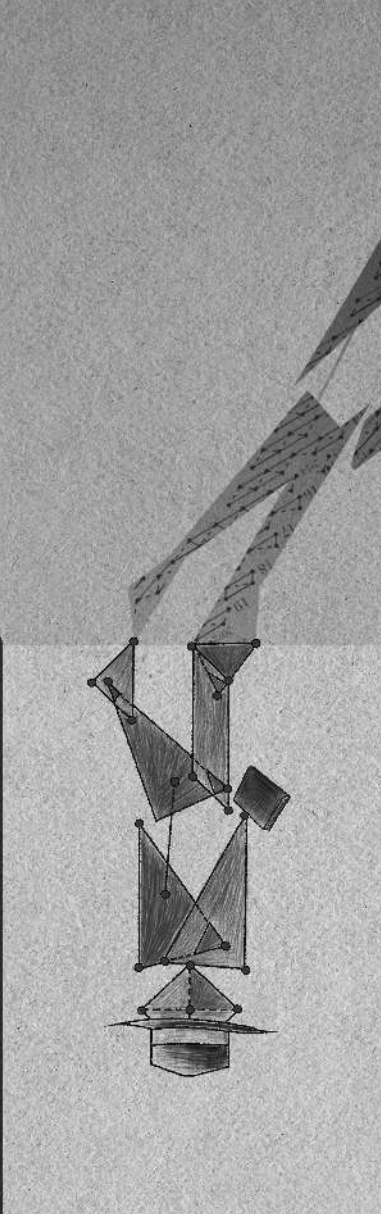


Cambridge University Press & Assessment
978-1-009-12329-7 — Stable Categories and Structured Ring Spectra
Edited by Andrew J. Blumberg , Teena Gerhardt , Michael A. Hill
Frontmatter
[More Information](#)

This comprehensive text focuses on the homotopical technology in use at the forefront of modern algebraic topology. Following on from a standard introductory algebraic topology sequence, it will provide students with a comprehensive background in spectra and structured ring spectra. Each chapter is an extended tutorial by a leader in the field, offering the first really accessible treatment of the modern construction of the stable category in terms of both model categories of point-set diagram spectra and infinity-categories. It is one of the only textbook sources for operadic algebras, structured ring spectra, and Bousfield localization, which are now basic techniques in the field, and the book provides a rare expository treatment of spectral algebraic geometry. Together the contributors Emily Riehl, Daniel Dugger, Clark Barwick, Michael A. Mandell, Birgit Richter, Tyler Lawson, and Charles Rezk offer a complete, authoritative source to learn the foundations of this vibrant area.

Cambridge University Press & Assessment
978-1-009-12329-7 — Stable Categories and Structured Ring Spectra
Edited by Andrew J. Blumberg , Teena Gerhardt , Michael A. Hill
Frontmatter
[More Information](#)

Cambridge University Press & Assessment
978-1-009-12329-7 — Stable Categories and Structured Ring Spectra
Edited by Andrew J. Blumberg , Teena Gerhardt , Michael A. Hill
Frontmatter
[More Information](#)



Cambridge University Press & Assessment
978-1-009-12329-7 — Stable Categories and Structured Ring Spectra
Edited by Andrew J. Blumberg , Teena Gerhardt , Michael A. Hill
Frontmatter
[More Information](#)

Cambridge University Press & Assessment
978-1-009-12329-7 — Stable Categories and Structured Ring Spectra
Edited by Andrew J. Blumberg , Teena Gerhardt , Michael A. Hill
Frontmatter
[More Information](#)

Mathematical Sciences Research Institute
Publications

69

Stable Categories and
Structured Ring Spectra

Mathematical Sciences Research Institute Publications

- 1 Freed/Uhlenbeck: *Instantons and Four-Manifolds*, second edition
- 2 Chern (ed.): *Seminar on Nonlinear Partial Differential Equations*
- 3 Lepowsky/Mandelstam/Singer (eds.): *Vertex Operators in Mathematics and Physics*
- 4 Kac (ed.): *Infinite Dimensional Groups with Applications*
- 5 Blackadar: *K-Theory for Operator Algebras*, second edition
- 6 Moore (ed.): *Group Representations, Ergodic Theory, Operator Algebras, and Mathematical Physics*
- 7 Chorin/Majda (eds.): *Wave Motion: Theory, Modelling, and Computation*
- 8 Gersten (ed.): *Essays in Group Theory*
- 9 Moore/Schochet: *Global Analysis on Foliated Spaces*, second edition
- 10–11 Drasin/Earle/Gehring/Kra/Marden (eds.): *Holomorphic Functions and Moduli*
- 12–13 Ni/Peletier/Serrin (eds.): *Nonlinear Diffusion Equations and Their Equilibrium States*
- 14 Goodman/de la Harpe/Jones: *Coxeter Graphs and Towers of Algebras*
- 15 Hochster/Huneke/Sally (eds.): *Commutative Algebra*
- 16 Ihara/Ribet/Serre (eds.): *Galois Groups over \mathbb{Q}*
- 17 Concus/Finn/Hoffman (eds.): *Geometric Analysis and Computer Graphics*
- 18 Bryant/Chern/Gardner/Goldschmidt/Griffiths: *Exterior Differential Systems*
- 19 Alperin (ed.): *Arboreal Group Theory*
- 20 Dazord/Weinstein (eds.): *Symplectic Geometry, Groupoids, and Integrable Systems*
- 21 Moschovakis (ed.): *Logic from Computer Science*
- 22 Ratiu (ed.): *The Geometry of Hamiltonian Systems*
- 23 Baumslag/Miller (eds.): *Algorithms and Classification in Combinatorial Group Theory*
- 24 Montgomery/Small (eds.): *Noncommutative Rings*
- 25 Akbulut/King: *Topology of Real Algebraic Sets*
- 26 Judah/Just/Woodin (eds.): *Set Theory of the Continuum*
- 27 Carlsson/Cohen/Hsiang/Jones (eds.): *Algebraic Topology and Its Applications*
- 28 Clemens/Kollár (eds.): *Current Topics in Complex Algebraic Geometry*
- 29 Nowakowski (ed.): *Games of No Chance*
- 30 Grove/Petersen (eds.): *Comparison Geometry*
- 31 Levy (ed.): *Flavors of Geometry*
- 32 Cecil/Chern (eds.): *Tight and Taut Submanifolds*
- 33 Axler/McCarthy/Sarason (eds.): *Holomorphic Spaces*
- 34 Ball/Milman (eds.): *Convex Geometric Analysis*
- 35 Levy (ed.): *The Eightfold Way*
- 36 Gavosto/Krantz/McCallum (eds.): *Contemporary Issues in Mathematics Education*
- 37 Schneider/Siu (eds.): *Several Complex Variables*
- 38 Billera/Björner/Green/Simion/Stanley (eds.): *New Perspectives in Geometric Combinatorics*
- 39 Haskell/Pillay/Steinhorn (eds.): *Model Theory, Algebra, and Geometry*
- 40 Bleher/Its (eds.): *Random Matrix Models and Their Applications*
- 41 Schneps (ed.): *Galois Groups and Fundamental Groups*
- 42 Nowakowski (ed.): *More Games of No Chance*
- 43 Montgomery/Schneider (eds.): *New Directions in Hopf Algebras*
- 44 Buhler/Stevenhagen (eds.): *Algorithmic Number Theory: Lattices, Number Fields, Curves and Cryptography*
- 45 Jensen/Ledet/Yui: *Generic Polynomials: Constructive Aspects of the Inverse Galois Problem*
- 46 Rockmore/Healy (eds.): *Modern Signal Processing*
- 47 Uhlmann (ed.): *Inside Out: Inverse Problems and Applications*
- 48 Gross/Kotiuga: *Electromagnetic Theory and Computation: A Topological Approach*
- 49 Darmon/Zhang (eds.): *Heegner Points and Rankin L-Series*
- 50 Bao/Bryant/Chern/Shen (eds.): *A Sampler of Riemann–Finsler Geometry*
- 51 Avramov/Green/Huneke/Smith/Sturmfels (eds.): *Trends in Commutative Algebra*
- 52 Goodman/Pach/Welzl (eds.): *Combinatorial and Computational Geometry*
- 53 Schoenfeld (ed.): *Assessing Mathematical Proficiency*
- 54 Hasselblatt (ed.): *Dynamics, Ergodic Theory, and Geometry*
- 55 Pinsky/Birnir (eds.): *Probability, Geometry and Integrable Systems*
- 56 Albert/Nowakowski (eds.): *Games of No Chance 3*
- 57 Kirsten/Williams (eds.): *A Window into Zeta and Modular Physics*
- 58 Friedman/Hunsicker/Libgober/Maxim (eds.): *Topology of Stratified Spaces*
- 59 Caporaso/M^cKernan/Mustață/Popa (eds.): *Current Developments in Algebraic Geometry*
- 60 Uhlmann (ed.): *Inverse Problems and Applications: Inside Out II*
- 61 Breuillard/Oh (eds.): *Thin Groups and Superstrong Approximation*
- 62 Eguchi/Eliashberg/Maeda (eds.): *Symplectic, Poisson, and Noncommutative Geometry*
- 63 Nowakowski (ed.): *Games of No Chance 4*
- 64 Bellamy/Rogalski/Schedler/Stafford/Wemyss (ed.): *Noncommutative Algebraic Geometry*
- 65 Deift/Forrester (eds.): *Random Matrix Theory, Interacting Particle Systems, and Integrable Systems*
- 66 Corvino/Miao: *Lectures on Mathematical Relativity*
- 67–68 Eisenbud/Iyengar/Singh/Stafford/Van den Bergh (eds.): *Commutative Algebra and Noncommutative Algebraic Geometry*
- 69 Blumberg/Gerhardt/Hill (eds.): *Stable Categories and Structured Ring Spectra*
- 70 Larsson (ed.): *Games of No Chance 5*

Volumes 1–4, 6–8, and 10–27 are published by Springer-Verlag

Cambridge University Press & Assessment
978-1-009-12329-7 — Stable Categories and Structured Ring Spectra
Edited by Andrew J. Blumberg , Teena Gerhardt , Michael A. Hill
Frontmatter
[More Information](#)

Stable Categories and Structured Ring Spectra

Edited by

Andrew J. Blumberg
Columbia University

Teena Gerhardt
Michigan State University

Michael A. Hill
University of California, Los Angeles



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press & Assessment
978-1-009-12329-7 — Stable Categories and Structured Ring Spectra
Edited by Andrew J. Blumberg , Teena Gerhardt , Michael A. Hill
Frontmatter
[More Information](#)

Andrew J. Blumberg
Department of Mathematics
Columbia University
andrew.blumberg@columbia.edu

Teena Gerhardt
Department of Mathematics
Michigan State University
teena@math.msu.edu

Michael A. Hill
Department of Mathematics
University of California, Los Angeles
mikehill@math.ucla.edu

Silvio Levy (*Series Editor*)
Mathematical Sciences Research Institute
Berkeley, CA 94720
levy@msri.org

Frontispiece image: Jing Hu 胡菁

The Mathematical Sciences Research Institute wishes to acknowledge support by the National Science Foundation and the *Pacific Journal of Mathematics* for the publication of this series.

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
One Liberty Plaza, 20th Floor, New York, NY 10006, USA

www.cambridge.org

Information on this title: www.cambridge.org/9781009123297

© Mathematical Sciences Research Institute 2022

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2022

A catalog record for this publication is available from the British Library.

ISBN 978-1-009-12329-7 hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet Web sites referred to in this publication and does not guarantee that any content on such Web sites is, or will remain, accurate or appropriate.

Contents

	<i>Contributors</i>	<i>page xi</i>
1	Introduction	1
	1.1 Goals of this book	1
	1.2 Summaries of the chapters	3
	1.3 Acknowledgements	4
2	Homotopical categories: from model categories to $(\infty, 1)$-categories	5
	2.1 The history of homotopical categories	5
	2.2 Categories of fractions and localization	8
	2.3 Model category presentations of homotopical categories	10
	2.4 Derived functors between model categories	25
	2.5 Homotopy limits and colimits	38
	2.6 Simplicial localizations	49
	2.7 Quasi-categories as $(\infty, 1)$ -categories	54
	2.8 Models of $(\infty, 1)$ -categories	60
	2.9 Model-independent $(\infty, 1)$ -category theory	67
	2.10 Epilogue	73
3	Stable categories and spectra via model categories	75
	3.1 Introduction	75
	3.2 Stable model categories	87
	3.3 Monoidal machinery	92
	3.4 Spectra for Sulu and Chekov	102
	3.5 Diagram categories and spectra	109
	3.6 Localization and the stable model structures on spectra	116
	3.7 Symmetric spectra	120
	3.8 Orthogonal spectra	129
	3.9 EKMM spectra	140
	3.10 Afterthoughts	145
4	Stable homotopy theory via ∞-categories	151
	4.1 Spectra	151
	4.2 Examples	157

x	Contents	
	4.3 Smash products	162
	4.4 Stable ∞ -categories	165
	4.5 Generalisations	170
	4.6 Stabilisation	173
	4.7 t -structures	175
5	Operads and operadic algebras in homotopy theory	183
	5.1 Introduction	183
	5.2 Operads and endomorphisms	186
	5.3 A_∞ , E_∞ , and E_n operads	192
	5.4 Operadic algebras and monads	198
	5.5 Modules over operadic algebras	204
	5.6 Limits and colimits in categories of operadic algebras	210
	5.7 Enrichment and geometric realization	215
	5.8 Model structures for operadic algebras	220
	5.9 Comparison/rectification theorems for operadic algebras	224
	5.10 Enveloping algebras, Moore algebras, and rectification	229
	5.11 E_n spaces and iterated loop space theory	236
	5.12 E_∞ algebras in rational and p -adic homotopy theory	240
6	Commutative ring spectra	249
	6.1 Introduction	249
	6.2 Features of commutative ring spectra	253
	6.3 Chain algebras and algebras over Eilenberg–Mac Lane spectra	258
	6.4 Units of ring spectra and Thom spectra	260
	6.5 Constructing commutative ring spectra from bipermutative categories	269
	6.6 From topological Hochschild to topological André–Quillen homology	273
	6.7 How do we recognize ring spectra as being (non) commutative?	283
	6.8 What are étale maps?	287
	6.9 Picard and Brauer groups	293
7	An introduction to Bousfield localization	301
	7.1 Introduction	301
	7.2 Motivation from categorical localization	304
	7.3 Local objects in categories	305
	7.4 Localization using mapping spaces	308
	7.5 Lifting criteria for localizations	311
	7.6 The small object argument	313
	7.7 Unstable settings	314
	7.8 Stable settings	317
	7.9 Homology localizations	323
	7.10 Model categories	330
	7.11 Presentable ∞ -categories	335
	7.12 Multiplicative properties	337

8	Spectral algebraic geometry	345
8.1	Introduction	345
8.2	The notion of an ∞ -topos	348
8.3	Sheaves on an ∞ -topos	350
8.4	Slices of ∞ -topoi	351
8.5	Truncation and connectivity in ∞ -topoi	354
8.6	Morphisms of ∞ -topoi	359
8.7	Étale morphisms	363
8.8	Spectra and commutative ring spectra	365
8.9	The étale site of a commutative ring	369
8.10	Spectral Deligne–Mumford stacks	373
8.11	Morphisms of spectral DM stacks	375
8.12	The category of spectral DM stacks	378
8.13	Étale and flat morphisms of spectral DM stacks	380
8.14	Affine space and projective space	383
8.15	Functor of points	385
8.16	Formal spectral geometry	388
8.17	Formal groups in spectral geometry	390
8.18	Quasicoherent sheaves	394
8.19	Elliptic cohomology and topological modular forms	398
8.20	The classifying stack for oriented elliptic curves	400
	<i>Bibliography</i>	403
	<i>Index</i>	421

Cambridge University Press & Assessment
978-1-009-12329-7 — Stable Categories and Structured Ring Spectra
Edited by Andrew J. Blumberg , Teena Gerhardt , Michael A. Hill
Frontmatter
[More Information](#)

Contributors

Emily Riehl
Johns Hopkins University, Baltimore, MD

Daniel Dugger
University of Oregon, Eugene, OR

Clark Barwick
University of Edinburgh, Edinburgh, UK

Michael A. Mandell
Indiana University, Bloomington, IN

Birgit Richter
University of Hamburg, Hamburg, Germany

Tyler Lawson
University of Minnesota, Minneapolis, MN

Charles Rezk
University of Illinois, Urbana-Champaign, Illinois

Cambridge University Press & Assessment
978-1-009-12329-7 — Stable Categories and Structured Ring Spectra
Edited by Andrew J. Blumberg , Teena Gerhardt , Michael A. Hill
Frontmatter
[More Information](#)
