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Chapter 15
Assessing a Student's Mathematical Knowledge by Way of Interview

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WITH
BRANDON PEOPLES

Deborah Loewenberg Ball conducted the following interview with Brandon Peoples, a sixth grader, on March 8, 2004 at the first MSRI Workshop on Critical Issues in Mathematics Education.

This interview assessment of a student's mathematical understanding, conducted live in front of the assembled workshop participants, provides an immediate and vivid case of student thinking and exemplifies the interview assessment, an important mechanism for accessing student thinking.

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1 Ball: [To audience] Brandon and I are going to pretend you're not in the room, so I'm going to stop talking to you and we don't actually care that you're there, so, uh, goodbye. And uh, just be as quiet as you can because we actually want to do the work that we're setting out to do here. So, okay?

[To Brandon] So, you know some of the stuff we were doing, before people came, we might go back to some of that just because some of it was interesting and we didn't really finish talking about it. But I wanted to just start by, talking to you just a little bit about school. Do you remember when I called you the other day? We had a few minutes and I asked you a few things about your school and what you're working on this year? So can we just go back to that, because we were – we didn't have very much time that day. I was just wondering, like what you're doing in math right now in school.

2 Brandon: Really, we're learning about fractions.

3 Ball: Mm-hmm.

4 Brandon: And, umm, like – like we're doing – we're going on to like more – to more advanced of it instead of just – we're just moving on up the line of the fr – of math.

5 Ball: So can you give me an example of something you've worked on, I don't know, recently in fractions? Can you think of something you've been working on?

Time: 00:02:00

6 Brandon: Okay.

7 Ball: Do we have plenty of paper around? Do we have blank paper?

8 Brandon: Yeah.

9 Ball: Where is it? Oh, okay. No, you don't have to change, I just wanted to make sure we had enough.

[Brandon starts to give his example.]

10 Ball: Okay, so what did you write?

[Brandon has written $\frac{1}{2} \div \frac{1}{2}$ on the overhead.]

11 Brandon: One-half divided by one-half.

12 Ball: And you've been working on this?

13 Brandon: Mm-hmm.

14 Ball: So, what have you learned to do with that?

15 Brandon: See, it's the part – you have to – you have to multiply these two in, but instead you flip it upside down . . .

16 Ball: Uh-huh.

17 Brandon: So you, instead of one-half it'll be two over one.

18 Ball: Okay, can you show me?

19 Brandon: Mm-hmm.

[Brandon writes $\frac{1}{2} \times \frac{2}{1}$ on the overhead.]

20 Ball: Okay. And after that there is another step that you take? Or what do you do next?

21 Brandon: You multiply –

22 Ball: Uh-huh.

23 Brandon: – and then you get your answer.

24 Ball: Can you do that?

25 Brandon: Mm-hmm.

[Brandon writes $\frac{2}{2} = 1$ on the overhead.]

26 Brandon: Which equals this to this or one whole.

27 Ball: Okay, umm, is there – did you make any pictures of that or anything like that or did you just kind of learn how to do it the way you just showed me?

28 Brandon: Well it's – I learned different ways. I tried to come up with different strategies, but this was the better – this just adding and stuff is a better idea.

29 Ball: Well maybe we'll talk about that a little bit more later. Do you remember – have any recollection or memory of when you first started to work on fractions at school? Like what grade it was?

30 Brandon: I think we started to – I think we started on in fifth grade –

31 Ball: Mm-hmm.

32 Brandon: because we didn't – we didn't learn – our teacher in fourth grade didn't – we really didn't learn fractions.

33 Ball: Mm-hmm. And what's the new stuff this year? Like this I guess, right?

34 Brandon: Yeah, and, umm, this thing of my other math teacher, he calls it cross-canceling.

35 Ball: Mm-hmm… You mentioned that on the phone. What is that exactly?

36 Brandon: It's like when instead of, like, going through all this reducing, instead it's like two over – two over four times, umm, three sixths.

[Brandon writes $\frac{2}{4} \times \frac{3}{6}$.]

37 Ball: Mm-hmm.

38 Brandon: See, you'll want it – you'll want to reduce that to maker it smaller instead so – and then you get – you come out with a fraction.

39 Ball: Mm-hmm. How do you do that?

40 Brandon: So I think you… And then you multiply.

[Brandon has written $\frac{\cancel{2}^{1}}{4} \times \frac{3}{\cancel{6}_{1}} = \frac{3}{4}$.]

41 Ball: Okay, so can you explain what you just did?

42 Brandon: Mm-hmm. Umm, two-sixths can be reduced to? I mean oops.

[Brandon corrects what he has written: $\frac{\cancel{2}^{1}}{4} \times \frac{3}{\cancel{6}_{3}} = \frac{3}{4}$.]

43 Brandon: Three – two sixths can be reduced to one-third, so I put a cross through the two, I put a one and cross this out and put a third. Multiplied one time – by three, and multiplied four times one.

44 Ball: But you have a three there now.

45 Brandon: Oh. Hold on, I think I didn't do it right? three-twelfths, so it should be? Hmm.

[Brandon crosses out $\frac{3}{4}$ and writes $\frac{3}{12}$ directly to the right of it.]

46 Brandon: Let me see it's three-twelfths but it can be reduced to I think it's one-fourth or one-third. How did I get to . . . ? Wait, hold on. Something isn't right.

[Brandon writes $\frac{1}{4}$ directly to the right of $\frac{3}{12}$.]

47 Ball: Is that chair too high?

48 Brandon: Yeah, I think so . . .

49 Ball: So, do you want to leave this right now or do you want to work on this some more right now?

50 Brandon: We can work on it more.

51 Ball: You want to go back to the beginning then and tell me what you were trying to show me?

52 Brandon: I think I was trying to redu – reduce it, but then I got the wrong answer. I redu – I got – I add – I multiplied it, got an answer, but the answer could be reduced too.

53 Ball: What was the original problem that you wrote down?

54 Brandon: Was two-fourths times three-sixths.

55 Ball: Uh-huh. And do you – do you already have a sense of what you think the answer's supposed to be to that?

56 Brandon: Yeah.

57 Ball: What is it you think it's supposed to be?

58 Brandon: Well, I thought it was three-fourths, but it wasn't real, so I, it was . . .

59 Ball: Why did you think it would be three-fourths?

60 Brandon: Because, umm, I put a one instead of a three.

61 Ball: Mm-hmm.

62 Brandon: I thought because, umm, two-sixths was supposed to be reduced to one-third, so –

63 Ball: Uh-huh.

64 Brandon: – but can re – it's redu – when it's reduced to one third.

65 Ball: But you wrote one fourth. Do you mean one-fourth or one-third?

66 Brandon: One fourth. Yeah.

67 Ball: One-fourth. So now you're no – which answer are you saying is correct? The three-fourths you originally wrote or the one-fourth you just wrote?

68 Brandon: One-fourth.

69 Ball: Okay. Maybe – let's save this one and maybe – maybe if there's some time when we've done some other work we'll come back to it. Is that okay? 'Cause right now we're just trying to get a sense of the different things you've been working on.

70 Brandon: Mm-hmm.

71 Ball: Do you ever use fractions any place besides in school?

72 Brandon: Mm-hmm, not really.

73 Ball: Does anybody? Do you know anybody who uses fractions anyplace other than in school?

74 Brandon: Mm-mm.

75 Ball: Uh-huh. So, why do you think you learn about fractions?

Time: 00:08:23

76 Brandon: I think so it's like – it's kinda – it kinda takes, umm – it's kinda in relation to percentages.

77 Ball: Mm-hmm.

78 Brandon: Because like – because like one-fourth would be to – in percentage, it would be 25 percent.

79 Ball: Mm-hmm.

80 Brandon: And in, umm, decimals it would be zero-point-two-five

81 Ball: Mm-hmm.

82 Brandon: So it's – I think it's one way we can tell, umm, how much is something.

83 Ball: Okay.

84 Brandon: Or – or like how much, or what is – what percentage of it is.

85 Ball: Okay.

86 Brandon: Like – it's like different things contain different things. So like one part of something would be a percent – a percentage or a fraction. Same thing.

87 Ball: Okay. Umm, I think we're going to shift over and do some other problems and questions and things like that and I just wanted to remind you of something I told you before we started today which is anything, you've kind of already doing it, but anything I ask you, you can draw, you can write, you can use words. Any – anything that you've ever used before you can do, or anything that you think you can use – the main thing I'm trying to understand is how you're actually thinking and what you're thinking about. So even if I don't, you know, suggest using these blocks, or I don't suggest using a

drawing, you're totally free to do that. You can use the board, we can use this thing. Okay, do you remember when I was talking about that?

88 Brandon: Mm-hmm.

89 Ball: And, um, it's really not a test, I'm actually just trying to do some mathematics problems with you and see the ways that you think about them, kind of like you've already started to do. So you're doing exactly what I was hoping you would do. And the more you can tell me about kind of how you're thinking and why you're thinking certain things, the more helpful that will be in what I'm trying to learn about watching you work. Okay?

90 Brandon: Mm-hmm.

91 Ball: So, umm, I think I'll start with, umm – can you hand me one of those green pieces of paper? And maybe we'll need clean paper to write on too, so can I have another one?

92 Brandon: Mm-hmm.

93 Ball: Thanks. Okay. Can you fold that piece of paper in half?

[Brandon folds paper in half.]

Okay. So can you just explain how you knew what – what to do when I asked you that? Like, how did you decide how to do it?

94 Brandon: Well, umm, you take it like this and then draw down the line so – I know that it would be half 'cause it's only two –

95 Ball: Mm-hmm.

96 Brandon: – it's only two uh, two major things. So like 'cause if it was like – if it was like this, then it would be in fourths [referencing his folded paper]. But since it's – I only see two sides then I know it's just half. They're both halves.

97 Ball: If I asked a little kid to do that, like a five year old, do you think they would be able to do it too? Or do you think they might make a mistake when they did it?

98 Brandon: I think they would get, like, how to fold it in half, but I don't think they could explain how would it be half.

99 Ball: Can you think of any mistakes a little kid might make? In folding it? 'Cause you did it right and I'm curious if you can think of anything a little kid might do that would be a mistake that you didn't make.

100 Brandon: They would probably – they would probably fold it a different way –

101 Ball: Mm-hmm.

102 Brandon: – 'cause they might not know what a half is, or what like, what would – what would this be.

103 Ball: Like, what if they did something like this? Can you picture a little kid doing that?

[Ball folds a sheet of paper incorrectly: One side is obviously larger than the other.]

104 Brandon: Yeah, 'cause –

105 Ball: Why would they do that?

106 Brandon: 'Umm, probably 'cause I think they probably might not know what half is. They might just fold it. They might just fold it in their way.

107 Ball: Mm-hmm.

108 Brandon: Instead of like, er, fold it not like – not like this but like one side would be bigger and one would be smaller.

109 Ball: So another thing you – you – you were trying to do what to make the parts be the same?

110 Brandon: Mm-hmm.

111 Ball: Okay [referencing the folded paper]. So can you write down how much of the piece of paper is on this side, or can you just tell me how much that is? Just this amount of the paper. How much of the paper is that?

112 Brandon: Wh – half or, umm, that's like fifty percent?

113 Ball: Can you write those down?

114 Brandon: Okay.

115 Ball: You can use this paper to write with.

[Brandon writes 50% on the left side of the paper.]

116 Ball: Okay. And what's the other thing?

117 Brandon: 50 percent.

[Brandon writes 50% on the right side of the paper.]

118 Ball: Okay. And you also said half. How would you write half?

119 Brandon: Half. Umm…

[Brandon writes on the right and left side of the paper.]

120 Ball: Okay. Is there any other way you could write it?

121 Brandon: Yeah.

122 Ball: Oh I see, you're writing it for each side, right? Is that what you're doing?

123 Brandon: Mm-hmm.

[Brandon writes 0.50 on the right and left side of the paper.]

124 Ball: Okay. You mentioned – A minute ago you mentioned that you could fold it into fourths. Can you show me that one? How you'd do that?

[Brandon folds the same paper into fourths.]

125 Ball: Okay, so what did you do to that?

126 Brandon: Umm, it was already like this from the half, so I folded that into – into right here –

127 Ball: Mm-hmm.

128 Brandon: – and turned it into fourths.

129 Ball: Why did it turn into fourths?

130 Brandon: Cause since it – it already had a line right here from the – from half, so if I fold it in this one it'll have another line right here and that separates them into fourths.

131 Ball: Okay. So what – how would you – how could you explain – how could you write down what amount that each of those portions of the paper are? Do you want that pen?

Can you put it up here? [Referring to the projector.]

132 Brandon: Oh, sure.

[Brandon moves paper up to projector. He writes 0.25 and $\frac{1}{4}$ on the top left quarter of the paper.]

Or umm?

[Brandon writes 25% on the top left quarter of the paper.]

133 Ball: Okay. Now when you think about, umm, the paper you just had – this one – are there any other fractions you could use to represent how much? Actually we should – let's fold this one. I think I should've just let you do it on the one you had. Okay, are there any other fractions you can use to write down how much this part of the paper is? What else could you write?

134 Brandon: Umm, you could write other – You could write, umm, one number and what would be half of it.

135 Ball: Like what?

136 Brandon: Like, umm, say for all of this could be – like, all of this could be thirty-eight, but, like only – only nineteen of it – this is nineteen and this is nineteen.

137 Ball: So how would you write that fraction?

138 Brandon: Ummm.

[Brandon writes $\frac{19}{38}$ on the right and left side of the paper.]

139 Ball: Okay, and – go ahead. Okay, and is there another one you can do?

140 Brandon: Hmm. I'm thinking of one.

[Brandon writes $\frac{32}{74}$ on the right and left side of the paper.]

141 Ball: So how did you do that one?

142 Brandon: 'Cause, umm, wait? No, actually no, it's thirty-seven. Thirty-seven.

[Brandon corrects both $\frac{32}{74}$ fractions to $\frac{37}{74}$.]

143 Ball: So how – how did you do this one?

144 Brandon: Umm. 'Cause this – all of this could be equal to – is seventy-four, but half of it would be thirty-seven.

Time: 00:16:44

145 Ball: So what's the general thing you're doing? You seem to have some way you can always write a fraction that's that amount. What's the general thing you're doing?

146 Brandon: It's like, umm, see the whole would be seventy-four, but umm... like that if you – Since it's one side is this and one side is that, when you add these two together it equals that, so – so right there you know that half of it is thirty-seven, the other half is...

147 Ball: Okay. Can you do similar things with one-f – with the fourths? What would be an example of something like that? Can you write another fraction for one-fourth?

148 Brandon: Okay.

149 Ball: How would you do it with fourths? Just a minute. Okay. [adjusts paper on the projector]. Okay, so what did you do this time?

[Using the paper folded into fourths, Brandon has written $\frac{2}{8}$ on the overhead.]

150 Brandon: Umm, two-eighths 'cause it takes four – it take four twos to equal eight, so two would be 25 percent –

151 Ball: Okay.

152 Brandon: – or one-fourth.

153 Ball: Okay. Let's go to a different... can you get another piece of paper? Let's do one more of these paper-folding things. Do you have a blank one over there somewhere?

154 Brandon: Yeah.

155 Ball: Could you fold the piece of paper in thirds?

156 Brandon: Thirds? I think so.

157 Ball: I'm sorry that chair is too tall. Do you remember how we adjusted it?

158 Brandon: Yeah...

159 Ball: Oh, I know. It's that lever over on your left – on your right. I think if you – If you pull on that… Are you uncomfortable? Okay.

[Brandon folds paper in thirds. He estimates the first fold, and sees that the next fold will not come out even with the paper's edge. So he then adjusts the first fold to correct before finishing.]

160 Ball: You can put it on the desk if you want. Okay, so how did you do this one?

161 Brandon: Umm… One – I put one right here and then folded it.

162 Ball: It seemed like this was a little bit more difficult than folding the half or the fourth. Why was that?

Time: 00:19:00

163 Brandon: 'Cause you – you can't exactly – See it – with thirds it's like you try to fold it right – try to fold it right – in right where you, umm, put the crease into it so you know that all the sides would be even, but with fourths all you have to do is just fold it like this –

164 Ball: Mm-hmm.

165 Brandon: – and – but half is just like this.

166 Ball: What do you get if you fold it like you did – remember when you had the half and you folded it this way? What do you get if you do that this time?

167 Brandon: You get sixths?

168 Ball: Okay. Can you write, umm, like [we just worked – we'll spend a minute on this] can you write what fraction would you put for that portion?

169 Brandon: I think this would be…

[Brandon writes $33\frac{1}{3}$ in each third of the paper.]

170 Ball: Thirty-three and a third is…

171 Brandon: Mm-hmm.

172 Ball: So there – this is…

173 Brandon: Because when – if you just – 'Cause all this is – I think it's one-hundred, but when you put thirty three into it, it's – it's – it – it'll go on and on with threes –

174 Ball: Mm-hmm.

175 Brandon: – so what would be better is to just put one-third.

176 Ball: Okay. Are there any other fractions you can write for this?

177 Brandon: I don't think… Mm-mm.

178 Ball: No? How much would you write for this much of it; for two of the sections?

179 Brandon: So that'll be six, so…Umm. I'm not sure.

180 Brandon: Umm.

[Brandon folds the paper into sixths as he is working on finding an answer.]

181 Brandon: I think sixteen.

182 Ball: You think what?

183 Brandon: Sixteen.

184 Ball: Sixteen?

185 Brandon: Mm-hmm.

186 Ball: How would you write it?

187 Brandon: 'Cause – since this is thirty-three, then I think it would be sixteen because I just thought that if fifteen – it's – it's almost, umm, half of thirty-three, so if this was sixteen… So add it all together they would equal – I think they would equal ninety-si –

[Brandon writes 16 into each sixth of the paper.]

188 Ball: Mm-hmm.

189 Brandon: No. Ninety-six?

190 Ball: Mm-hmm.

191 Brandon: So in – if I put seventeen then they'll be one-hundred and something, so I'll be over.

192 Ball: Okay. So let's just keep these available for a minute. So you have this one – here's the one you made in halves,

193 Brandon: Mm-hmm.

194 Ball: okay, here's the one you made in fourths, and this one has what? Thirds and sixths?

195 Brandon: Mm-hmm.

196 Ball: Okay, so I'm going to write a fraction down and then I'd like to see if you can show me what part of one of these pieces of paper you could use for that fraction.

197 Brandon: Mm-hmm.

198 Ball: So give me a clean piece of paper please. I'll show you – we'll start with one that's easy.

199 Ball: So here's one you already did, just you see what I'm saying. If I write that fraction, can you – here, let's put these on your side. Can you pick up a piece of paper and show me which – how to use one of those pieces of paper to show one-half?

[Ball writes $\frac{1}{2}$ on the paper.]

200 Brandon: This one?

201 Ball: Okay. So now I'm going to write a different one, okay? Can you use one of your pieces of paper to show that?

[Ball writes $\frac{2}{4}$ on the paper.]

202 Brandon: I think this one solves both of those.

203 Ball: Okay, how?

204 Brandon: Because when one-half is two-fourths – when it's reduced – so it's the same thing, only just the numbers are bigger.

205 Ball: Okay. What about this?

[Ball writes $\frac{2}{3}$.]

206 Brandon: I think [when it] – this in th – in thirds . . .

207 Ball: Mm-hmm

208 Brandon: . . . this in thirds, so it would be these two – these two parts would be two-thirds

209 Ball: Okay. What about . . . ?

[Ball writes $\frac{4}{4}$.]

210 Brandon: That would equal a whole.

211 Ball: Okay. Okay so how do you know that was a whole?

212 Brandon: Because, umm, all fractions are pieces of a whole, so – just – I knew that half wouldn't be, so – but all of those just equal into one.

213 Ball: Okay. So what if I wrote down this fraction? Could you show me that with the paper?

[Ball writes $\frac{3}{2}$.]

214 Brandon: Wait, how would it go into it?

215 Ball: Could you show me that much paper with the . . . 'Cause you've been showing me one-half of a piece of paper, you showed me two-thirds of pa – of a piece of paper. Can you show me that much paper?

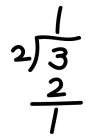
216 Brandon: Mm-hmm.

217 Ball: Can you put your paper up here?

218 Brandon: Mm-hmm.

219 Brandon: You would have to put two into three in order to find what – how mu – how mu – how much a piece of pach – paper would be. So it'd be one and a half.

[Brandon has written out the long division of 3 by 2:]



220 Ball: Okay. So how would you show that with the paper?

221 Brandon: It's the whole and this part would be a half.

222 Ball: Okay. How would you read that number that I wrote? Can you read that number?

223 Brandon: Three-twos, I think.

224 Ball: Okay. So, umm, let's, uh, work on the board for a few minutes. It's getting kind of boring sitting here. Umm, where's our chalk? I thought we had some chalk somewhere. Here. So, umm, here's where we have a bunch of fractions we were just talking about. Can you, umm, take, let's say one-half and two thirds. Could you write them up there?

[Brandon writes $\frac{1}{2}$ and $\frac{2}{3}$ on the chalkboard.]

225 Ball: Okay. So now which one of those two do you think is great – is larger?

226 Brandon: Larger? Umm. I think one-hal – I think two-thirds?

227 Ball: Why do you think two-thirds is larger?

228 Brandon: Because half would just be half of something, but one-and-a-half is – is half a third, so – but it's two – so, two would be bigger.

229 Ball: Okay. What about if we add, uh, three-fourths to that? Where would you put three-fourths?

230 Brandon: Let me see.

[Brandon writes $\frac{3}{4}$ to the right of $\frac{1}{2}$ and $\frac{2}{3}$.]

231 Ball: Okay. So what are you saying about three-fourths?

232 Brandon: I think this would be a little bit bigger.

233 Ball: Than?

234 Brandon: Than … Three-fourths would be bigger than two-thirds 'cause, umm, it's just – this is just over one-and-a-half by a half, but this is over – this – 'cause this is two thirds. Two-thir – I mean two fourths would be half, but three is over two so it's over half by one …

Time: 00:27:20

235 Ball: Okay. All right, let's sit back down again for a second. I'm going to show you some pictures of some different things. We'll get away from the paper

folding for a minute. And then maybe we'll do a little more comparing, like we were – remember we were doing it before this morning.

236 Brandon: Mm-hmm.

237 Ball: So, umm, like this for example. Can you tell me if this whole thing is the whole?

[Ball shows Brandon this figure:]

238 Brandon: No.

239 Ball: Okay. So can you tell me what that fraction would be?

240 Brandon: It would be . . .

241 Ball: The shaded part is what I'm talking about.

242 Brandon: Oh.

243 Ball: Sorry.

244 Brandon: Sixty percent?

245 Ball: Fifty percent?

246 Brandon: Sixty . . .

247 Ball: . . . Percent?

248 Brandon: Mm-hmm.

249 Ball: Why do you think it's sixty percent?

250 Brandon: 'Cause five – it's five whole things and all those equal to one, so each one is worth twenty percent.

251 Ball: Mm-hmm.

252 Brandon: So since it's three of those it's sixty.

253 Ball: Okay. Is there a fraction you could write for that?

254 Brandon: Mm-hmm.

[Brandon writes $\frac{60}{100}$.]

255 Ball: Okay. What if somebody wanted to write a fraction that had a five in the denominator since, like you said, there's five parts? What fraction could you write if that was a five in the denominator?

[Brandon writes $\frac{3}{5}$.]

256 Ball: Okay. What if there was a ten in the denominator? Could you make a fraction that had a ten in the denominator?

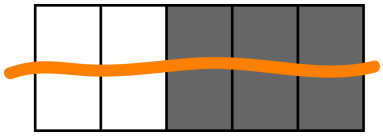
257 Brandon: Ten? Would you – would you call these ten?

258 Ball: No, these would – You have five parts. How could you make it so that, umm, you represent it as a number of tenths? You can change the drawing if you want to.

259 Brandon: I think you can dr – you draw a line through this.

260 Ball: Okay, go ahead. You can do that.

[Brandon draws a line through the figure:]



261 Ball: Okay. So now could you write a fraction that was over ten? To –

262 Brandon: Yeah.

263 Ball: – represent the shaded part?

264 Brandon: Mm-hmm.

[Brandon writes $\frac{6}{10}$.]

265 Ball: Okay. How did you decide how to do that?

266 Brandon: 'Cause if – when I put a line through it, it makes it ten so this, even though these – all this is – is six, it's six of them shaded with the line going through it. And without the line it's – it's just, umm, three over five.

267 Ball: Okay. Umm, of these three fractions you've written, which one's the largest?

268 Brandon: Three-fifths?

269 Ball: Why is it the largest?

270 Brandon: 'Cause three-fifths is like – like we said earlier, it – 'cause one-hundredths are really small…

271 Ball: Mm-hmm

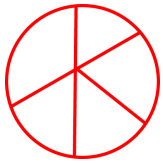
272 Brandon: I mean it – In – I – my opinion it's not – it's not about the numerator, I think it's about the denominator.

273 Ball: Tell me – I know you were telling me that earlier, so keep going 'cause we didn't really get to talk about that.

274 Brandon: Okay. So this would –

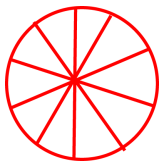
275 Brandon: That would be fifths

[Brandon draws:]



276 Brandon: and – this is tenths,

[Brandon draws:]



277 Brandon: so – so it's six of that, even though the numer – the numerator's bigger than this numerator, it – my opinion is that the denominator – how big the denominator determines how – how big the fraction – the whole fraction is.

278 Ball: So, what were we talking about earlier? Do you – is that paper around still? We were comparing, umm, let's pursue this a little bit. Is that one-half and seven-eights, or something like that? Or one-quarter and seven-eighths?

279 Brandon: Oh. They were on the – they were on those little cards.

280 Ball: Oh yeah, that's right. Is it over by you?

281 Brandon: I'm not sure.

Ball and Brandon transition from working on the projector to working on the chalkboard. In this next segment Ball continues to lead Brandon through a series of activities where he draws pictorial representations of selected fractions to explain his understanding of the numerator and the denominator.

---

282 Ball: Here they are. Yeah. Here, why don't – You want to put them up on the board?…Okay. So I asked you, I think, to put them up and put the smaller one on the, uh, right. I meant on the left, yeah. So you put them up, like… oh okay, is that what you meant before?

283 Brandon: No.

284 Ball: Okay.

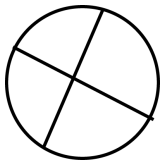
285 Brandon: 'Cause this would be…

[Brandon places the $\frac{7}{8}$ card on the left of the $\frac{1}{4}$ card on the board.]

286 Ball: Okay, you put them sm – okay, right, sorry. Okay. So then tell me again what you were explaining, because I didn't f – I didn't completely – we didn't get to really finish talking about it.

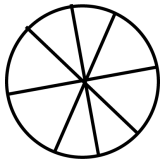
287 Brandon: Umm. What I understand is that fourths – fourths would be like that

[Brandon draws:]



288 Brandon: and eights is like this.

[Brandon draws:]



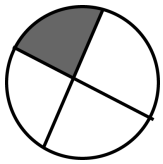
289 Ball: Okay.

290 Brandon: So s – Like, I said, umm, I think that f – the d – the, umm, denominator determines how big the whole fraction is.

291 Ball: Okay. So where does that lead you with these two fractions?

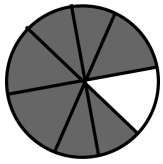
292 Brandon: 'Cause fourths – 'cause, like, fourths are bigger than eights so –

[Brandon colors the circle:]



293 Brandon: so even though it's just . . .

[Brandon colors the circle:]



294 Brandon: So I think that –

295 Ball: So that's your seven-eighths . . .

296 Brandon: Uh-huh. And this is one-fourth.

297 Ball: Okay. But when I look at your drawings, it looks like you've shaded more for your picture of seven-eights than for one-fourth. That's the part I'm not completely understanding.

298 Brandon: 'Cause – because it's – you have fourths – I meant – I mean eighths is – eighths are a lot smaller, so seven of them would have to – you have to shade in 'cause you couldn't put seven into four.

299 Ball: Okay. So then you have this piece of paper from earlier you – remember where you divided the paper into fourths and you told me that one of these could also be – you could write it as two-eighths, right?

300 Brandon: Mm-hmm

301 Ball: So what if we say instead of one-fourth, I take – 'cause you said that would be the same, right?

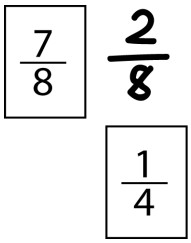
302 Brandon: Mm-hmm

303 Ball: Do you s – do you still think that's the same?

304 Brandon: Right.

305 Ball: So what if I put two-eighths here instead?

[Ball moves the $\frac{1}{4}$ card down and writes $\frac{2}{8}$ to the right of the $\frac{7}{4}$ card.]



306 Ball: Now how would you compare seven-eighths and two-eighths? 'Cause then the denominators aren't different?

307 Brandon: I think that seven-eighths would be bigger.

308 Ball: Okay.

309 Brandon: Because they're – they both have eighth, but, umm, si – I mean seven would be bigger than two. They have the same denominator.

310 Ball: Okay. So in general, when they have the same denominator, how do you compare them?

311 Brandon: By the numerator.

312 Ball: But when the no – denominators are different, what is it you do?

313 Brandon: I compare them by the denominator.

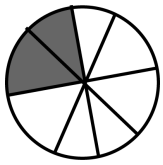
314 Ball: Okay. So then this is the part I'm not – I don't think get completely, 'cause if I understood you correctly, you said two eights was the same as one-fourth. Or were you not saying that?

315 Brandon: Umm. It's –

316 Ball: Do you have a picture of two-eighths up on the board? Can you make a picture of this? [She points to the $\frac{2}{8}$ written on the board.]

317 Brandon: Okay.

[Brandon draws on chalkboard:]



318 Ball: Okay.

319 Brandon: So...

[Brandon finishes his drawing.]

320 Ball: Okay. So now this is your picture of one-fourth. Can you label it?

321 Brandon: Mm-hmm.

322 Ball: Your picture – this is a picture of... Okay. So now, what I'm trying to understand is when you compare these two [pointing to Brandon's drawings of $\frac{1}{4}$ and $\frac{2}{8}$], what do you conclude about which one's bigger?

323 Brandon: How do I determine it?

324 Ball: Yeah. How do you decide? Like, which one is bigger? 'Cause I thought you were saying they were the same 'cause you labeled this one one-quarter and then you labeled this one two-eighths, so I thought you were saying those are different ways to write the same amount? [Showing Brandon the paper he had written on previously showing that $\frac{1}{4}$ is the same as 0.25]

325 Brandon: No, it's 'cause this would be bigger, but it can be – what I'm saying is that it can go smaller.

326 Ball: Say more about – what do you mean, "it can go smaller"?

327 Brandon: Like... I can't really decide on how big they are when they're reduced.

328 Ball: Mm-hmm. When you look at these two pictures, which one do you think is greater: two-eighths or one fourth?

329 Brandon: Umm... One-fourth?

330 Ball: Why do you think one-fourth?

331 Brandon: Umm. 'Cause it has – it has bigger chunks into it to make fourths, so – but these are all, like li – sorta small, so just one out of four is bigger than two out of eight.

332 Ball: Okay. Let's, umm, let's go over and use this line. Remember when I talked to you on the phone and you said you don't usually use the line so much?

333 Brandon: Oh a number line?

334 Ball: Yeah. Or do you – did you say you did use it?

335 Brandon: I don't think we used it.

336 Ball: Okay. I'm just going to mark a few points and then we can go from there. So I'm going to call that point zero, and then I'm going to call this one one, okay? And then I'll call this one two, and that's all I'm going to put on there for right now, okay?

[Ball draws on the board:]

337 Brandon: Mm-hmm.

338 Ball: So. Like where would three be if we were to put it on?

[Brandon adds a 3 to the number line:]

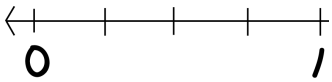
339 Ball: Okay. Now do you think there's some numbers between these?

340 Brandon: Mm-hmm

341 Ball: Like what?

342 Brandon: Umm . . .

[Brandon begins to draw on chalkboard. He adds three lines between 0 and 1.]



343 Brandon: So I think that this part would be zero . . .

344 Ball: Mm-hmm

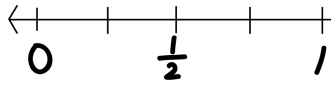
345 Brandon: And this would be – I think it would be – so what number would you declare it to be whole?

346 Ball: Well why don't we – what would you call this number? Do you a n – a name – what you would call this [pointing to the line Brandon has drawn in

the middle, between 0 and 1]? If this is one – this still has to be one and that's zero so what would this be? [Again, referring to the middle line.]

347 Brandon: I think this would be half.

[Brandon writes:]



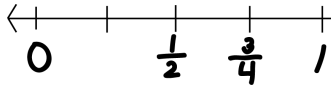
348 Ball: Okay. So then…

349 Brandon: Just half.

350 Ball: Just half. And why did you think that would be one-half?

351 Brandon: Because this would be, umm, this would be…

[Brandon writes:]



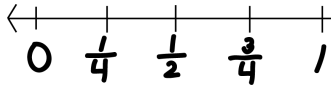
352 Brandon: So it's – so like this is half [pointing to the middle line]…

353 Ball: Mm-hmm

354 Brandon: This is like – this – this side is bigger – I mean the same – they have the same side, but this is closer to one, so…

355 Ball: Okay. Then what's that one there then?

[Brandon writes:]



356 Brandon: One-fourth.

357 Ball: Okay. And how did you decide to label this one one-fourth and this one three fourths?

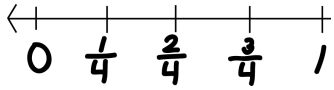
358 Brandon: 'Cause one fou – I mean… 'Cause four of these equal one, so this is just one section of it, so it'd be one, and this would be half point, and this is fourths, and then this would equal one.

359 Ball: What did you mean four of them equal one? What did you mean by four of them?

360 Brandon: Four is… So – I probably should change it. I need an eraser.

Time: 00:38:34

[Brandon changes his number line:]



361 Ball: Why did you change that?

362 Brandon: 'Cause, umm, I wanted them all to be fourths . . .

363 Ball: Oh.

364 Brandon: . . . so we wouldn't get confused, so . . .

365 Ball: How did you decide the denominator should be fourths? Why not fifths or sevenths or something like that?

366 Brandon: Well I think . . . Because in some numbers you can't – you can't put the numerator into the denominator . . .

367 Ball: Mm-hmm

368 Brandon: . . . so – but this would be easier.

369 Ball: So, was one-half wrong where you had it? I mean is – is this point – could you call this one-half also?

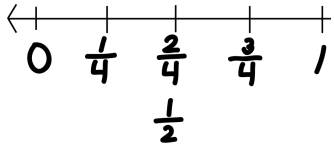
370 Brandon: Mm-hmm

371 Ball: So why don't you put it back, like underneath it?

[Brandon had erased the $\frac{1}{2}$ he had written and replaced it with $\frac{2}{4}$. Ball was now asking that he add $\frac{1}{2}$ back to his number line, positioning it just below the $\frac{2}{4}$.]

372 Brandon: Okay.

[Brandon writes:]

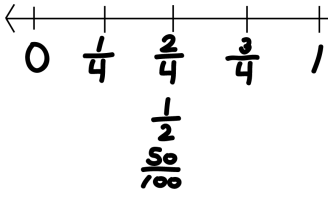


373 Ball: Is there any other fraction you could call that point besides two-fourths and one-half?

374 Brandon: Umm, yeah. I call it – you can call it fifty-out-of-a-hundred, or . . .

375 Ball: So why don't you write fifty-out-of-a-hundred too, then.

[Brandon writes:]



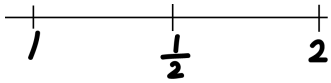
376 Ball: So this is – goes back to the thing you were telling me earlier in general. Like you can take some number and then half of it and then . . .

377 Brandon: Mm-hmm

378 Ball: Okay. So, uh, what number would be here? [She draws a mark on the number line, half way between the 1 and the 2.] Here, I'll use a different color from you. What number would you say would be here?

379 Brandon: This would be half point 'cause it's the same size or bigger, so, umm . . .

[Brandon writes:]



380 Brandon: So it's like – it's not exactly right here [pointing to a space between $\frac{1}{2}$ and 2] where it's like three out of four and then it's not exactly one out of four [pointing to a space between 1 and $\frac{1}{2}$], so it's half.

381 Ball: Are you saying it's the same number as that number? [She points to the $\frac{1}{2}$ mark between the 1 and the 2 and refers back to the $\frac{1}{2}$ mark between the 0 and 1.]

382 Brandon: Yeah.

383 Ball: 'Cause you have one-half there.

384 Brandon: Mm-hmm.

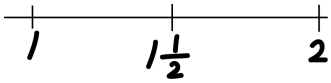
385 Ball: But this number is greater than one. See, it's past the one [indicating that the new mark between 1 and 2 lies to the right of the 1 mark].

386 Brandon: Yeah.

387 Ball: So, is it just one-half?

388 Brandon: It'll be one-and-a-half.

[Brandon writes:]

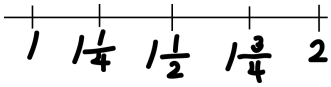


389 Ball: Oh, it'd be one-and-a-half. Okay, so if we repeat the same thing you did before, could you label that one and that one?

390 Brandon: Mm-hmm.

391 Ball: What would that be?

[Brandon writes:]

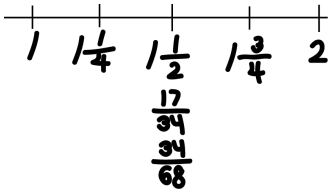


Time: 00:40:42

392 Ball: Okay. So, umm, is there anything else you could write – you know how you wrote different ways to write one-half? Are there for one-and-a-half?

393 Brandon: Yeah.

[Brandon writes:]



394 Ball: Do you need the one in front of it, or you don't need the one in front of it?

[Brandon shakes his head in the negative.]

395 Ball: So this is just seventeen thirty-fourths?

396 Brandon: Mm-hmm. It's – that's half.

397 Ball: Okay.

398 Brandon: Mm-hmm.

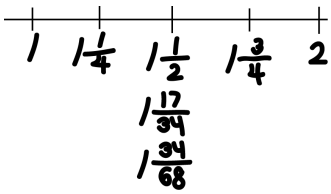
399 Ball: But it's greater than one, right? So . . .

400 Brandon: Mm-hmm.

401 Ball: So . . .

402 Brandon: It's one.

[Brandon writes:]



403 Ball: Okay. All right. So, umm, can you think of any other fractions that you could put up here? Like, is there any fraction that goes between one-fourth and two-fourths, for example?

404 Brandon: Umm, mm-mm. No.

405 Ball: No? Er, have we put all the fractions up here that we can?

406 Brandon: Yeah.

407 Ball: Well what if we wanted to put one-third up there? You think it couldn't – we couldn't put it on there?

408 Brandon: Sure, but we would probably have to put them into thirds instead of fourths.

409 Ball: Okay, so you can do that. Here, let's – what color were you using? Pink? Here, just use this.

410 Brandon: Okay.

411 Ball: Can you – can you mark it so that you'd be able to put thirds one there?

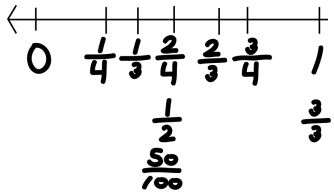
412 Brandon: Do I need to erase this?

413 Ball: You don't have to erase it.

414 Brandon: Okay.

415 Ball: You can kind of ignore what you've done; now concentrate on this. Okay, so what would you label those two points?

[Brandon writes:]



416 Brandon: This section would be one-third. This would be two-thirds. And then this would be three-thirds right here.

417 Ball: Okay.

418 Brandon: Whole.

419 Ball: If you wrote this with – in halves, what number would you write there? So you wrote three-thirds, could you write some number of halves there? Something over two?

420 Brandon: Mm-mm

421 Ball: Could you write something over four there?

422 Brandon: Over four? Yeah.

423 Ball: What would you write over four?

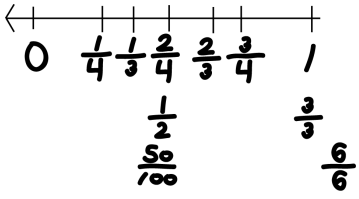
424 Brandon: For half?

425 Ball: . . . yeah. For one. Like instead of three thirds, could you write a fraction that's . . .

426 Brandon: That's . . .

427 Ball: . . . over four, like that?

[Ball adds $\frac{6}{6}$ to the number line:]



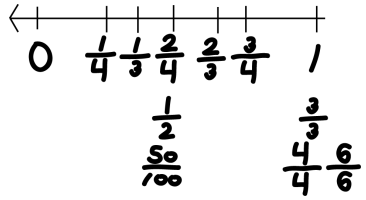
Time: 00:42:53

428 Brandon: No, 'cause this is all one, so three-thirds or four-fourths, or . . .

429 Ball: Okay. You can write four-fourths.

430 Brandon: Or . . .

[Brandon writes:]



431 Ball: Okay. That's supposed to go, like here? It's the same?

432 Brandon: Mm-hmm.

433 Ball: Okay. So now I'm going to give you some cards with fractions on them, and you don't have to find exactly where they would go, but I'd like you to – here I'll show you. Where're those ones with the magnets that we had? These? Okay. So here I just want you to put it sort of in the right place where you think it would go. Like is it more than one, is it less than one? Is it more than a half, is it less than a half? It'll be hard to tell exactly where it should go, but about where do you think that fraction goes? What is it? Can you read it to me?

434 Brandon: Eight-ninths.

435 Ball: Okay. So wh – approximately where do you think you would put that? For example, is it – which side of one does it go on?

436 Brandon: It goes right here because if it's – it's not – it's not – it's – it's not nine-ninths . . .

[Brandon places the $\frac{8}{9}$ card directly to the left of the 1 on the number line.]

437 Ball: Okay.

438 Brandon: . . . so it wouldn't be one.

439 Ball: So nine-ninths would be one? And how do know it's more than a half? 'Cause you put it – like here's a half. You put it quite far over from a half.

440 Brandon: Yeah. 'Cause four-and-a-half is half of nine, so – but this is eight so it's over –

441 Ball: Okay.

442 Brandon: it's over . . .

443 Ball: All right. Where would you put this one? Can you read that?

444 Brandon: Three-fifths.

445 Ball: Yeah. Where would you put that one?

446 Brandon: [right] two, three, four… Hold on.

447 Ball: I mean again roughly. Like is it close to one? Is it close to zero? Is close to a half?

448 Brandon: So these lines right here would be representi – representative –

449 Ball: So about where do you think it goes? Which side of one-half does it go on?

450 Brandon: I think it's a li – it's a little over half.

[Brandon places the $\frac{3}{5}$ card directly to the right of the $\frac{1}{2}$ on the number line.]

451 Ball: Okay.

452 Brandon: 'Cause two-and-a-ha – two-and-a-half is five – is half of five, so…

453 Ball: Okay.

454 Brandon: …but it's three so it – it would be over.

455 Ball: Where would you put that one?

456 Brandon: Right here.

[Brandon places the $\frac{19}{19}$ card directly under the 1 on the number line.]

457 Ball: What is that?

458 Brandon: Nin – nineteen-nineteenths.

459 Ball: Okay. Why does that go there?

460 Brandon: 'Cause it's whole.

461 Ball: Okay. And where would you put this one? What is that fraction?

462 Brandon: Three-twos?

463 Ball: Okay. Where would you put it?

464 Brandon: So, okay. So which one is the whole number? Which…

465 Ball: It's divided into parts that are halves. So…

466 Brandon: To find this answer – to find this answer you would have to divide two into three. So…

[Brandon writes on chalkboard:]

$$2\overline{)3}\quad 1\quad \frac{2}{1}$$

467 Brandon: So it would be one-and-a-half.

468 Ball: Okay. So where would you put it?

469 Brandon: [Gap in audio] – put this right here.

470 Ball: Okay. I have a – one or two more to show you.

[Ball goes for more cards.]

471 Ball: Oh. Where would you put that one. What is that?

472 Brandon: Zero-fourths.

473 Ball: Yeah. Where would you put that?

474 Brandon: I'd put this right here.

[Brandon places the $\frac{0}{4}$ card directly under the 0 on the number line.]

475 Ball: Why?

476 Brandon: Because, umm, it's not a percentage yet, so its –

477 Ball: It's not a what?

478 Brandon: I mean it's not a part of it.

479 Ball: Uh-huh.

480 Brandon: It's not like – the numerator doesn't have a number on top of it.

481 Ball: And what does that tell you?

482 Brandon: It tells me that it's not part of a fraction yet, so it would just be – it would just be zero.

483 Ball: Okay. Do you want to a really strange one, and then we can stop with this? What does that say?

484 Brandon: Two – eight – two-hundred-eighteenths over two-hundred-six?

485 Ball: Now I just want to know roughly, do you think it's less than one? More than one? Is it more than two?

Time: 00:47:05

486 Brandon: I think it's more than one 'cause it's – 'cause two-hundred-and-six can't go into two-hundred-eighteen two times, so . . .

487 Ball: Okay. So where would you put it, about?

488 Brandon: I would put it – just randomly I would put it right here.

[Brandon places the $\frac{218}{206}$ card directly to the right of the $1\frac{1}{2}$ on the number line.]

489 Ball: Why did you put it so close to two? Like how did you decide to put it all the way over there?

490 Brandon: 'Cause I'm not really sure 'cause it's not really – it's not – I'm not sure if it's over half or before half.

491 Ball: So what if I asked you to write a fraction, let's say, that was – well let me write it in a different spot. What if here where you've been writing these

other fractions I asked you to write a fraction that was the same as one that had two-hundred-six in the denominator? What would you put in the numerator if you wanted it to be the same amount as all these other numbers here?

[Brandon writes $\frac{206}{206}$.]

492 Ball: Why did you write that?

493 Brandon: If I wanted it to be whole, then it would have to be two-hundred-and-six.

494 Ball: Okay. So now if you look at this one, does that help you know about how big that one is, or not really?

495 Brandon: Not really.

496 Ball: Okay. Why don't – You want to switch and do something else for a while?

497 Brandon: S – sure.

Time: 00:48:18

498 Ball: Did you say you hadn't been working with number lines or you had worked with number lines?

499 Brandon: We haven't.

500 Ball: How – was that hard to do, what we were doing?

501 Brandon: No, 'cause – 'cause most of – I think – in my opinion it's sort of like common sense.

502 Ball: Uh-huh.

503 Brandon: It's like – if it's one then it would just be – if you put, like, four points – it depends on how much points you put.

504 Ball: Mm-hmm.

505 Brandon: If you put five then we'll know that it'll take a certain amount – five to equal one, or four – you put four points then it'll be –

506 Ball: Well, you know this thing we were having over here about the eighths?

507 Brandon: Mm-hmm.

508 Ball: I wonder how that looks on here. So you said, umm, you were trying to talk about two-eighths and one-fourth. So where would you put two-eighths on this number line?

509 Brandon: Well, let me see.

[Brandon divides the sections between 0 and 1 on the number line into eighths.]

510 Ball: Okay, so . . .

511 Brandon: The white points would be – these are eighths.

512 Ball: Okay. So can you label them?

513 Brandon: Mm-hmm

[Brandon adds more fractions to the number line, but the number line begins to appear cluttered.]

Time: 00:50:01

514 Ball: I think we're getting too many things on here, maybe.

515 Brandon: Probably.

516 Ball: It's hard to keep track of. Umm, maybe let's just make a clean one 'cause I think we have too many things on here and I think y – it was hard to see what you were doing. Let's come back – well let's go over here and we'll just work on a little piece of it and see if we can talk about this eighths. So… There's zero, and I'll just do it between zero and one this time, okay?

517 Brandon: Mm-hmm.

518 Ball: So I'm just going to put back a couple things we had. We had one half, and we had th – uh, what was this?

519 Brandon: Umm, three-fourths

520 Ball: Three-fourths. We also had two-fourths. And we had one-fourth. That's all I'm going to put on for right now, okay? Or maybe we'll put a couple of your whole ones.

521 Brandon: Mm-hmm.

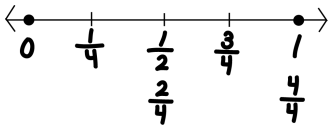
522 Ball: What did you have? Four-fourths?

523 Brandon: Yeah.

524 Ball: Okay. So is that enough now just to get oriented?

525 Brandon: Mm-hmm.

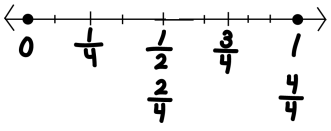
[Ball has drawn:]



526 Ball: So now try to make the eighths. Okay, do you want a colored one? So can you make that so you can represent eighths? Just do it carefully 'cause you were right, you wanted to make eight parts but I think you lost track a little bit.

527 Brandon: Two…

[Brandon divides the section between 0 and 1 on the number line:]



528 Ball: Okay. So, umm, this point right here [pointing to the mark] is what?

529 Brandon: It's half.

530 Ball: It's half?

531 Brandon: Mm-hmm.

532 Ball: And what's the pink one you just drew? What's that?

533 Brandon: That's half point.

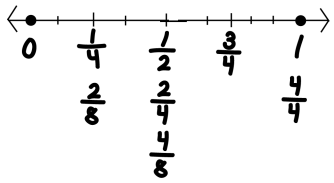
534 Ball: That's a half point? So in eighths what would you write?

535 Brandon: Four-eighths.

536 Ball: Okay. So what about this one right here?

537 Brandon: This is two-eighths.

[Brandon has written:]



538 Ball: Okay. So now you've got two-eighths and one-fourth at the same point, right?

539 Brandon: Yeah.

Time: 00:51:58

540 Ball: So how does that go back with what we were talking about with the pictures? 'Cause here, I think you were telling me that one-fourth was more, 'cause the fourths were bigger chunks.

541 Brandon: Mm-hmm.

542 Ball: But here you've got them at the same point, so I'm curious about that.

543 Brandon: Because – 'cause you put, like – if – it depends, like – where – how – how big are the points, so if they were in fourths, then one-fourth would be bigger . . .

544 Ball: Mm-hmm.

545 Brandon: . . . because eighths are like – fourths – some fourths are like this, but then eighths are like this.

546 Ball: But didn't you –

547 Brandon: So – but the space between them would be bigger.

548 Ball: Right. But you have that, right? See here you have one-eighth and then two-eighths. You have two-eighths right here at the same place. Is that in the wrong place, that two eighths?

549 Brandon: No. One-fourth is – one-f … Because I put them – this is in eighths, but this – if you put them in fourths then it would be right here.

550 Ball: Okay. So are they the same or they're not the same?

551 Brandon: They're not the same.

552 Ball: And why, on this drawing, do they come out looking like they're the same? On this number line?

553 Brandon: 'Cause if you s – if you put them in fourths then this would be one …

554 Ball: Mm-hmm.

555 Brandon: [counting the fourths along the number line] … two, three, and then …

556 Ball: What you have, right? You have one-fourth, two-fourths, three-fourths.

557 Brandon: All right, and then one whole …

558 Ball: Right. So here you have – when you divide it into eighths it looks like you've got one-eighth, two-eighths – is this right? – three-eighths, and the you already wrote four-eighths –

559 Brandon: Four …

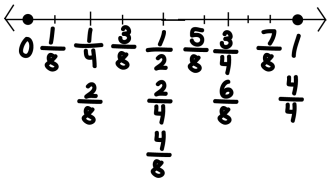
560 Ball: – then you would have five-eighths …

561 Brandon: Six …

562 Ball: … then six-eighths, and then seven eighths? Is that – is that correct?

563 Brandon: Mm-hmm.

[Ball has written:]



564 Ball: But then you do have – you have four-eighths at the same place as one – well what's bigger: one-half or four-eighths?

565 Brandon: I mean, the numbers are bigger, but they're both the same cause they're both the same 'cause they're both half.

566 Ball: Okay. So those you see as the same. Is two-fourths the same also?

567 Brandon: Mm-hmm.

568 Ball: Now back to this. You think these are different though –

569 Brandon: Yes –

570 Ball: – is that right?

571 Brandon: – yes because – I put – these are in fourths …

572 Ball: Mm-hmm.

573 Brandon: . . . what d – in a number line, what determines how big the fraction is that if – how big the space is.

574 Ball: Mm-hmm.

575 Brandon: 'Cause – from here [pointing to zero and referring to the distance between 0 and $\frac{1}{4}$] it's like – this is like one-fourth and two-fourths and three-fourths and one whole [counting up the number line by fourths], so the space – the space betwe – these are eighth [pointing to the distance between 0 and $\frac{1}{8}$], so – but this is one-fourth so they're in – since it's in eighths, the spaces in between it – it is smaller, so that's why one-fourth would be bigger [i.e. because the space between 0 and $\frac{1}{4}$ is bigger than the spaces between the eighths, one-fourth is bigger than two-eighths].

576 Ball: Okay. Let's – let – why don't we leave our number lines and drawings one the board and we'll switch gears a little? We've probably done enough of that for a while.

Time: 00:54:40

[Brandon and Ball: walk back to the projector.]

577 Ball: Okay. Did we get s – do we have any clean paper left?

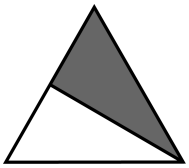
578 Brandon: Yes.

579 Ball: Okay. Now I'm just going to show you some pictures that – some of them are easier and some of them are harder . . .

580 Brandon: Mm-hmm.

581 Ball: . . . and I'd like you to try to think about how much of the whole it is and what fraction you could use to express that, okay? So, how 'bout this one?

[Ball shows Brandon a figure:]



582 Ball: Let's put a piece of paper under it in case you want to write something. So the triangle – the big triangle is the whole and I'm interested in what you think about the shaded part.

583 Brandon: It's half. [He writes $\frac{1}{2}$ under the picture.]

584 Ball: Okay. And how did you decide that?

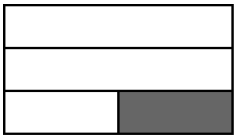
585 Brandon: 'Cause it's bo – it's evenly split –

586 Ball: Mm-hmm.

587 Brandon: – so one side is the same as this side, so . . .

588 Ball: Okay. What about this one? So the whole is this whole rectangle –

[Ball shows Brandon a figure:]



589 Brandon: Mm-hmm.

590 Ball: – okay?

591 Brandon: So would this be in – what's – is it – would this be in fourths, or . . . ? I'm not sure.

592 Ball: What are you not sure about? What is your question?

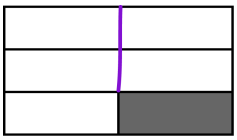
593 Brandon: Because these don't have a line in between the middle, but these two do, so . . .

594 Ball: You can do – you – you need to do something to the picture, you can.

595 Brandon: Okay.

596 Ball: If that helps you decide what you want to call that.

[Brandon draws a line through the rectangle:]



597 Ball: Okay. What did you do?

598 Brandon: I put a line though it.

599 Ball: And why – why did you want to do that?

600 Brandon: 'Cause this part doesn't have a line through it so it would – so – but this part does, so it wouldn't make sense for just this part to have a line through it and not the rest.

601 Ball: Okay. So now would you be able to decide how much the shaded part is?

602 Brandon: Mm-hmm.

603 Ball: What?

604 Brandon: It's one – it's one-sixth.

605 Ball: Can you write that?

Time: 00:56:36

[Brandon writes $\frac{1}{6}$.]

606 Ball: Okay. Okay, this one – this one might be a little bit trickier. Maybe not, I don't know.

[Ball shows Brandon a figure:]

607 Ball: So the whole is the whole rectangle, okay? And what you want to try to figure out is how much is shaded.

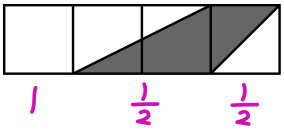
608 Brandon: Mm-hmm.

609 Ball: You can talk about it as you think about it, if you want to.

610 Brandon: I – I'm not really sure 'cause – I mean t – this – if this was a fraction [pointing to the second square of the picture], this [pointing to the last square of the picture] would be half of it 'cause it's sh – and this would be half, so . . . But this isn't shaded, so I guess this is one and –

611 Ball: Why don't you keep a little record of what you're saying? So you're saying this is one . . .

[Brandon writes:]



612 Brandon: Yeah. And that would be the half – that would be half.

613 Ball: Mm-hmm.

614 Brandon: And this would be half, so . . .

615 Ball: Okay. But you're – what are not sure about then?

616 Brandon: 'Cause – 'cause this isn't shaded, so – but this i – this part is shaded. This is half shaded.

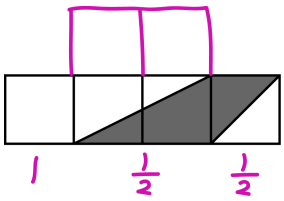
617 Ball: Okay. So if you just looked at these two squares right here?

618 Brandon: Mm-hmm.

619 Ball: How much of the whole are these two? Forget about the shading. So you have this whole rectangle. What if – how much of the whole are these two together?

620 Brandon: It's two squares.

[Brandon draws:]



621 Ball: Mm-hmm.

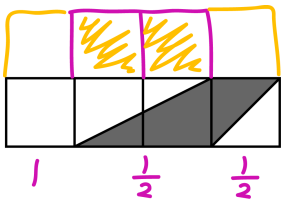
622 Brandon: Okay.

623 Ball: A – and how much of the whole is that? What fraction of the whole are those two squares?

624 Brandon: Which fraction is shaded?

625 Ball: Which-uh, yeah. Like if we shaded this… And there's your original rectangle. What fraction of the whole would be shaded?

[Ball has drawn:]



626 Brandon: Two-fourths.

Time: 00:58:27

627 Ball: Two-fourths, okay. But you just said only half of that is shaded, right? So what's half of two-fourths?

628 Brandon: Two – I mean half is two-fourths. One-fourth?

629 Ball: Okay. So here you've got one-fourth is shaded and one-fourth isn't?

630 Brandon: No.

631 Ball: No?

632 Brandon: I'm not really s – I'm not sure.

633 Ball: Okay. Why don't – should we leave that one? What about this one?

[Ball shows Brandon a figure:]

634 Brandon: I'm guessing they're six. They're six 'cause –

635 Ball: Do you want to draw some lines again?

636 Brandon: Sure.

[Brandon draws:]

637 Ball: Okay. So now would be able to say how much of the – this whole is shaded?

638 Brandon: This part is, like – so all of these are eighths, so I think – so five eighths?

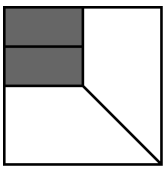
639 Ball: Okay. Does it matter that they're not next to each other?

640 Brandon: Mm-mm

641 Ball: How would you write five-eighths?

[Brandon writes $\frac{5}{8}$.]

642 Ball: Okay [puts a new example on the projector].



643 Brandon: Those would be – that would be half 'cause there're four segments of it –

644 Ball: There're what?

645 Brandon: There're four pieces of it and they all come together to be one.

646 Ball: Can you draw that? Can you – you want to put any lines on to show me what you mean? Or you're saying one, two, three, four [pointing to the different areas on the picture]

647 Brandon: Yeah. It – I mean they're different shapes, but there are four segments of it and they come together to make one square, so –

648 Ball: Uh-huh.

649 Brandon: – but two of the segments are shaded, so it would be two-fourths, or half.

[Brandon writes $\frac{2}{4}$ under the figure.]

650 Ball: Okay, how did you decide that? Because . . .

651 Brandon: Because since it's four of – segments then – and two are shaded so two out of four would be half.

652 Ball: Earlier when you were folding the paper you made the big point to me that, umm, like a little kid might fold them and not mi – might not make the parts the same size.

653 Brandon: Mm-hmm.

654 Ball: Do you think these are the same size there?

655 Brandon: Mm-mm.

656 Ball: You don't think they're the same size. Which ones do you think are different?

657 Brandon: These two – I mean these two from this – from there [pointing to the two small rectangles that comprise the shaded square] or these two from this – from this … [pointing to the unshaded quadrilaterals along the bottom and right side].

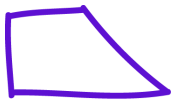
658 Ball: Okay. So how come you're – how come you're going to call those each one-fourth then if they're not the same part – the same size?

659 Brandon: 'Cause one – they're all one piece but there a line that puts them into – that separates them 'cause, it's like … This is one,

[Brandon draws:]

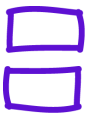
660 Brandon: this is one,

[Brandon draws:]



661 Brandon: and then this part is …

[Brandon draws:]



662 Brandon: they're all just kind of – there's a line that separates them –

663 Ball: Mm-hmm

664 Brandon: – That puts them into fourths, but since they're different sizes they're four – they're still four segments, so – and two of them are shaded, so it's two-fourths.

665 Ball: Okay. Do we have more paper? Are we running out?

666 Brandon: Yeah.

667 Ball: We are running out?

668 Brandon: Mm-hmm. Well we still have a few.

669 Ball: Okay. We're okay then. Okay. So if I write this fraction. Here, what is that?

[Ball has written $\frac{3}{6}$.]

670 Brandon: Half. Three-sixths or half.

671 Ball: Okay. Well actually I was going to ask you about that. Earlier you said something about reducing fractions.

672 Brandon: Mm-hmm.

673 Ball: How would you reduce that fraction?

674 Brandon: Find a number that c – that goes into both the numbers evenly.

675 Ball: Mm-hmm. So can you show me?

676 Brandon: So three – I picked three. Three can go into three one, and three go – can go into six two times.

[Brandon has written $\frac{1}{2}$ directly to the right of $\frac{3}{6}$.]

677 Ball: Okay. What if somebody didn't believe you that one-half was the same amount as three sixths? Is there some way you could show them?

678 Brandon: Mm-hmm. Or you could just – if you multi – multiply these like – no, that's wrong . . .

679 Ball: You can scratch it out if you want.

680 Brandon: Okay.

681 Ball: Okay. So how could you – what if somebody didn't think those were the same? Are you saying that they're the same?

682 Brandon: Mm-hmm.

683 Ball: How could – what if somebody didn't believe you? 'Cause, I mean, a lot of people might say those numbers are a lot bigger.

684 Brandon: Umm. I think – I'm not s – I'm not sure. There're so many ways, but . . .

685 Ball: Well what's a good way, do you think?

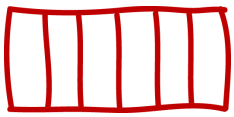
686 Brandon: Umm. You could – I could draw – draw, umm, a picture to explain it.

687 Ball: Mm-hmm.

688 Brandon: Or something that it – that'll show that three-sixths is the same as half.

689 Ball: So what do you think – what is one way that you could do it?

[Brandon draws:]



690 Ball: Okay.

691 Brandon: Those are sixths so . . .

[Brandon colors the rectangle:]

692 Ball: Okay.

693 Brandon: There's the fraction for how much of it is shaded then it would be three-sixths

[Brandon writes under $\frac{3}{6}$ the rectangle.]

694 Brandon: and all together it's six, so three would be half and the other three would be half.

695 Ball: Okay. All right. So why do you call that reducing?

696 Brandon: Reducing? Umm, 'cause you – if – 'cause some numbers can't be reduced, or some fractions can't be reduced, but redu – called reduce when it's reducing when you make it smaller, but if it's reduced then it's already small.

697 Ball: Okay, but are you actually making the fractions smaller? Like is one-half smaller than three-sixths?

698 Brandon: I mean, they're both the same, like, form, but they're – they're different numbers

699 Ball: Okay. S – can you write down a fraction that can't be reduced? Can you think of one?

700 Brandon: Mm-hmm.

[Brandon writes $\frac{5}{6}$ on the overhead.]

701 Brandon: Five-sixths.

702 Ball: Okay, why can't that be reduced?

703 Brandon: Because there's no number than can into five and six evenly.

704 Ball: Mm-hmm. Umm, can you reduce fractions that are a little less familiar? Like, for example, if I wrote a number like . . .

[Ball writes $\frac{24}{42}$.]

705 Ball: What does that fraction say?

706 Brandon: Twenty-four-s – er – uh – er – twenty-four-forty-twos?

707 Ball: Yeah. Can you reduce that?

708 Brandon: Yeah.

709 Ball: How would you do it? How could you tell so quickly that you could?

710 Brandon: Because they end and start with a number that – that – that a number can go into.

711 Ball: Okay. So what do you – what do you – what do you think you'd do to do it?

712 Brandon: One is that they're both even.

713 Ball: Okay, and why does that tell you it can be reduced?

714 Brandon: Well, if they're even, and some numbers that are odd can be reduced too, but . . .

715 Ball: Okay, but if they're even you're sure they can be reduced?

716 Brandon: Yeah.

717 Ball: Why is that?

718 Brandon: 'Cause if it ends with, like, a number with two, four, six –

719 Ball: Mm-hmm –

720 Brandon: – it can be reduced to . . . some could be even, but some numbers are odd that can be –

721 Ball: Okay.

[Brandon writes $\frac{6}{7}$.]

722 Brandon: Six-sevenths.

723 Ball: Okay. How did you do that?

724 Brandon: 'Cause I picked a number that can go into t – twenty-four and forty-two, and it –

725 Ball: What number did you pick?

726 Brandon: Six.

727 Ball: You picked six?

728 Brandon: Wait no. Actually it's . . .

729 Brandon: Well actually it's . . .

[Brandon corrects his work: $\frac{\not{6}^{4}}{7}$.]

730 Brandon: Four-sevenths.

731 Ball: Okay. What number did you pick?

732 Brandon: Six.

733 Ball: Okay. And what did you do?

734 Brandon: I… six can go into twenty-four four times, and six can go into seventy-two seven times.

735 Ball: Okay. All right. Umm, let's see. Are you tired? Do you want something to drink?

736 Brandon: Mm-mm.

737 Ball: No? Okay.

738 Ball: They're not being very good are they? Aren't they supposed to be quiet?

739 Brandon: They were quiet.

740 Ball: For a while. Maybe they'll be a little too quiet now. Okay. Well actually I was going to show something sort of tricky. I need another piece of paper. We're going to run out very fast. So just now you were reducing fractions…

741 Brandon: Mm-hmm.

742 Ball: …but, umm, sometimes when I've been teaching kids, they do other things to reduce fractions and I wanted to show you something that I saw a student do, and I want to know if you think you could do it this way. So I showed him this fraction –

[Ball writes $\frac{13}{43}$.]

743 Ball: – and he said, "Well, I c – I know that that's – I can reduce that just by crossing out, or canceling the numbers that are in the ones place"

[Ball crosses out the threes, changing the fraction to $\frac{1\not{3}}{4\not{3}}$.]

744 Brandon: Mm-hmm.

745 Ball: "… and so I can just do that and it will be one fourth." Is that a correct way to reduce fractions?

746 Brandon: I can't really say if it's right or wrong cause I've never tried it.

747 Ball: Mm-hmm. What do – what would you try to – if you had to try to figure out, like, if that was a good method, or if it really was a method, what would you do to try to decide?

748 Brandon: Umm. I would see – I would see what number can go into those and see if it comes out to one fourth.

749 Ball: Mm-hmm. And does any number come to mind that you could do to divide into both of those to try to reduce it?

750 Brandon: No.

751 Ball: You can't think of one, or there isn't one, or what?

752 Ball: What're you trying?

[Brandon has written $13\overline{)43}$.]

753 Brandon: I'm trying to see if thirteen can go into forty-three.

754 Ball: Can you think of a number that you can use to divide into thirteen that you could also divide into forty-three?

755 Brandon: Umm. Three?

756 Ball: Three would go into 13?

757 Brandon: No. I'm not – I don't think – I'm not sure, 'cause – no, I don't think any number can go into thirteen evenly.

758 Ball: Uh-huh. So then what does that tell you about this? [i.e. 13/43]

759 Brandon: Umm. I'm not sure. I can't find a number that'll go into thirteen and thirty-three evenly – I mean forty-three evenly.

760 Ball: Okay. So what's your view about this method right now? You're saying you don't know, right? Is that what you're saying? Or are you thinking it doesn't – it's not a good idea?

761 Brandon: I'm thinking it's not a good idea.

762 Ball: Uh-huh. Why a – why are you thinking that?

763 Brandon: 'Cause – I don't know how, but I can't – there – I don't think there's a number that can go into thirteen and forty-three evenly.

764 Ball: Okay. Have you done some adding and subtracting of fractions?

765 Brandon: Mm-hmm.

766 Ball: And you did – you showed me you were doing some dividing also, right?

767 Brandon: Mm-hmm.

768 Ball: What else? Have you multiplied fractions too?

769 Brandon: Mm-hmm.

770 Ball: You want to do a few of those now? Okay. Can I have a, umm – can you hand me one of those pens. And I think we're okay with paper. Yeah. Thank you.

771 Ball: How would you do that problem? You can use your own. Here.

[Ball has written:]

$$\frac{2}{3} + \frac{2}{3} =$$

772 Brandon: It would equal four-thirds.

[Brandon writes his answer:]

$$\frac{2}{3}+\frac{2}{3}=\frac{4}{3}$$

773 Ball: How'd you do it?

774 Brandon: You added them, but some frac – some – along the line there's going to be a problem that – where the denominators aren't the same . . .

775 Ball: Mm-hmm.

776 Brandon: . . . so you would have to make the denominators the same.

777 Ball: Uh-huh. But if they're the same, then what?

778 Brandon: Then you can add.

779 Ball: Okay. Well what if somebody said the answer to this was actually four-sixths, what would you say? Do you see how somebody might get that?

780 Brandon: Mm-hmm.

781 Ball: What would you say about that?

782 Brandon: It's not correct.

783 Ball: Why?

784 Brandon: Because, umm. 'Cause you don't – you don't add the denominator.

785 Ball: Okay.

786 Brandon: I think you just add the numerator.

787 Ball: All right. So let – want to try one where the denominators are different? Can you show me how you do that? Are you going to write one? Okay.

[Brandon writes $\frac{4}{6}$ and then $\frac{2}{3}$ directly under it.]

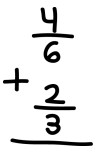
788 Ball: So you're doing four-sixths plus two-thirds?

789 Brandon: Mm-hmm.

790 Ball: Okay. Can we – let's – You want to write it horizontally and then show me how you would do it, or is this how you would normally write it, vertically?

791 Brandon: This way.

[Brandon writes:]



792 Ball: Okay. So what would you do?

793 Brandon: I would find a number that can go into three and six evenly.

794 Ball: Mm-hmm.

795 Brandon: The smallest number that can go . . .

796 Ball: So like what?

797 Brandon: Three.

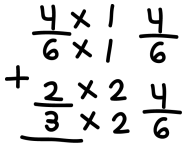
798 Ball: Okay.

799 Brandon: Wait, no. I'm getting confused . . . like six. 'Cause si – times two and then . . . You – whatever you do, whatever you multiply to get – get to the number, you do to the numerator, so . . .

800 Ball: Okay.

801 Brandon: . . . it's times would be times two by the numerator.

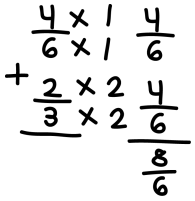
[Brandon has written:]



802 Ball: Mm-hmm.

803 Brandon: Then you would add.

[Brandon writes:]



804 Brandon: But that can be – that can be put into another number.

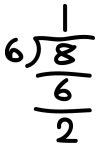
805 Ball: Okay, what did you get for that?

806 Brandon: Eight-sixths.

807 Ball: Okay.

808 Brandon: But that's a improper fraction. When – it's when the – the numerator is bigger than the denominator. So what – I would divide six into eight.

[Brandon writes:]



809 Brandon: Two – So you would get one-and-two-sixths, or one-and-one-third.

810 Ball: Okay. And why do you divide it to find out how to write it another way?

811 Brandon: Umm.

812 Ball: You've done that actually a few times. You did that with the three-over-two a couple times, right?

813 Brandon: Mm-hmm.

814 Ball: Why are you dividing it?

815 Brandon: 'Cause, umm – 'cause there's a – a different way that the numerator can be – it can be smaller than the denominator. It can – it'll be back to a whole number and a fraction of the whole.

816 Ball: Okay. I'm going to write a addition problem, okay? How 'bout . . . But you prefer written this way?

[Ball writes the fraction problem for Brandon both horizontally and vertically:]

$$\frac{3}{5}+\frac{2}{3} \qquad \begin{array}{r} \frac{3}{5} \\ +\frac{2}{3} \\ \hline \end{array}$$

817 Ball: So either way. How would you do that? There the denominators aren't the same, right?

818 Brandon: Mm-hmm.

819 Ball: So what would you do in order to add those?

820 Brandon: Find the, uh, the smallest number that three and five could go into.

821 Ball: Okay, so what would that be?

822 Brandon: Fifteen.

823 Ball: All right.

[Brandon writes:]

$$\begin{array}{rr} \frac{3\times3}{5\times3} & \frac{9}{15} \\ +\frac{2\times5}{3\times5} & \frac{10}{15} \\ \hline & \frac{19}{15} \end{array}$$

824 Ball: Okay. So let's try a subtraction problem, okay? What if I wrote down four-and-one-third minus two? What would the answer to that be?

[Ball has written:]

$$4\frac{1}{3}-2=$$

825 Brandon: One-and – one-and-two-thirds?

826 Ball: How'd you decide that?

827 Brandon: 'Cause minus two . . .

[Brandon writes:]

$$4\frac{1}{3}$$
$$-2$$

828 Brandon: 'Cause, umm . . . 'Cause it's – it's not whole number minus whole number so it – so it – it's a fraction, so it would . . . So if you minus two, you get two, but it's – it's part of one fraction so it – one-and-two-thirds – and two-thirds – the two-thirds – two-thirds plus one-third would equal three.

829 Ball: I don't understand where you got the two-thirds from. I don't see where this came from.

830 Brandon: 'Cause that plus that would equal two.

831 Ball: That – this one-and-two-thirds –

832 Brandon: That's two-third plus – yeah one-and-two-thirds plus one-and-three-thirds would equal two.

833 Ball: Okay. And then what?

834 Brandon: What?

835 Ball: And then – so then you're have two. That one – one-and-two-thirds plus one-third equals two.

836 Brandon: That would equal two, so that would equal a number so that's why I got that cause it's different.

837 Ball: What if you had a fraction that you were subtracting on the bottom as well? So what if you had four-and-one-third minus, umm, two-and-one-half? Then what would you do?

838 Brandon: Hmm. Find the number that two and third – two and three can go to, 'cause you can't – you can't subtract.

839 Ball: So what would that be?

840 Brandon: It'd be six.

841 Ball: Okay.

842 Brandon: But you don't do anything to the whole numbers.

843 Ball: Okay. So now what?

844 Brandon: And then you subtract.

845 Ball: Okay, so how would you do that?

846 Brandon: Umm. Hold on, you can't subtract yet 'cause two is – two – you can't subtract two from – you can't subtract three from two so –

847 Ball: Mm-hmm

848 Brandon: – you would cross out four and make it a three.

849 Ball: Mm-hmm.

850 Brandon: And would I turn this in – and would you turn this into a twelve?

851 Ball: How – what are you trying to decide?

852 Brandon: Umm, cross this out, make the three, and put the whole number right here.

853 Ball: Okay.

854 Brandon: And then you subtract.

855 Ball: Okay, so what would you get?

856 Brandon: You would get nine – you'd get nine . . .

857 Ball: It's kind of hard to see what you've got here. You look like you've got twelve-sixths minus three-sixths, and you have three minus two. Do you want me to rewrite it?

858 Brandon: Yeah.

859 Ball: Three-and-twelve-sixths, that's what you have?

860 Brandon: Mm-hmm.

861 Ball: And you have, umm, two-and-three-sixths.

[Ball has written:]

$$3\frac{12}{6}$$

$$2\frac{3}{6}$$

862 Brandon: Hmm. That's a hard question 'cause if you subtract it, it would be nine – it would be one-and-nine-sixths.

863 Ball: Yeah.

864 Brandon: So it'll be a whole number and an improper fraction.

865 Ball: Why did you put the one here? I mean I think I have an idea about why you put the one, but I wasn – I'd like to hear you explain it to me.

866 Brandon: 'Cause –

867 Ball: Here, I mean.

868 Brandon: Here – here it – you put three, 'cause three . . .

869 Ball: You crossed off the four and made it a three. Then why did you write a one there?

870 Brandon: Because I borrowed, umm, a whole from the four 'cause you can't sub – you can't subtract three from two, so I just crossed out the three, made it – crossed out the four and made it a three and put the one that was taken from the four and put it by the two.

871 Ball: Okay. So what would you say if I said to you you're actually working in sixths here, right? So a whole is six-sixths, is that right?

872 Brandon: Mm-hmm.

873 Ball: So if you have six-sixths then you wouldn't be moving a ten over there, right, you'd be moving six-sixths over there. Can you try doing it that way?

874 Ball: Alright… we'll make it clear. So here you have… Actually let's start where we were. So we had four-and-two-sixths, right?

875 Brandon: Mm-hmm.

876 Ball: And we were trying to subtract two-and-three-sixths?

877 Brandon: Mm-hmm.

[Ball has written:]

$$4\frac{2}{6}$$
$$-2\frac{3}{6}$$

878 Ball: That's after you found a common denominator. So what I'm saying is you said you were borrowing a ten, but the whole here would be six-sixths, right? So what if we try again and say you're going to make that a three and now you've got a whole that you can put together with the two-sixths, but what is that whole…

[Ball has written:]

$$3\,4\frac{2}{6}$$
$$-2\frac{3}{6}$$

879 Brandon: I would cross the two out and put a six.

880 Ball: Well you have two-sixths already and now you're having six more sixths that you got from this one. Now how many six-sixths would you have all together?

881 Brandon: Umm, six?

882 Ball: Well you have six-sixths from this whole?

883 Brandon: Mm-hmm.

884 Ball: …but you already had two-sixths, so how much is six-sixths plus two-sixths together?

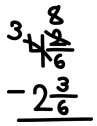
885 Brandon: One-and-two-sixths?

886 Ball: Okay, but why don't you write it as an improper fraction because you're going to be subtracting. So how much would that be? Six-sixths plus two-sixths?

887 Brandon: Mm, it would be eight-sixths.

888 Ball: So write an eight here instead of the two.

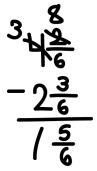
[Brandon writes:]



889 Ball: Okay, now can you subtract?

890 Brandon: Uh-huh.

[Brandon writes:]



891 Ball: Okay. So, now walk back 'cause I kind of helped you with that one a little bit. Can you try to explain what I was trying to show you? See if you understand what I was trying to show you?

892 Brandon: 'Cause –

893 Ball: Originally what you did is – it seems like you borrowed, like, a ten, the way you normally would borrow, but since we're working with fractions, I tried to show you a different way to think about the whole. Can you try to explain it back to me?

894 Brandon: Umm, what you would do is that…

895 Ball: Okay.

896 Brandon: …is that you can't borrow, like ten, so you would cross, making it eight, so – wait how…

897 Ball: Why did we get the eight from, though?

898 Brandon: I'm not sure. I'm not sure.

899 Ball: Okay. Okay. So what we were talking about in part is if you borrow when you're working with fractions, you have to think what the whole is, right?

900 Brandon: Six?

901 Ball: It's six-sixths, so what we did is we took the six-sixths from this whole and we put it together with the two-sixths we already had. Have you seen that before?

902 Brandon: Probably, but I probably got confused on it.

903 Ball: Mm-hmm.

904 Brandon: So, okay we took – what we – we took the whole from the three and we added it with the two sixths.

905 Ball: Mm-hmm.

906 Brandon: So two plus six would be eight –

907 Ball: Mm-hmm.

908 Brandon: so it would be eight-sixths.

909 Ball: Okay. How're we doing? We're almost out of time I think. You want to do one more thing, or are you tired?

910 Brandon: Oh no, we can . . .

911 Ball: You're not tired; you could do this all day? Yeah? Okay? Every time I kid with you they get badly behaved. Okay. Umm.

912 Ball: So I was thinking of telling you a problem, a story. Have you really ever done anything like that at all, or have you just been working with a lot of numbers and drawings?

913 Brandon: Mm-hmm.

914 Ball: Okay. So here's the story I'm going to tell you, but I'll have to tell it to you probably even more than one time. Maybe we want some paper to keep track. I'll tell you the story first. So imagine your teacher, Carol . . .

915 Brandon: Mm-hmm

916 Ball: Imagine she bought a cake, but she only bought half a cake. Okay, she didn't buy a whole cake, she just bought half a cake. Ever see it in the store where they sell half-cakes sometimes? She bought half a cake. Okay? And then she ate half of that. Can you make a picture to show me how much she ate?

917 Brandon: Mm-hmm.

[Brandon draws:]

918 Brandon: This would be half of the cake, so if she ate half of that, she ate half this part.

919 Ball: Okay. So how much did she eat of the whole cake? If there had been a whole cake, how much would she have eaten of the whole?

920 Brandon: One-fourth out of it.

921 Ball: One-fourth of the whole cake, and how much of her cake?

922 Brandon: Half.

923 Ball: Half of her cake. Now she was full then . . .

924 Brandon: Mm-hmm.

925 Ball: . . . and she decided to split the rest of her cake between you and one of the other kids in the class, but equally. So this is the part she didn't eat right here.

[Ball colors Brandon's drawing indicating that the shaded portion is what is left uneaten:]

926 Ball: Okay, so how could she divide that part equally and give you each an equal piece?

[Brandon draws:]

927 Ball: And how much would guys each be getting?

928 Brandon: What?

929 Ball: How much of the orig – how much of the whole cake would you each be getting?

930 Brandon: Umm, a half of that piece.

931 Ball: A half of this piece is the remainder, right?

932 Brandon: Mm-hmm.

933 Ball: So how much cake is that?

934 Brandon: Like this?

935 Ball: Yeah, how much cake is that?

936 Brandon: Out of the whole – out of this –

937 Ball: Out of the whole cake.

938 Brandon: One – I mean one-third. If Carol hadn't eaten this part then it would just be one out of third.

939 Ball: Okay. So you're getting one-third of her – the cake she bought?

940 Brandon: Yeah.

941 Ball: And the other student?

942 Brandon: Mm-hmm. But since she ate it, since she ate one-third of it then there's only two-thirds of it remaining.

943 Ball: But she ate half of it, not a third of it.

944 Brandon: Oh yeah. So I – I ate and the other friend ate one-half of – the half of her whole cake, or the whole cake

945 Ball: Right, so what's one-half of a half?

946 Brandon: I'm not sure.

947 Ball: Okay. So you were thinking of these as three equal pieces or three different pieces.

948 Brandon: Three different pieces.

949 Ball: Okay. All right. Want to try one more?

950 Brandon: Mm-hmm.

951 Ball: Okay. Somebody showed you this problem and said you shouldn't find a common denominator, or you shouldn't find an exact answer.

[Ball shows Brandon the problem:]

$$\frac{19}{21} + \frac{52}{55}$$

952 Brandon: Mm-hmm.

953 Ball: They just wanted to know about how big is the answer to that. Like is it about – is the total of those two numbers together about one-half? Is the total about one? Is the total about one-and-a-half? About two? What would you say?

954 Brandon: Umm, I –

955 Ball: Just approximately.

956 Brandon: About – about one.

957 Ball: How did you figure that out?

958 Brandon: 'Cause – wait no. Actually it would be seventy-out-of – seventy-out-of-eighty, I think. 'Cause since this number is five then I can go up so it would be sixty . . .

959 Ball: Mm-hmm

960 Brandon: . . . and this is approximately twenty. And that is approximately twenty, and this is approximately fifty, so fifty plus twenty would be . . .

961 Ball: You can write it out.

[Brandon writes $\frac{70}{80}$.]

962 Brandon: Okay. So this would be seventy and this would be eighty.

963 Ball: Okay, so if you were to write a fraction that was a whole with twenty-one in the denominator, what would it be?

964 Brandon: It – to make it a whole?

965 Ball: Mm-hmm

966 Brandon: Twenty-one over twenty-one.

967 Ball: Okay. Is that pretty close to that?

968 Brandon: Mm-hmm.

969 Ball: And if you were going to make a fraction that had fifty-five in the denominator that was one, what would you write?

970 Brandon: Fifty-five.

971 Ball: Is that pretty close to that?

972 Brandon: Yeah, but – yeah. Mm-hmm.

973 Ball: So this one's pretty close to one and this one's pretty close to one. When you add them together what're you going to get?

974 Brandon: Two.

975 Ball: You're going to get pretty close to two. Do you see why?

976 Brandon: Yeah. 'Cause –

977 Ball: Can you explain it to me?

978 Brandon: Yeah, 'cause this is almost one. It's approximately and this approximately one, so . . . If we were to round then it – they would be one plus one will, will equal to two.

979 Ball: Okay. So maybe we should stop. I think we've been working a long time and I wanted to ask you a couple questions. What was – of all the stuff we did, and when you look back – we have tons of paper up here now – what do you think was, umm, the most fun that we did, or was anything fun?

980 Brandon: Yeah. I – I like the number line –

981 Ball: Uh-huh.

982 Brandon: – 'cause, umm, I kn – I probably did it few time, but I don't really work with number lines so it was fun to learn to do something new.

983 Ball: Uh-huh. So that was pretty new for you?

984 Brandon: Mm-hmm.

985 Ball: And of the things we did, what was – what did you find – was there anything that you found difficult that we did?

986 Brandon: Umm, yeah. Well no, not actually. Mm-hmm.

987 Ball: No?

988 Brandon: ’Cause I – ’cause probably in the past I probably said, “I don’t remember this,” but sort of like if I see it then, yeah I’ll probably remember or . . . from a long time ago.

989 Ball: Mm-hmm. Were there things you found that you had to think about, umm, very hard, that you hadn’t been thinking about for a while?

990 Brandon: Yeah.

991 Ball: What – do you remember an example today of something you had to think really hard about?

992 Brandon: Umm. About the subtracting – about the subtracting. About, umm, for – for, umm, like if a number – if – if it’s – if you have a whole number and a fraction, but the numerator of the fraction can’t subtract the numerator of the other fraction, you have to borrow from the other one – from the whole.

993 Ball: Mm-hmm. And you think you had learned that before, or you don’t think you’ve learned that before?

994 Brandon: I – yeah I learned it, but – but then I get confused with it, so I use it a different way –

995 Ball: Uh-huh.

996 Brandon: – the incorrect way.

997 Ball: And how did it feel to have all these people out there? What did you think about that?

998 Brandon: Nothing, ’cause we were basically just focusing, like on the board or on the chalkboard –

999 Ball: Mm-hmm.

1000 Brandon: – so it wasn’t a big deal.

1001 Ball: Uh-huh. Except when they laughed, of course. Right? Umm, do you have any questions you wanted to ask me, or anything you wanted to, I don’t know, anything? Anything about what we did or anything specific?

1002 Brandon: Mm-mm.

1003 Ball: No? Do you want to stop?

1004 Brandon: Mm-hmm.

1005 Ball: So you’re – there’s going to be a little presentation for you, to thank you for this, so at this point people are going to pay attention to you again. But the director of the Mathematical Sciences Research Institute – I’m going to introduce him to you and he’s going to make a little presentation to you. Is that okay?

1006 Brandon: Mm-hmm.