Trends in Commutative Algebra
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Preface

Over the last fifteen years, commutative algebra has experienced a striking evolution. During this period the outlook of the subject has been altered, new connections to other areas have been established, and powerful techniques have been developed. To foster further development a year-long program on commutative algebra was held at MSRI during the 2002–03 academic year, starting with an introductory workshop on September 9–13, 2002. This workshop concentrated on the interplay and growing connections between commutative algebra and other areas, such as algebraic geometry, the cohomology of groups, and combinatorics.

Six main speakers each gave a series of three talks during the week: David Benson, David Eisenbud, Mark Haiman, Melvin Hochster, Rob Lazarsfeld, and Bernard Teissier. The workshop was very well attended, with more than 120 participants. Every series of main talks was supplemented by a discussion/talk session presented by a young researcher: Manuel Blickle, Ana Bravo, Srikanth Iyengar, Graham Leuschke, Ezra Miller, and Jessica Sidman. Each of these speakers has contributed a paper, or in some cases a combined paper, in this volume.

David Benson spoke on the cohomology of groups, presenting some of the many questions which are unanswered and which have a close relationship to modern commutative algebra. He gave us many convincing reasons for working in the "graded" commutative case, where signs are introduced when commuting elements of odd degree. Srikanth Iyengar gives background information for Benson's notes.

David Eisenbud spoke on a classical subject in commutative algebra: free resolutions. In his paper with a chapter by Jessica Sidman, he visits this classic territory with a different perspective, by drawing close ties between graded free resolutions and the geometry of projective varieties. He leads us through recent developments, including Mark Green's proof of the linear syzygy conjecture.

Mark Haiman lectured on the commutative algebra of n points in the plane. This leads quite rapidly to the geometry of the Hilbert scheme, and to substantial combinatorial questions (and answers) which can be phrased in terms of common questions in commutative algebra such as asking about the Cohen–Macaulay

property for certain Rees algebras. Ezra Miller writes an appendix about the Hilbert scheme of n points in the plane.

Mel Hochster gave three lectures on tight closure, telling eleven reasons to study tight closure. Hochster presents tight closure as a test for ideal membership which is necessary, but not sufficient, except for certain rings such as regular rings. Graham Leuschke's appendix gives examples of computation of tight closure.

The theory of multiplier ideals has been expanding rapidly in the last few years and bears a close relationship to commutative algebra, particularly tight closure. Rob Lazarsfeld and Manuel Blickle present a gentle introduction to this theory, with emphasis on the important theorems and concepts, applications, and examples.

Resolution of singularities has long played a crucial role in algebraic geometry and commutative algebra. Bernard Teissier talked about new ideas for understanding resolution coming from the simplest of all polynomials: monomials and binomials. Toric geometry of course enters into this story in a crucial way. Ana Bravo provides a summary of results on SAGBI bases which enter into this story.

The editors of this volume, who formed the organizing committee for the year program, would like to thank the many people who made the year possible, and thank the speakers for their wonderful contributions. A special thanks to David Eisenbud, the director of MSRI, without whom none of this would have been possible. We thank Michael Singer, the acting director of MSRI during the academic year when the program took place, for his generous help, and for the loan of Eisenbud to participate in our program. The great staff at MSRI were unfailingly helpful, friendly and professional. We thank the MSRI editor, Silvio Levy, for all his work on this volume. Finally, we thank the National Science Foundation for its support of institutes of mathematics in general, and of MSRI in particular.

We hope the papers in this volume will be a springboard for further learning and research for both experts and beginners.

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Mark Green
Craig Huneke
Karen Smith
Bernd Sturmfels

Note: The lectures this volume is based on were videotaped. They are available on streaming video and for downloading at www.msri.org/publications/video or www.cambridge.org/0521831954.