandwidth into many frequency channels, each defined by a particular basis function. These channels are orthogonal with out the use of guard

bands. Detection is performed by matched filters. Both the frequency partitioning and the matched filter detection can be performed all-optically, radically increasing the network's capacity.



**Figure 12.** Encoding of orthogonal waveforms onto individual channels. Different spectral windows, if shaped properly, can be made to overlap, making it possible to use the full spectral bandwidth.

**Conclusion.** Even though dramatic improvements have been made during the last 10 years to combat absorption, dispersion and nonlinear effects in optical fibers it is also apparent that we need to do more if we are going to realize the ultimate bandwidths which are possible in glass optical fibers. One very powerful way to make a system transparent to fiber impairments is to encode amplitude and phase information which will be immune to the negative effects of, for example, dispersion and nonlinear interactions.

### References

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