

This book surveys the state of the art in the theory of combinatorial games, that is, games not involving chance or hidden information. Enthusiasts will find a wide variety of exciting topics, from a trailblazing presentation of scoring to solutions of three piece ending positions of bidding chess. Theories and techniques in many subfields are covered, such as universality, Wythoff Nim variations, misère play, partizan bidding (a.k.a. Richman games), loopy games, and the algebra of placement games. Also included are an updated list of unsolved problems, extremely efficient algorithms for Taking and Breaking games, a historical exposition of binary numbers and games by David Singmaster, chromatic Nim variations, renormalization for combinatorial games, and a survey of temperature theory by Elwyn Berlekamp, one of the founders of the field.

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