

Section 5

What Do Different Assessments Assess?

The Case of Fractions

“How do I understand thee? Let me count the ways.” Well, that’s not what Elizabeth Barrett Browning said, but if she were a mathematician, teacher, or mathematics educator referring to any topic in mathematics, she might have. And if she were an assessment specialist she might have noted that there are countless ways to explore and document those understandings.

This section provides two detailed explorations of one mathematics topic, fractions. Broadly speaking, it addresses two main issues: what does it mean to understand fractions (at, say, the sixth-grade level), and what is the potential of various kinds of assessments to reveal those understandings?

Let us start with procedural fluency. Certainly, one expects sixth graders to be fluent in adding, subtracting, multiplying, and dividing fractions. They should be able to convert fractions to decimals, and place both fractions and decimals on the number line; consequently, they should be able to compare the magnitudes of various fractions. They should have a sense of magnitude, and be able to answer questions like: “Which of the numbers 0, 1, or 2 is the sum $\frac{7}{8} + \frac{12}{13}$ closest to?”

A next level of performance consists of being able use one’s knowledge of fractions and be able to explain why what one has done makes sense. Here are some items from the *Mathematics Framework for California Public Schools* [California 2006, pp. 77–78]:

Your after-school program is on a hiking trip. You hike $\frac{3}{4}$ of a mile and stop to rest. Your friend hikes $\frac{4}{5}$ of a mile, then turns around and hikes back $\frac{1}{8}$ of a mile. Who is farther ahead on the trail? How much farther? Explain how you solved the problem.

Jim was on a hiking trail and after walking $\frac{3}{4}$ of a mile, he found that he was only $\frac{5}{8}$ of the way to the end of the trail. How long is the trail? Explain.

These are applications. One level deeper involves understanding what fractions are and various representations of them. The following item from the *California Framework* [California 2006, p. 78] begins to explore this territory:

Draw a picture that illustrates each of the following problems and its solution. Explain how your drawings illustrate the problems and the solutions.

$$1. \quad \frac{3}{4} \times \frac{1}{2} \qquad 2. \quad \frac{3}{4} + \frac{1}{2} \qquad 3. \quad 2 \times \frac{3}{4}$$

But this just scratches the surface. Does the student understand that all of the n -ths in the fraction m/n must be the same size? (Recall the sample assessment item from Chapter 1.) That if the numerator of a fraction is kept constant and as the denominator increases, the magnitude of the fraction decreases? Can the student work with representations of fractions on the number line? As parts of a “pie chart”? When the “whole” is, say, a pie and a half? Can the student explain why $\frac{2}{3}$ is equivalent to $\frac{4}{6}$, using any of these representations? Even this is just a first step into the domain of fraction understanding: the literature on what it means to understand fractions is immense. (A Google search on the phrase “understanding fractions” gives 22,000 hits.)

Once one has a sense of the terrain, there is the question of how one finds out what any particular student knows. This is enormously complex — and fascinating, as the three contributions in this section demonstrate. In Chapter 14, Linda Fisher shows the kinds of insights that well-crafted assessment tasks can provide into student thinking. She presents a collection of tasks that have been used by the Silicon Valley Mathematics Assessment Collaborative, and discusses the ways in which student responses reveal what they understand and do not. Such information is of use for helping teachers develop deeper understandings of student learning, and (when one sees what students are or are not making sense of) for refining curricula. In Chapter 15, Deborah Ball cranks up the microscope one step further. No matter how good a paper-and-pencil assessment may be, it is static: the questions are pre-determined, and what you see in the responses is what you get. At the MSRI conference, Ball conducted an interview with a student, asking him about his understandings of fractions. Like a written assessment, the interview started off with a script — but, when the student’s response indicated something interesting about his understanding, Ball was in a position to pursue it. As a result, she could delve more deeply into his understanding than one could with a test whose items were fixed in advance, and also get a broad and focused picture of how his knowledge fit together. In Chapter 16, Alan Schoenfeld reflects on what such interviews can reveal regarding the nature of student understanding, and about the ways in which skilled interviewers can bring such information to light.

[California 2006] *Mathematics framework for California public schools, kindergarten through grade twelve*, Sacramento: California Department of Education, 2006.

Chapter 14

Learning About Fractions from Assessment

LINDA FISHER

Assessment can be a powerful tool for examining what students understand about mathematics and how they think mathematically. It can reveal students' misconceptions and gaping holes in their learning. It can also reveal the strategies used by successful students. Student responses can raise questions for teachers: Of all the strategies that students are exposed to, which ones do they choose to help make sense of a new situation? Which can they apply accurately? Which aspects of these strategies might be helpful for other students? Assessments can also, with guidance and reflection, provide teachers with a deeper understanding of the mathematical ideas and concepts that underlie the rules and procedures frequently given in textbooks. A good assessment raises questions about changing or improving instruction. The diagnostic and curricular information afforded by assessments can provide powerful tools for guiding and informing instruction.

Over the past seven years, the Mathematics Assessment Collaborative has given formative and summative assessments to students and used their responses in professional development for teachers and to inform instruction. This chapter will share some examples of how assessment can reveal student thinking and raise issues for instructional planning and improvement.

Learning from Formative Assessment

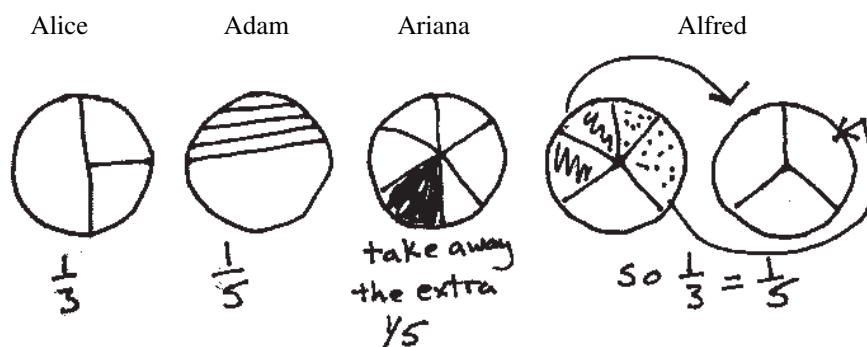
A group of teachers engaged in lesson study¹ was interested in the idea of focusing on students' use of mathematical representations. The teachers had looked at lessons and representations from a variety of sources, including an intriguing lesson from *Singapore Primary Mathematics 3B* which uses bar models (diagrams which use rectangular "bars" to represent quantities in a problem),

¹Lesson study is a process in which teachers jointly plan, observe, analyze, and refine classroom lessons which are known as "research lessons." See <http://www.lessonresearch.net/>

triangles, pentagons, hexagons, squares, and circles to represent part-whole relationships. The middle part of that lesson tries to get students to see the following generalization: when the numerators of a collection of fractions are the same, the size of the portion represented by each fraction is inversely related to the numerical value of the denominator. The lesson study group at first considered designing its research lesson as an introduction to bar models. After lively debate, it opted to investigate what representations students would use to solve a problem for themselves without further instruction. The lesson study group then went into several classrooms to collect data about the representations that students already used.

The teachers asked students the question, “Which is greater, one fifth or one third? We are curious about how you think this out. So please show us in words and pictures.” As students finished their thinking, the teachers selected student work and asked students to come to the board and explain their representations.

The lesson study teachers were surprised that all but one student used circles to represent fractions. Here are four typical representations used by third graders:



Their work suggests that Alice and Adam do not have the understanding that when one third (or one fifth) are represented as one of three (or five) parts of a whole, that each part has equal size. Ariana, and many other students, had difficulty drawing fifths. Her solution was to ignore the extra “one fifth.” That piece *is* one fifth, but not one-fifth of the “one” represented by the circle. Instead, the piece is one-fifth of the unit represented by the unshaded pieces in her drawing. Does Ariana know that? Finally, some students, like Alfred, used a matching strategy. Alfred explained, “Two fifths is about the same size as one third. The next two fifths is the same size as one third, so there is one piece left in each circle. So one third and one fifth are the same size.” Alfred apparently doesn’t see any problem in saying that one-third and one-fifth of the same object are equal. Notice that while all the drawings attempt to represent

denominators, none of these third-grade students attempted to shade the part of the representation that corresponded to the “one” in one fifth and one third.

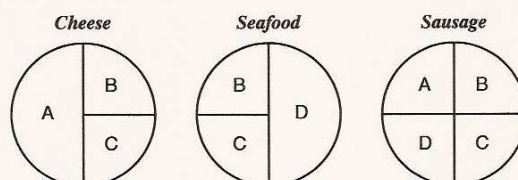
This type of assessment provides many insights for the classroom teacher about the complexity of understanding fractions and what types of experiences students need to make sense of the concepts that make up this understanding. It raises issues about the difference between identifying fractional parts in a diagram in a textbook and being able to make a representation of fractional parts accurate enough to help in solving a particular problem. The student work raises many pedagogical issues. How can teachers help students to develop the idea of equal parts? How can teachers get students to understand the idea of a unit—that you can’t remove a piece of the object that represents the unit and have the remaining object represent the same unit? Is it appropriate to use odd denominators at this grade level and have students struggle to draw equal pieces or should students only be given amounts that are convenient to draw? Do students learn enough from examples and explanations or is there a quantifiable benefit from confronting and discussing misconceptions? What is gained and what is lost by these approaches? These are the types of questions raised by good assessments. Such assessments provide evidence of gaps in student understanding that are not revealed through computational exercises. They provide windows into what is missing when instruction focuses solely on procedures.

Looking at Summative Assessment: Sharing Pizza

A common way to introduce fractions to students is in the context of sharing. Consider the grade 5 task Sharing Pizza. This task gives students the opportunity to identify fractional parts, combine common unlike fractions, draw representations of fractions, and use fractions in a sharing context.

Sharing Pizza

Aretha, Beth, Carlos, and Dino go into a pizza shop and order three different pizzas. They divide the pizzas so that they each end up with the same amount to eat. Aretha can’t eat seafood. The other friends like all the pizzas.



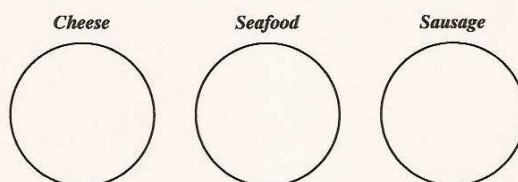
Aretha gets all the pieces labeled A. Beth gets those labeled B. Carlos gets those labeled C. Dino gets those labeled D.

1. What fraction of the *Cheese* pizza does Aretha eat? _____

What fraction of the *Sausage* pizza does Aretha eat? _____

How much pizza does Aretha eat? _____

2. Complete the diagram below to show how **five** friends—Aretha, Beth, Carlos, Dino, and Erica—would divide the **three** pizzas. Remember that each person gets the same amount to eat. Remember that Aretha can't eat seafood, but the other friends like all three pizzas.



How much pizza does Aretha eat this time? Explain. _____

This task was one of five given in a summative test to approximately 11,000 fifth graders in March, 2001. On this task 68% of the students were able to calculate $\frac{1}{2} + \frac{1}{4}$ in Part 1. However, only 33% were able to demonstrate any proficiency with any of the mathematics in Part 2.

Some students could give responses to Part 2 of the task—share the three pizzas among five friends—such as this:

Student A



How much pizza does Aretha eat this time? Explain. Aretha eats $\frac{3}{5}$ of the pizzas. Since there's 5 friends, I divided every pizza into fifths. Since there are 3 whole pizzas I knew each one would get $\frac{3}{5}$ of pizza.


Page 1


Sharing Pizza Test 5: Form A

Student A can clearly articulate the part-whole relationship and describe each student's portion. The student is easily able to deal with the constraint that Aretha doesn't like seafood, without it affecting her overall serving size.

Student B has also solved the task, by partitioning each pizza into ten pieces:


Student B



How much pizza does Aretha eat this time? Explain. Aretha eats $\frac{6}{10}$.
The pizza's were 10 pieces. All together there were 30 pieces. Everyone had 6. 

Student C's response shows the importance of digging beneath the surface of the answers that students provide. Giving constructed-response questions provides an opportunity for students to reveal what they know and what they do not. Student C writes " $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$," a correct calculation, although not one that expresses Aretha's share of the pizzas. However, look at the rest of the student's work:

Student C



How much pizza does Aretha eat this time? Explain. $\frac{7}{10}$
 $\frac{1}{2} + \frac{1}{5}$

Here the student shows no evidence of knowing that unit fractions with the same denominator must each be represented by parts of equal size. The slice labeled A in the cheese pizza is not a representation of one fifth. Student C cannot adequately deal with the constraint of Aretha not liking seafood. The student does not seem to have noticed that Aretha's, Carlos's, and Dino's shares are not represented as having the same size. A and C both get a half-pizza and another piece. C's half is composed of a fourth of the cheese pizza and a fourth of the seafood pizza, and A gets half of the sausage pizza. But the portions being added to those halves are not the same size: A gets a share that looks like one sixth of the cheese pizza, and C gets approximately one eighth of the sausage pizza. D gets an even smaller "piece of the pie." Student C appears to

be using ideas similar to those of the third graders, Alice and Adam: fractions of the form $1/n$ are represented by any one of n pieces of an object, but each piece is not necessarily of the same size. In this case, a correct computation may not include an increased understanding of fractional parts and their underlying relationships.

Teachers who only look at the written answer may think that Student D has a firm grasp of the material:

Student D

How much pizza does Aretha eat this time? Explain. _____
I counted the cheese and sausage.
I got $\frac{3}{5}$.

A response like this can be misleading, however. Good assessments allow teachers to probe further into issues of student understanding. Looking at Student D's drawing (below), one might wonder: Does the student have a grasp of equal-size pieces, or is the student just counting parts? How has the student made sense of "Aretha can't eat seafood"? Is this approach correct? What should be the "whole" for this task? This response also raises questions about experience with fractions: How often has Student D had the opportunity to grapple with the idea of the whole being several objects rather than one object?



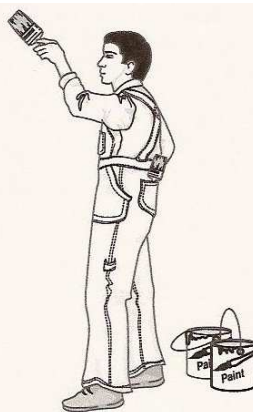
Assessments allow us to carefully examine work across grade levels for trends. If we see many misconceptions from third grade still appearing in fifth grade, this raises important questions about instruction and instructional materials. Are the representations presented in textbooks helping students to develop the intended ideas? Would different representations help? Should we look at some of the materials from Japan and China, which quickly move from fractions of one object like a pizza, or of a collection of objects, like marbles, to fractions of quantities like distances or cups of sugar?

Middle School Work with Fractions

What happens in middle school when ideas of part-whole relationships are not fully developed? How do student conceptions progress as students move through the curriculum? Consider the grade 7 assessment task Mixing Paints.

Mixing Paints

Wayne is mixing paint.



He makes six quarts of brown paint by mixing equal quantities of yellow paint and violet paint. The violet paint is made from one-third red paint and two-thirds blue paint.

1. How much red paint does he use? _____ quart(s)
2. How much blue paint does he use? _____ quart(s)
3. What percentage of the brown paint is made from blue paint? _____

Explain how you figured it out.

On the surface, this task appears trivial. The first sentence says that there are three quarts each of yellow and violet paint; the second sentence says that the three quarts of violet paint were made from one quart of red paint and two quarts of blue paint. Two of the six quarts of paint used to make the brown paint were blue, so the answer to Question 3 is $33\frac{1}{3}\%$. Thus, from our perspective as adults the numbers are easy and the mathematics appears straightforward.

Yet, in a sample of almost 11,000 seventh graders, more than 50% of the students scored *zero points* on this task. That is, more than half the students were unable to identify the quantity of either red or blue paint. About 44% of the students could only find the quantity of red and blue paint. Only 18% of the students could do this and convert the amount of blue paint into a percentage

of the total amount of brown paint — they had difficulty identifying the whole, which was the six quarts of brown paint. (Generally, students wrote that the answer to Question 3 was 66%. This may have been a conversion of the “two thirds” in the statement that the violet paint is two-thirds blue paint.)

Assessment items such as Mixing Paints provide a way for teachers to gain a sense of how students are thinking, and to see how students are (or are not) making sense of part-whole relationships, and what kinds of representations might help students better grasp those relationships. How do various representations facilitate student learning and deepen student thinking over time? Mathematics Assessment Collaborative teachers have come to appreciate the bar models used in Singapore and Russia, which help students visualize fractional relationships and make sense of rates. A benefit of bar models is that they can be modified to represent percents and other mathematical topics.

A question related to the part-whole issue explored in Mixing Paints is, “How do students’ difficulties in understanding part-whole relationships and identifying the whole come into play when students are trying to make sense of percents, particularly percent increase or percent decrease?” Consider this grade 8 task:

Traffic

The daily average of motor vehicles using a certain freeway during 1997, 1998, and 1999 is shown in the table below.

	Year		
	1997	1998	1999
Average number of motor vehicles each day	54,800	61,700	73,400

1. In 1997, 15% of the motor vehicles traveling along the freeway were taxis. Calculate the average number of taxis traveling along the freeway each day in 1997.

Show your work. _____

2. Find the percentage increase in the average number of motor vehicles using the freeway each day from 1997 to 1999. Round your answer to the nearest whole number.

Explain how you figured it out. _____

3. From 1999 to 2000, there was an average increase of 20% in the number of motor vehicles using the freeway. Find the daily average of motor vehicles using the freeway in 2000. Round your answer to the nearest hundred. _____

Explain your reasoning.

In a sample of more than 6,000 eighth graders, 41% scored no points on this task. Slightly less than 50% of the students could successfully find 15% of the traffic in Part 1. Overall, less than 10% of the students could calculate accurately using percents. Many students who scored zero points tried to divide instead of multiply in Part 1:

Student E

1. In 1997, 15% of the motor vehicles traveling along the freeway were taxis. Calculate the average number of taxis traveling along the freeway each day in 1997.

Show your work.

$$\begin{array}{r} 3653.\bar{3} \times \\ 15 \overline{) 54800} \end{array}$$

Other students learn rules for dealing with percent problems, such as “divide” or “move the decimal point.” But what understanding of the underlying concepts do they show when they use those rules? Consider the work of Students F and G:

Student F

2. Find the percentage increase in the average number of motor vehicles using the freeway each day from 1997 to 1999. Round your answer to the nearest whole number.

Explain how you figured it out.

19%

I subtracted the avg. number of vehicles in 1999 by the number of vehicles in 1997 and got 18,600 vehicles (difference).

$$\begin{array}{r} 73,400 \\ - 54,800 \\ \hline 18,600 \end{array}$$

18.6

Student G

2. Find the percentage increase in the average number of motor vehicles using the freeway each day from 1997 to 1999. Round your answer to the nearest whole number.

Explain how you figured it out.

74%

percentage increase = $\frac{\text{increase}}{\text{whole}}$ I figured it out by taking the number of vehicles in 1997 and dividing it by the number of vehicles in 1999.

$$\frac{54,800}{73,400} = 0.74$$

$$\begin{array}{r} 73,400 \overline{) 54,800.0} \\ \underline{51,380} \\ 28,420 \\ \underline{29,360} \\ 4,840 \end{array}$$

$$\begin{array}{r} 73,400 \times \\ \underline{51,380} \end{array}$$

0.74

74%

Page 8 Traffic Test 8: Form A

Often interesting assessment tasks can provide insight into the strategies used by successful students. For example, consider the task Fractions.

Fractions

Tom has four number cards.



1. He arranges his four cards to make fractions less than 1.
Using each number card only once, make two fractions that have the same value.

$$\frac{\square}{\square} \quad \text{and} \quad \frac{\square}{\square}$$

2. Find a way to use two number cards to make a fraction less than $\frac{1}{2}$.

$$\frac{\square}{\square}$$

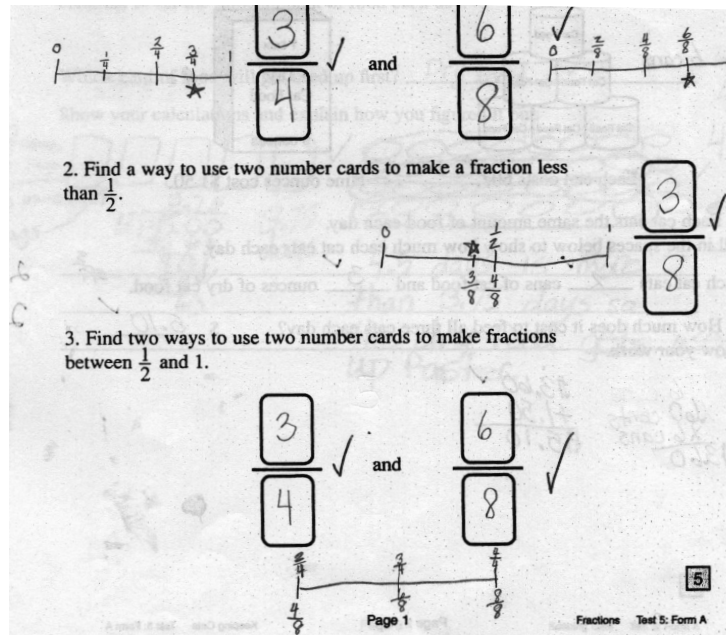
3. Find two ways to use two number cards to make fractions between $\frac{1}{2}$ and 1.

$$\frac{\square}{\square} \quad \text{and} \quad \frac{\square}{\square}$$

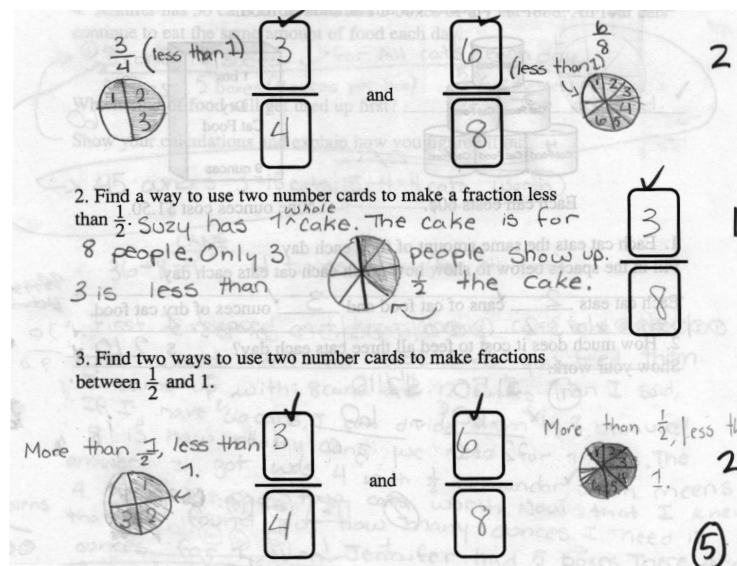
Although students were not asked to explain their answers, many students provided clear drawings. These drawings help to provide insights into what they know and understand, and how they make sense of fractions.

Some students used number lines to locate fractions and to judge relative sizes; others used pie diagrams. Note that the student whose work is shown at

the bottom of this page also uses words to indicate that all the constraints in the problem are being met by the solution.



Fraction comparison via number lines, reduction to common denominator



Fraction comparison via pie diagrams

A few students reduced fractions to lowest terms to check size, but still relied on drawings. Another strategy was to convert fractions to decimals.

3/6 ✓ and 4/8 ✓

$\frac{3}{6} \div \frac{3}{3} = \frac{1}{2}$
 $\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$

2. Find a way to use two number cards to make a fraction less than $\frac{1}{2}$.

3. Find two ways to use two number cards to make fractions between $\frac{1}{2}$ and 1.

3/6 ✓ and 6/8 ✓

Fraction comparison via pie diagrams, reduction to lowest terms

3/4 ✓ and 6/8 ✓

$\frac{3}{4} = 0.75$
 $\frac{6}{8} = 0.75$
 $\frac{6}{8} \div 2 = \frac{3}{4}$
 $\frac{3}{4} \times 2 = \frac{6}{8}$

use two number cards to make a fraction less than $\frac{1}{2}$.

$\frac{4}{8} = \frac{1}{2}$ $\frac{1}{2} = 0.50$

$\frac{3}{8} = 0.38$

ways to use two number cards to make fractions between $\frac{1}{2}$ and 1.

1.

3/4 ✓ and 6/8 ✓

$0.75 = \frac{3}{4}$
 $\frac{1}{2} < \frac{3}{4}$
 $\frac{3}{4} < 1$
 $\frac{6}{8} = 0.75$
 $\frac{6}{8} > \frac{1}{2}$
 $\frac{6}{8} < 1$

Page 1 Fractions Test 5: Form

Fraction comparison via decimals

Some students have a clear understanding that “half of” a quantity means dividing that quantity by 2, and they use that understanding to compare fractions, writing, for example: “Half of eight is four and three is less than four, so $\frac{3}{8}$ is less than $\frac{1}{2}$ ($= \frac{4}{8}$).”

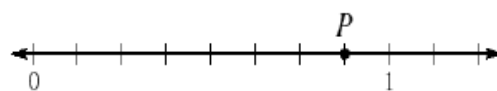
As noted above, students’ responses to assessment tasks yields insights into their thinking and into the advantages and disadvantages of particular curricula. As some of the previous examples indicate, giving students experience with a wide variety of strategies and representations provides them with a range of tools they can use to make sense of problem situations.

In summary, high quality assessments can provide teachers with useful information about student misconceptions related to: the role of equal parts in the definition of fractions; using and interpreting representations of fractions that display the whole; and understanding equality. These assessments can provide a window into student thinking, and help teachers think about their own instructional practices and concrete ways in which they might be improved. Good assessments help teachers reflect upon issues like: “What does it mean to understand an mathematical idea and how is assessing it different from looking at computational fluency? What is the value of having students draw representations themselves versus interpret representations provided in textbooks? How is the learning different?” They also give teachers the chance to look across grade levels and see how student thinking is or is not getting deeper and richer over time. Mathematically rich performance assessments provide insight into the strategies used by successful students. Of all the tools available to a student, which one does the student use to solve a complicated problem? When a teacher sees this, how can he or she turn these strategies into tools for all students?

Multiple-Choice and Constructed-Response Tasks: What Does a Teacher Learn from Each?

Two of the released items from the California Standards Test are given below.

1. What fraction is best represented by the point P on this number line?



- A. $\frac{1}{8}$ B. $\frac{1}{5}$ C. $\frac{3}{4}$ D. $\frac{7}{8}$

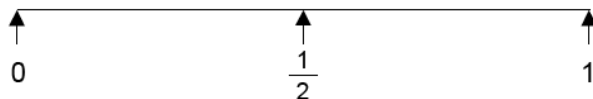
2. Which fraction represents the largest part of a whole?

- A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$

Let us view these tasks through the lens developed in the first part of this chapter. The question is: What does student work on these tasks tell the classroom teacher? For example, if you know that your student missed the first task, that tells you that the student may be struggling with number lines and the relative size of numbers represented as fractions. If the teacher knows that the student picked answer A, the teacher might understand that the student is making sense of partitioning into equal parts but is not understanding the role of the numerator. Yet the teacher can't be sure; perhaps the student merely guessed or picked that answer for a different reason. The reality of high-stakes testing is that the teacher does not see the task or receive information about which distractor the student selected. Feedback comes in the form of an overall rating: "your student is a basic (one step below proficient)" or "your student is not doing well in 'decimals, fractions, and negative numbers.'" This provides the teacher with little information that could be used to think about changing instruction or helping the student in question.

Now consider a very similar task in a constructed-response format.

Here is a number line.



1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.
2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.
3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? _____

Explain how you figured it out.

A great deal can be learned when such tasks are used for purposes of formative (as well as final) assessment. First, as discussed above, the written prompts provide an opportunity for individual students to reveal their thinking, and for collective scores to reveal the strengths and weaknesses of the current curriculum. A scoring rubric can assign points for particular competencies. (For an example of such a rubric, see Hugh Burkhardt's chapter in this volume.) Individual scores on the problem can indicate individual strengths and weaknesses, and cumulative distributions of scores can reveal areas of the curriculum where

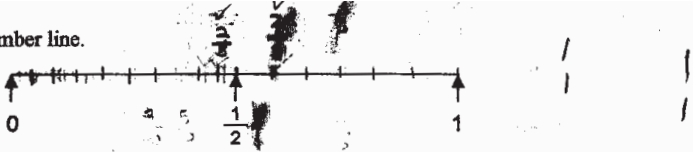
students need help. School districts working with the Mathematics Assessment Collaborative generate graphs showing student score distributions. This allows the district to pinpoint places where curricular attention is needed, and to examine progress over the years. (See Elizabeth Stage's chapter in this volume for a similar discussion of what tests can reveal.)

Beyond the statistical data, actual student work can be used for purposes of professional development. Teachers can look at student work and try to infer the strategies used by students — thus developing a better understanding of what the students have learned. Consider, for example, what might be gained from a group of teachers looking at student work as follows:

Examine the work done by students H, I, and J in the next three figures.

- How would you characterize the strategies being used by these students?
- What did students need to understand about fractions to use these strategies? Could you give a name to these strategies?

Here is a number line.



1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.
2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.
 I made all the fractions to have the same denominator and I put the fractions either before $\frac{1}{2}$ or beyond $\frac{1}{2}$. I got $\frac{12}{30}$ for $\frac{2}{3}$, $\frac{20}{30}$ for $\frac{2}{5}$, and $\frac{15}{30}$ for $\frac{1}{2}$. I compared those two numbers with $\frac{1}{2}$ or $\frac{15}{30}$.
3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{5}$ ✓ = ✓ 1 1
 Explain how you figured it out.
 I made the fractions to have the same denominator and I compared them. $\frac{2}{3}$ was $\frac{3}{30}$ off of $\frac{1}{2}$. $\frac{2}{5}$ was $\frac{10}{30}$ off of $\frac{1}{2}$. So $\frac{2}{5}$ is closer. ✓ (6)

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line. 11

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line. 11

Because $\frac{2}{3}$ is 66% and 66% is close to 60% so I estimated where 60% was. And $\frac{2}{5}$ is 40% so I got 40%.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? 11

$\frac{2}{5}$

Explain how you figured it out. 2

Because $\frac{1}{2} = 50\%$ and $\frac{2}{3}$ is 66% and $\frac{2}{5} = 40\%$ so 40% (40%) is closer to 50% (1/2).

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line. 1

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line. 1

I divided the top part of the line into thirds and marked $\frac{2}{3}$ and divided the bottom part of the line into fifths and marked $\frac{2}{5}$.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? 1

$\frac{2}{5}$

Explain how you figured it out. 2

On the number line the dash that marks $\frac{2}{3}$ is further from $\frac{1}{2}$ than the dash that marks $\frac{2}{5}$.

In discussions of such student work, teachers become increasingly attuned to diagnosing student thinking — to seeing not only what students got right or wrong, but how and why they got those things right or wrong. These understandings can lead to more effective teaching and improved student learning.