

Critical Issues in Mathematics Education  
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# Developmental Mathematics: For whom? To what end?

By Katherine Stevenson





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“As a culture with its own language, tools, and beliefs, math is well-positioned to help students develop each of these. The fundamental “belief” in mathematics is that truth is accessible and can be understood through logical reasoning.”

— PHILIP URI TREISMAN  
UNIVERSITY OF TEXAS



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“**B**egin at the end.  
to improve education,  
put the learner first.”

— LEE ZIA

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# DEVELOPMENTAL MATHEMATICS: FOR WHOM? TO WHAT END?

Critical Issues in Mathematics Education · Volume 11 · Workshop 12  
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*On the cover:* Katherine Yoshiwara of Los Angeles Pierce College during her CIME 2015 talk “Developmental mathematics at community colleges”.

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# INTRODUCTION

With its 2015 workshop, the *Critical Issues in Mathematics Education* (CIME) series entered its second decade of providing opportunities for mathematicians to collaborate in finding ways to improve the teaching and learning of mathematics. The focus this year was on an issue that touches all segments of mathematics education: developmental mathematics. By hosting these workshops the Mathematical Sciences Research Institute seeks to help publicly identify and support critical issues and advances in mathematical education, and lend credibility and legitimacy to such efforts, just as it does when hosting workshops in all areas of mathematical research.

The 2015 workshop addressed developmental mathematics at two- and four-year colleges and universities, as well as the broader dynamics of mathematics remediation at all educational levels. The critical nature of this issue is underscored by the failure rate crisis in entry-level mathematics courses throughout higher education in the United States, a crisis that includes developmental mathematics. Recently published reports suggest that this failure rate will reduce the number of U.S. graduates in the natural sciences and in engineering, and cause a shortfall in trained professionals needed to replace the large number of projected retirees over the next 20 years. Moreover, for students choosing to study non-STEM fields (i.e., those outside of science, technology, engineering and mathematics) developmental math has become a barrier blocking all but a minority from achieving the dream of an associate's or a bachelor's degree.

The workshop addressed student learning and how that learning is impacted by the “tectonic shifts” that are currently driving, or forcing, change in developmental math on a national level. It was organized around six central questions:



*Statue of Shiing-Shen Chern, one of MSRI's three founders.*

1. How do we teach content in ways that acknowledge and leverage each student's prior learning experiences? In particular, how do we take advantage of a student's maturity while refining his or her learning habits where necessary?
2. How can developmental mathematics instruction move students through remedial topics while simultaneously providing support for more advanced study, including the development of practices needed for research and meaningful mathematical work?
3. What strategies support the diverse needs of students in developmental mathematics programs? Can these strategies inculcate basic skills for some while still building the foundation necessary for a STEM major for others? How can we successfully address equity issues for students from groups underrepresented in STEM fields? How can developmental mathematics instruction blend synchronous and asynchronous instruction to generate utmost proficiency?
4. What is the proper balance between addressing the comprehensive needs of students mentioned in the preceding point while keeping course offerings and instruction concise?
5. What are the characteristics, training, and practices of a successful developmental mathematics teacher?
6. What support services enhance the success of a developmental mathematics program?



Consistent with the CIME tradition, the conference fostered a conversation among participants about the goals of developmental mathematics and the contributions made toward its efficacy. Workshop participants included mathematicians and educators from community colleges, universities, and K–12 schools, as well as scholars, education journalists, and university and college administrators whose focus lies in mathematics education. This group of professionals comprised what Estela Bensimon of the Center for Urban Education calls *institutional agents*, or people who “use their resources, power, knowledge, and influence on behalf of students” by striving to make entry-level mathematics effective and relevant. In bringing these agents together, the conference provided a starting point for the mobilization of a professional network whose communal expertise is needed to address the fundamental questions facing developmental mathematics: What are the next steps for action? How do we apply those steps to scale?

As the daughter of a mathematician and a social worker, the importance of developmental math education seems part of my DNA. It was an honor to be a co-organizer of the workshop, and it has been a privilege to write this summary. This booklet is based on the ideas of the organizing committee as realized by the workshop contributors. I have tried to weave the wealth of information communicated during the event into coherent narratives on six main themes:

1. Developmental mathematics for whom?
2. Developmental mathematics to what ends?
3. Tectonic shifts shaping developmental mathematics.
4. Non-cognitive issues related to developmental mathematics.
5. Equity issues related to developmental mathematics.
6. K–12 connections to developmental mathematics.

My goal was to present the views of contributors in their own voices while selectively blending them, where warranted, with others on related themes. I hope this method reinforces an urgent message: We know what to do to improve developmental math education; the time has come for immediate action.

I would like to thank the many people who reviewed this document. These include the speakers, organizers, and contributors to the workshop as well as my dear friend and informal editor Joyce Mitamura. To them I owe a debt of gratitude for help in matters of exposition and organization. The flaws that remain are my own.



*Conference organizers from right: Duane Cooper, Robert Megginson, Katherine Stevenson, Mark Hoover, Richard Sgarlotti.*

# DEVELOPMENTAL MATHEMATICS: FOR WHOM?

*Based on talks by Philip Uri Treisman, Katherine Stevenson, and Thomas Bailey as well as contributions from Pamela Burdman.*

Developmental math is “a hurdle for the majority,” [Burdman 2013] and for those from underserved populations, it is a painfully high one. According to recent data, approximately 68% of students entering two-year colleges and about 35% of students at four-year colleges in the United States take some form of developmental math [Hodara 2013]. The percentage of minority students and students of lower socioeconomic status is disproportionately high in this group, with limited success at all levels of mathematics. This has been the state of affairs for more than 20 years, with the brunt of the impact hitting community colleges.

Uri Treisman is a mathematician and established leader in examining how learning outcomes for students from underserved populations can be improved, particularly in mathematics [Treisman 1992; also see “New Mathways” in the Appendix, page 55]. He opened the workshop by questioning beliefs and myths related to who gets placed into developmental math and why.

The following questions were explored by Treisman as well as several other speakers during the course of the workshop. At the workshop’s opening session (seen below), these topics were summarized and supported with arguments from recent scholarly work.



*CIME opening session  
in Simons Auditorium  
on Wednesday,  
March 18, 2015.*



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### WHY IS THE PROBLEM FOCUSED ON TWO-YEAR COLLEGES? ARE STUDENTS IN TWO-YEAR COLLEGES MORE LIKELY TO ENROLL IN REMEDIAL COURSES BECAUSE THEY HAVE WEAK ACADEMIC BACKGROUNDS?

*Based on Treisman's introductory plenary remarks, available at <https://www.msri.org/workshops/758/schedules/19521>.*

The average two-year college student does have a weaker academic background than the average four-year student. This is simply a result of the differing admissions criteria of the two systems. However, work of Attewell and Monaghan demonstrates that this relative weakness does not account for the full magnitude of difference in remediation levels between the two systems. Using both logistic regression and propensity matching, they found that community college students are 19% more likely to require remedial courses than their counterparts enrolled at non-selective and minimally selective four-year universities. They write, “This is a particularly stark difference considering that we are comparing groups which are balanced in terms of high school math classes taken, high school grades, and SAT scores” [Attewell 2015]. Thus, the comparatively weaker academic backgrounds of students in two-year colleges *does not fully account* for their increased enrollments in developmental math courses.

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### IS THE CURRENT PLACEMENT SYSTEM EFFECTIVE?

*Based on Thomas Bailey's talk “Recent reforms in math assessment and developmental education: Connecting remediation to college level programs of instruction”. Available at: <https://www.msri.org/workshops/758/schedules/19539>.*

Thomas Bailey, Director of Columbia University's Community College Research Center (CCRC), is a pre-eminent researcher and expert on community college student success. While the placement systems at four-year universities have not been well-studied, Bailey calls the current community college placement systems, which depend on high-stakes testing, ineffective at meaningfully separating students into developmental and college-ready categories. Moreover, he points out that students are routinely under- rather than over-placed [Bailey et al. 2010; Scott-Clayton 2012]. Bailey lists several practical reasons why this is so:

1. The lack of consensus on definition of “college-ready” and requisite SAT scores.
2. The wide variation in grading standards, which makes it difficult for college faculty to trust high school grades even with students' transcript data in hand.
3. The fact that knowledge and skills are continuous and non-linear, rather than discrete and fixed, and come in cognitive and non-cognitive forms.
4. The skills that are required diverge from students' academic and career goals.

Bailey believes that equitable reforms in developmental math must include early assessment using multiple measures. Until recently, multiple measures have been only rarely used, and when they have been mandated (as in California's community colleges), they haven't been implemented in meaningful ways.

WHY ARE STUDENTS FROM UNDERSERVED POPULATIONS OVERREPRESENTED IN DEVELOPMENTAL MATH? ARE MINORITY STUDENTS AND STUDENTS WITH LOWER SOCIOECONOMIC STATUS LESS WELL PREPARED?

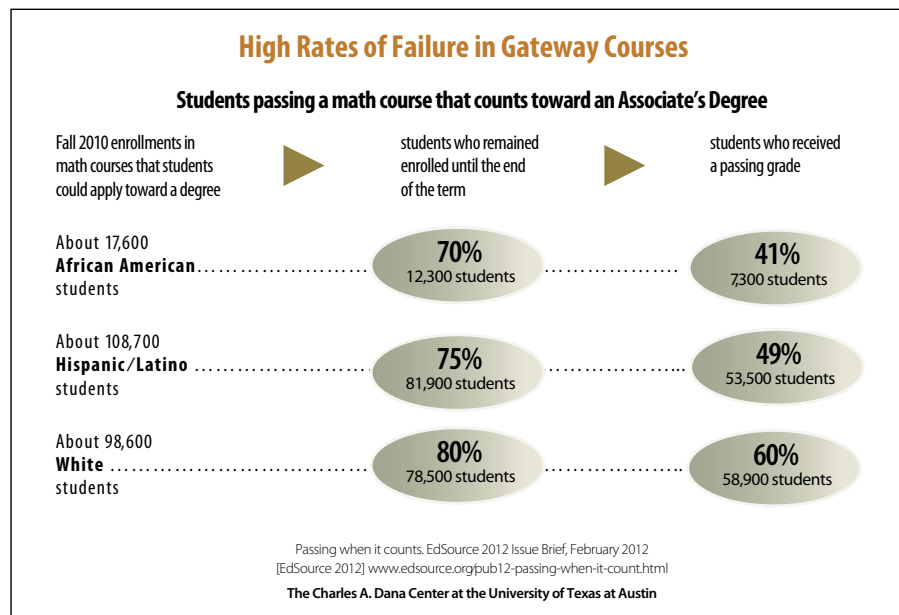
*Based on Uri Treisman's introductory plenary remarks.*

Minority students and students of lower socioeconomic status are overrepresented in developmental math and struggle disproportionately throughout their mathematical academic work. In fact,

“Though enrollment in remedial education is consistent across all racial groups, under-represented minority students were more likely to be placed at lower levels: 61% of African Americans and 52% of Latino students were placed into Arithmetic or pre-Algebra, while only a third of white and Asian students were placed that low.” [Perry et. al 2010]

Thus, there is an apparently ineffective testing system disproportionately placing minority students into lower level developmental math classes.

Moreover, when any student is placed into developmental math, evidence suggests that the sequence does a poor job of improving student outcomes. The main reason is not that students



*Adapted from Treisman, slide 8, attributed to EdSource, 2012.*

fail developmental courses, but that many assigned to developmental math either don't enroll, or they begin the sequence but don't complete it [Bailey, Jeong, and Cho 2010]. Once again, underserved and minority students suffer the most. Data from the California Community College System indicate that African American and Hispanic/Latino students pass remedial math classes at rates considerably below those of white and Asian students.

With a broken placement system and a sequence of courses that fails to serve its purpose, it is very difficult to pinpoint why underserved and minority populations are overrepresented in developmental math. However, as Attewell [Attewell et al., 2006] found, "African American students are significantly more likely to enroll in college remedial courses than are white students with the same academic skills and preparation and social background."

Interestingly, however, if one holds all other factors equal (e.g. students' academic background and other covariates) socio-economic status "is *not* a significant determinant of taking remedial coursework, independent of high school academic background" [Attewell et al., 2006].

The comparatively weaker academic backgrounds of students from underserved populations *does not fully account* for their increased enrollments and lack of success in developmental math courses. In Sections 4 and 5 of this booklet we discuss equity issues in more detail as well as the importance of non-cognitive aspects of learning in closing the achievement gap.



*A workshop session in Simons Auditorium on Thursday, March 19, 2015.*

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IS DEVELOPMENTAL MATHEMATICS WELL DEFINED? IS IT ONE THING OR IS  
DEVELOPMENTAL MATHEMATICS WELL DEFINED? IS IT ONE THING OR SEVERAL?

*Based on Katherine Stevenson's talk "The good shepherd".*

*Available at: <https://www.msri.org/workshops/758/schedules/19523>.*

The purpose, content, and reality of developmental math varies from context to context between and within the university system and its intersegmental articulation agreements. For some, the purpose of developmental math is to help students pass transferable or general education math classes, while for others it serves a broader liberal arts goal. The purpose drives the content, which is then tempered by the reality.

California is home to the largest public education structure in the country. Its systems abound with contradictions: It deploys the most innovative developmental math interventions yet simultaneously evinces strong resistance to those interventions; it has established one of the most successful intersegmental articulation agreements yet is also beset by procedural dysfunction obstructing these same agreements; it has some of the most resilient students yet also many of the most vulnerable. As such, examining California's higher education systems can shed light on how developmental mathematics is realized in different contexts, and how the realities of those contexts shape its purpose and content.

California has three public post-secondary education systems: the University of California (UC) with nine undergraduate campuses and more than 230,000 students; California State University (CSU) with 23 campuses serving more than 400,000 students; and the California Community College System (CCCS) with 112 campuses and more than 2.1 million students. The UC system selects incoming freshmen from among the top 12.5% of public school high school graduates (it also takes transfers, based on somewhat different criteria) and the CSU takes CCCS students from among the top third. The CCCS takes 100% of all high school graduates, and 85% of its entering students place into developmental math. Both the UC and CSU use GPA to restrict eligibility and both require three years of high school mathematics as a

condition for entry. Traditionally that meant algebra 1, geometry, and algebra 2, though under the Common Core State Standards, that material can also be covered through a three-year content-integrative sequence.

The UC system assumes that all entering students are proficient in math and English, so developmental math does not technically exist in the UC system. Some campuses provide additional assistance to students and groups of students that need support [e.g., Treisman's work at UC Berkeley



*Left: Bem Cayco and Wade Ellis (West Valley College); Bruce Yoshiwara (Los Angeles Pierce College) in back.*

with minority students], but by and large, students must fend for themselves.

With a system-wide six-year graduation rate of 80%, it seems that most students muddle through.

The CSU system lives in a different reality from the UC system. Plummeting passage rates in college-level math courses more than 25 years ago caused the CSU to cease assuming students were proficient in math and English. All campuses moved to a common exam, the Entry Level Math (ELM) test, to place admitted students in or out of developmental math. That system remains in place today and there is a common cutoff between proficient and developmental students across campuses. Individual campuses can set their own thresholds to determine which students require one versus two terms of developmental math (or 1–3 quarters on quarter systems).

By order of the CSU Chancellor, all students must complete their developmental math work within one year or be “stopped out” of CSU. While strict, the system adds some urgency, which is often envied by the community college system. Indeed, the CSU has been successful with this approach. About a third of admitted students require remedial coursework. Approximately 85% of these students complete their developmental math work within the requisite year [Sullivan 2015]. This compares quite favorably to less than a third who complete developmental math at the community college level (over a longer time period) [Burdman 2015b].

CSU’s developmental math programs guarantee that a minimum level of mathematical competency can be expected of all students. This “safety net” has allowed math departments and others to feel comfortable creating alternate college-level general education quantitative reasoning courses with names such as “Ideas in Math and Statistics” that avoid the high failure rates of algebra-based classes like college algebra or pre-calculus. If the ELM safety net were removed, math faculty would likely want to restore an algebra-based course as the default GE requirement for all students. Thus, CSU is unlikely to change its current developmental math structure.

The CSU experience mirrors that of many four-year universities, but Treisman and Stevenson point out that the “dirty secret” in this segment is shifted rather than solved. The main dilemma in four-year universities is not what to do about developmental math, but rather what to do about STEM students who cannot pass college algebra. Treisman calls college algebra “the most failed class in America”, and notes that it systematically culls a significant percentage of STEM students each year in the U.S.

Developmental math presents the greatest challenge at the two-year college level. As mentioned above, 85% of the 2.1 million CCCS students are placed into developmental math classes upon entry. With such an enormous system, the purpose of developmental math education inevitably becomes diffuse and blurred. Some view it as the same quantitative reasoning “safety net” used at CSU, while others view it purely as preparation for a transferable or college-



*Katherine Stevenson is a professor of mathematics and director of developmental mathematics at California State University, Northridge. Stevenson co-chaired the CIME 2015 workshop.*

level math class. Opinions vary from one extreme to the other, with administrative clashes occurring as students move between campuses or transfer from one system to another. For example, since budget cuts have led to a scarcity of classes, today's California community college students often migrate within the system to pick up needed classes. These students face a myriad of placement tests and processes because each college has been free to define its own test, cutoff score, and sequence of courses. As a result, articulation is poor and some students lose precious time and money.

Another challenge for the CCCS lies in its transfer agreements with the CSU and UC systems. To maintain their "safety net," the four-year systems require that any transferable math class have a prerequisite that includes intermediate algebra. Therefore, intermediate algebra is the default terminal developmental math course in the CCCS. Regrettably, there is a lack of uniformity across the state, and even within regions, on which learning outcomes define intermediate algebra. Moreover, since 2003, when the CSU changed its placement test (the ELM), the CSU has in effect required less algebra for entering freshmen than it does for community college transfer students. This mismatch is typical of the dysfunctional relationships between two- and four-year colleges described by Burdman:

"University limits on transferability are indicative of the predicament faced by community colleges: As key transit points for students moving through the higher education system, from high schools to universities, the colleges are crucibles for an array of expectations that are often confusing, ambiguous, or altogether conflicting. The effort to satisfy those requirements may ultimately hinder their effectiveness in serving students" [Burdman 2015b].

California's example demonstrates how developmental mathematics manifests itself quite differently in different academic systems. Yet, because students flow between the systems, it also points out the importance of having the whole of the mathematical community participate in the developmental math discussion. Obstacles to success in reforming developmental math observed by Bailey must be turned into first steps for improving it by having the mathematical community work toward:

1. consensus on a definition of "college-ready";
2. consistency in grading standards within regions;
3. placement via multiple measures that see knowledge and skills as continuous and acknowledge the importance of non-cognitive skills;
4. contextualization of "college-ready" within academic and career goals.



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# DEVELOPMENTAL MATHEMATICS: TO WHAT END?

*Based on talks by Uri Treisman, Wade Ellis, and Thomas Bailey  
as well as contributions from Pamela Burdman.*

The underlying assumption of developmental mathematics is that students need to do or know something first before starting college-level math. The intended end result of the existing developmental math model is that students who pass developmental math will benefit by being more likely to pass their subsequent college-level math classes and thereby go on to graduate. Yet, as was described in Section 1, developmental math students in two-year colleges face one to four semesters of developmental math and may spend long periods of time repeating courses they have either failed or withdrawn from. Often, having passed one course, they simply do not enroll in the subsequent course in their lengthy sequence, and ultimately leave college without a credential [Bailey et al. 2010]. The results are dismal success rates: Burdman points out, “Nationally, only 32% of students assigned to developmental math ever complete a college-level math course.” [Burdman 2015c]; and The Carnegie Foundation claims, “Only 15% of students who place at the lower level of developmental math will complete a college level course within two years.” The existing model costs too much in terms of public and private dollars and the human cost in lost time and opportunity is staggering.

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## THE QUESTIONS ARE:

- *Do the benefits outweigh the costs of developmental math?*
- *Is mathematics building a foundation for math learners or is it exacerbating the achievement gap between under- and over-served communities?*

As highlighted below, the answers may depend on the specific context.

One way to measure the benefits of developmental math is to look at the academic achievements of students who successfully complete it. In their 2004 article, Bettinger and Long find that “after accounting for selection, remediated students are less likely to drop out suggesting that the courses may increase persistence. However, they take longer to complete their degrees and are slightly more likely to transfer to lower-level colleges” [Bettinger and Long 2004]. They note that developmental math may serve as a re-sorting of post-secondary students, and their results suggest that developmental math may provide a moderate benefit to students. However, the work of [Attewell, et al. 2006] based on NELS:88<sup>1</sup> data, paints a more negative picture. These authors contrast the academic success of students who successfully completed all their developmental courses in math with students who did not ever enroll in developmental math courses. The student data is controlled for skills and coursework intensity during high school and for socioeconomic background, and the researchers ask whether students who successfully completed developmental math had better or worse outcomes than equivalent students who did not undertake remedial coursework at all. The comparison excludes the 70% of students who took developmental math between 1988 and 2008

<sup>1</sup> National Education Longitudinal Study of 1988. See <https://nces.ed.gov/surveys/nels88/>.

but either failed a course or withdrew. Thus, the authors measure the benefits of developmental math under the best circumstances. Nevertheless, they conclude that, “there appeared to be no significant difference between students who completed remedial math and students who never took remediation in mathematics” [Attewell et al. 2006, p. 914]. Thus, whatever the costs of developmental math, the benefits within the two-year college context do not appear to be validated by this longitudinal study [Bailey 2009].

If the benefits of developmental math are in doubt, the costs are not. The existing model costs the nation an estimated \$7 billion annually [Scott-Clayton et al. 2014]. However, beyond dollars spent, there are additional costs associated with taking long sequences of developmental math classes. Attewell et al. state that when students’ academic performance in high school is controlled for, “students who take two or more remedial mathematics courses in a two-year college have about a 3% lower likelihood of graduating with a degree” [Attewell et al. 2006, Table 4, p. 910]. This means that if one compares two students with similar high school GPAs, where one took several developmental mathematics courses at a two-year college and the other did not, the first student would have a 3 percentage point lower likelihood of graduating compared with the second. This significant cost to students is attributable to developmental math. At four-year universities, after controlling for high school preparation and other covariants, [Attewell et al. 2006] find that, “taking remedial coursework in mathematics [at a four-year university] might have no effect on graduation or possibly a weak negative effect on graduation.” In summary, the cost incurred by students taking sequences of developmental math classes at the four-year level is neutral to slightly negative, and for the millions of students at the two-year level it is clearly negative on balance.

The negligible benefits and significant costs of developmental math at the two-year college level put developmental math at a clear deficit. Such a conclusion demands fundamental change in the connection between college-level mathematics and its prerequisites in two-year colleges, and the change requires a shift in the way developmental math courses are approached. These are examined in Sections 4 and 5. There appears to be a need for more research on developmental education at the four-year level, but it appears that those systems fail at the next step: algebra-based general education classes. In the next section, we further examine the “tectonic shifts” that push all segments of the mathematical community to move toward curricular improvement and modernization.

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# WHAT ARE THE TECTONIC SHIFTS CURRENTLY DRIVING, OR FORCING, CHANGE IN DEVELOPMENTAL MATHEMATICS NATIONALLY? HOW WILL THESE BE FACED?

*Based on talks by Uri Treisman, Katherine Stevenson, Lee Zia, and Peter Trapa, and discussions with David Eisenbud and other National Science Foundation (NSF) Math Institute directors.*

**T**he mathematics community finds itself in a tight spot, trapped between the growing need for a scientifically and mathematically literate society and the public's perception that much of science and math is not relevant to their daily lives. While many point to the problem of producing insufficient numbers of STEM majors to fill the booming tech industry, the problem actually runs much deeper. The fact is that most jobs (from administrative assistants to provosts) now require the use of quantitative information to make decisions, yet a significant part of the population does not feel comfortable doing so. This gap has contributed to the erosion of the public's trust in the academy, and there is now a boom in legislation on topics such as articulation, remediation, time to degree, teaching loads, workplace development and outcomes-based funding. This legislation and popular opinion are forcing the mathematical community to act nationally, publicly, and regionally to define what mathematics students bound for college should know and to build successful math pathways for our students in K–14. This must be done now, before it is done for us.

## THE COMPLETION AGENDA AND INCREASE IN DEMAND COLLIDE WITH HIGH FAILURE RATES AND RECESSION

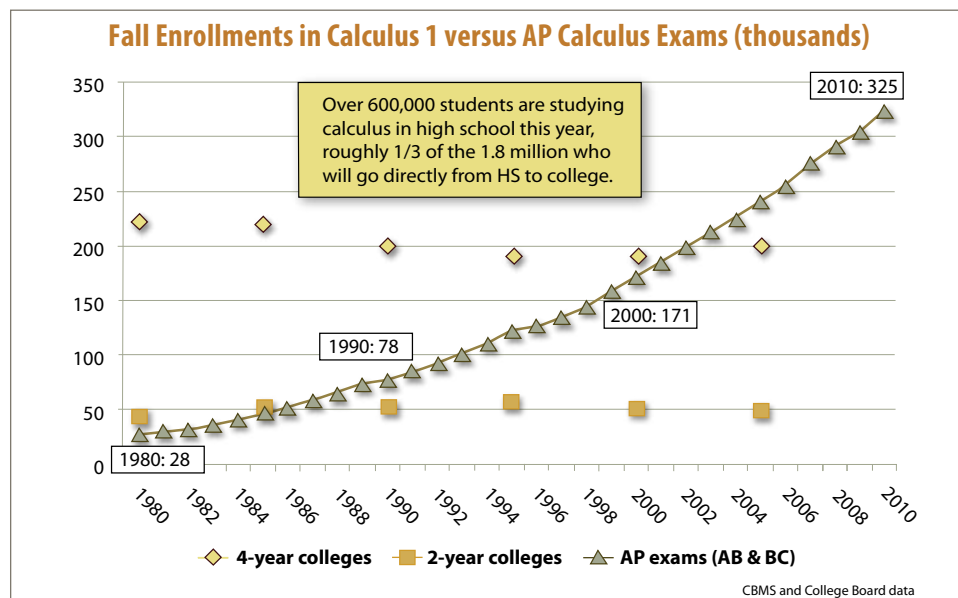
In recent years, increasing amounts of the time, talent and treasure of mathematics departments go towards teaching math at the level of college algebra (see the table below).

**Higher Education Mathematical Course Enrollment**

	4-Year Institutions				2-Year Institutions			
	1995	2000	2005	2010	1995	2000	2005	2010
College algebra and below	57%	58%	56%	54%	81%	81%	81%	80%
Calculus	37%	35%	37%	38%	10%	8%	7%	7%
Advanced courses	7%	6%	7%	8%				
Other courses (2-year)					12%	10%	12%	12%
<b>Total enrollment (in thousands)</b>	<b>1469</b>	<b>1614</b>	<b>1607</b>	<b>1971</b>	<b>1348</b>	<b>1273</b>	<b>1580</b>	<b>1887</b>

Based on Treisman, table S.2 slide 5. Adapted from the CBMS 2010 Census Report.

Moreover, while it appears that there is a growing animosity towards mathematics, work by David Bressoud points out that a steadily increasing number of students seem to be gravitating toward mathematics in high school.



Based on Treisman, slide 7 from Bressoud 2011.

As a matter of fact, the U.S. Obama administration has made college completion, with a focus on STEM, a second-term priority [White House 2014]. The focus is on equitable access by broadening participation for low-income students.

“With the growing demand for college-educated workers, a college education is one of the surest ways into the middle class. To help more students afford and graduate from college, the Administration has taken steps to address these challenges – doubling Federal investments in Pell Grants and college tax credits, reforming student loans, and taking new steps to reduce college costs and improve value. But while the President continues to push for changes that keep college affordable for all students and families, we can and must be doing more to get more low-income students prepared for college, enrolled in quality institutions, and graduating.”<sup>1</sup>



*Lee Zia, Deputy Division Director of the National Science Foundation's Division of Undergraduate Education.*

Despite the enthusiasm and the innovations, the majority of students fail to thrive in math and fail to continue in STEM fields. Bressoud points out that of the high school students who graduated in 2004 and earned credit for calculus in high school, 17% took remedial math in college, 31% took pre-calculus, and 32% never took calculus, see chart, page 17 [Bressoud 2011, slide 24]. The work of Bailey and others suggests that preparation, articulation and placement are to blame.

Unfortunately, the booming interest and failure in STEM has coincided with diminished funding at the state level for public education. Cuts in funding to public education mean fewer lower-level math classes taught by less experienced or less qualified instructors. “Crowded classrooms, waiting lists of thousands and fewer course offerings are some of the conditions students face as they arrive on campus ready to begin the fall semester,” said Chancellor Jack Scott of the California Community Colleges in 2010. The result is a level of “service” that the public finds unacceptable.

The frustration has led to legislation whose reach extends beyond funding. According to the National Conference of State Legislatures, in 2014 state legislatures passed laws on articulation (California), teaching loads (North Carolina), remediation (in five states), time to degree (in five states), and workplace development (in seven states). Outcomes or performance-based funding has arrived in Minnesota, New Mexico, South Dakota, and Utah. The impact of the legislation goes beyond high schools and community colleges and touches all segments. This puts pressure on the whole mathematical community to become engaged and work collectively and catalytically to achieve goals that are impossible for any one segment to tackle alone.

<sup>1</sup>“Increasing College Opportunity for Low-Income Students: Promising Models and a Call to Action” (White House, 2014), p. 2. Available at <https://www.whitehouse.gov/sites/default/files/docs/>



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### MATH IS READY FOR “REBRANDING”

Uri Treisman believes that American post-secondary mathematics education needs nothing short of a “rebranding” of the discipline. Rather than thinking about crises in mathematics, he suggests that the mathematical community move from its quiet position on the sidelines to a prominent place in the forefront of innovation. He lays out arguments for why mathematics is well-placed to be the field most effective in terms of curricular improvement and modernization, most supportive of successful learning outcomes in the required general education, and most responsible for upward social mobility. It follows that if mathematics were to achieve such startling results, then the dilemma of developmental math would resolve itself, as a corollary.

According to Treisman, “Mathematics can become the exemplar among disciplines in improvement, in identifying areas of consensus in a highly heterogeneous higher education landscape, and in developing and scaling innovation.” In mathematics, as compared to other subjects, creating, sharing, and implementing course materials is already part of the culture and relatively easy to do. Mathematicians are accustomed to sharing test banks, textbooks, and problems sets, and recently such materials have become a blend of online and hardcopy materials from both open source and proprietary suppliers. For example, WebWork is a commonly used open-source homework system in which exercises are authored by individual mathematicians and mathematics departments and shared openly. Departments without the resources (or will) to run their own computer servers are turning to for-profit companies that combine open source online homework systems with open-source learning management systems to provide discounted online access combined with professional technical support.

Even within assessment, mathematics has advantages. It is relatively easy in math to define course goals and student learning outcomes that are specific both in terms of topic and expectations of student work. Common examinations and grading rubrics are established (if not ubiquitous) practices that help make assessment feasible across multi-section courses or at different campuses. It is reasonable in math to ask instructors to record test scores in disaggregated format (problem by problem, student by student) without creating an undue burden. This allows challenges in specific learning outcomes to be identified quickly and precisely, making it possible to design and test potential fixes from one term to the next. Such focused, data driven course improvement encourages buy-in from faculty and improves student learning.

Mathematics has a further advantage in that it lends itself to several burgeoning learning technologies. In particular, hybridized learning fits well with mathematics’ sequential nature. Just-in-time prerequisites can be delivered asynchronously and adaptively to students while new material is delivered to the whole class or groups of students. Such formats allow the classroom to retain the momentum and cohesiveness of the group, while allowing all students the possibility of corrective action for prerequisite gaps. This puts within reach Bloom’s

“mastery learning to scale,” which he showed was a fundamental part of classroom practices that can improve student learning by two standard deviations (Bloom 1984). This blend, appropriately mediated by instructors, has been developed and proven effective at scale. The successfully scaled innovations in developmental math are less about the “disruptive change” touted by fads like massive, online, open-source courses (MOOCs) and more about *adapting* and *coordinating* effective teaching practices and resources so that instructors can more easily pursue a productive path rather than simply follow the familiar one. The Appendix lists several examples.

Mathematics is also well-positioned to be supportive of student success in the essential general education learning outcomes. The Association of American Colleges and Universities (AAC&U) has identified four essential learning outcomes:

- broad and integrative knowledge of histories, cultures, science and society;
- well-honed intellectual and adaptive skills, including analytic inquiry, communication fluency, quantitative fluency, engaging and working across differences, problem solving, and ethical reasoning;
- in-depth engagement with unscripted problems relevant to both work and citizenship, U.S. and global;
- signature work that shows the results of each student’s efforts related to a problem or project, extending over at least a semester.

As a culture with its own language, tools, and beliefs, math is well positioned to help students develop each of these. The fundamental “belief” in mathematics is that truth is accessible and can be understood through logical reasoning. Teaching mathematics with this cultural context in mind raises the subject matter above the procedural level. It opens an intellectual dialog between teachers and students about what learning and using mathematics entails. Moreover, historians, scientists and social scientists increasingly use quantitative reasoning to analyze and compare situations, structures or cultures, so mastering math content and practices sharpens the analytical and communication skills of those with interests beyond STEM. Finally, substantial work on problems relevant to society requires critical thinking, and often math can help identify problems and solutions. For example, Attewell et al. (2006) quantify discrepancies in the likelihood of students from different populations being placed into developmental math courses. In doing so they effectively demonstrate the need to fundamentally change developmental math for the sake of equity.

Treisman’s final step in rebranding math turns a weakness into a potential strength. He argues that mathematics is well suited to promote upward social mobility precisely because it is currently a barrier. While it is meritorious for any content area to work toward student success, if math does so it will translate instantaneously into greater social mobility.

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### HOW REBRANDING CAN HAPPEN

Large-scale change happens because some trusted individual or group decides to shepherd it through. The mathematical community, its associations, and institutions are well positioned to make mathematics the most successful discipline in terms of curricular improvement and modernization. The community already has the infrastructure to confront the problem, but according to Treisman it must also confront the hard work of:

- assessing existing and developing resources, and then identifying and scaling out the use of those resources;
- assessing existing and developing teaching practices that allow students to refresh prerequisite knowledge, while moving forward with new content;
- defining measurable student learning outcomes that are specific in terms of mathematical content and practice, as well as rubrics to assess them.

In particular, it is necessary to address each of these points in the context of the student's proposed academic pathway (STEM, business, humanities). General education mathematics cannot be so general that students see no connection to their field of study.

Much of the conference was dedicated to such innovations in high schools, community colleges, and comprehensive universities. These efforts vary from local endeavors, to system-wide projects, to nationwide movements [See the Appendix]. Yet the issues surrounding developmental math are not restricted to those segments. High school students hope to attend comprehensive and research (R1) universities, and community college students wish to transfer to them. Placement, preparation, and articulation form barriers to a smooth transition, and when students fall off the road to academic success, the public notices, and questions arise for all segments. There exists a real need for stewardship within the discipline and across its segments.

*Scott Peterson, Oregon State University (with microphone), in discussion with others at a CIME workshop.*



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### HOW CAN AND WHY SHOULD CHAIRS, PROVOSTS, AND OTHER ADMINISTRATORS AT INSTITUTES AND RESEARCH (R1) UNIVERSITIES MOVE CHANGE FORWARD IN DEVELOPMENTAL MATH?

*Based on a talk by Peter Trapa (University of Utah), with Duane Cooper (Morehouse College), and John Squires (Southern Regional Education Board).*

*Available at <http://www.msri.org/workshops/758/schedules/19528>.*

Institutions like R1 universities, mathematics institutes, and the NSF play an essential role in bringing issues of importance to the public eye, as well as bringing together experts who are best situated to implement change. This role is analogous to that of guiding and supporting research in mathematics. Peter Trapa, Chair of Mathematics at the University of Utah, spoke directly to the practical reasons why R1 institutions are increasingly compelled to take an interest in issues surrounding developmental math, even if such institutions offer few developmental courses themselves. Typically the lowest level of math is college algebra, one of the most failed classes on any campus. The attitude of faculty members at R1 math departments used to be that students unprepared for college algebra are someone else's problem: Either they should not have been admitted, or they should be "fixed" by someone else (e.g., developmental math programs at community colleges).

This view is changing in reaction to the "completion agenda" mentioned on page 17, which has shifted the role of higher education from propagating democracy to a much more vocational mission. States view public education as an investment in their economy, which will be recouped by future tax revenue from college graduates employed in high wage industries. With this view, it does not take long for politicians to conclude that 50-60% six-year graduation rates are not good investments. As a consequence, university administrators quickly identify classes with low success rates, like college algebra. If a department chair responds to concerns about such classes in an unconstructive way, perhaps saying "It's not our problem!", this will cause the core research mission of his or her department to suffer. That is to say, if research departments do not pay attention to these fundamental issues, funds will be diverted away from research and towards such courses. Thus, if only for the selfish reason of protecting research programs, these departments must engage with the issues that are important to the whole mathematical and educational community.

Another area of concern for R1 math departments is funding formulas. In particular, how does developmental math relate to "new" funding models like outcomes- or performance-based funding? Over the past thirty years, R1 public universities have seen huge increases in enrollment. Since the funding model has been tied to head-count, this has meant that resources dedicated to math departments continued to grow as more and more students have filtered through general education and STEM classes. As noted above, the majority of such growth has meant at levels below calculus. In the last fifteen years, departments have faced shrinking state dollars invested in public education and diminishing federal funding for basic research. As a result, budget funds tied to enrollment increases have been directed to "less profitable" parts of a department, like the training of new PhDs who require specialized faculty and small class sizes. Moreover, graduate students are often employed as teaching assistants or given research scholarships. There is thus significant budgetary pressure to redirect funds

from pre-calculus programs to graduate programs. In some cases, college algebra students have suffered in large classes taught by less experienced instructors with little oversight.

As noted above, the public noticed the shifting resources and put pressure on universities in the form of legislation on teaching loads, time to degree and outcomes- or performance-based funding. Under performance-based funding, the amount of resources coming to departments from tuition dollars is tied to the number of students who “successfully complete the program”. But what is successful completion? Is it the number of majors who graduate in four or six years, or is it the number of students who complete their general education math requirements in one or two years? Universities that have taken the former interpretation have ended up cutting the tuition dollars to their math departments by 50% or more. This is because math is typically a small major with a large service component. Ironically, this funding model, which was intended to protect students, ends up directing more dollars toward math majors, while diverting it away from graduate programs *and* math instruction at the levels below calculus. As a result, less and less attention is being paid to courses at the level of college algebra and pre-calculus, thus exacerbating the existing crisis.



*From left: Duane Cooper (Morehouse College) and Umesh Nagarkatte (Medgar Evers College, CUNY)*

In the face of this shifting legislative and fiscal landscape, it is important for chairs at R1 math departments and other math leaders, such as institute directors, to consider how they can use their political weight to help the mathematical community make improvements in developmental math. Department chairs at R1 universities and directors of mathematical institutes are generally well respected by the people in their communities and by state and federal representatives. They are often asked to attend conferences on issues such as developmental math. If math chairs and institute directors are, as a matter of habit, engaging in discussions that concern the broader mathematical community, then when called upon to join more open venues where such dialogs take place, they can do so in an informed way, and, where possible, in ways that support their colleagues at other levels. For such intersegmental discussions to work, it is important to start with modesty, patience, and a willingness to engage. No one part of the educational system has all the problems or all the solutions.

For these quite practical reasons, math chairs, institute directors, and mathematical organizations from all segments should work nationally, publicly, and regionally to find solutions. They should work nationally to convene groups that seek to find consensus on what mathematics students should know coming into and exiting colleges and universities. They should facilitate coherent and relevant teacher training and course redesign. They should work publicly to educate the general population on why we need a scientifically literate population, what mathematics that requires, and why it is required. And finally, they should work regionally to facilitate the establishment of a hub-and-spoke structure within the mathematical community that is inclusive of all members: R1, K–12, comprehensives, and community colleges. The whole community should be in the habit of collaborating on consensus building, professional development, and maintaining a rich intellectual life for all members.

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 EXAMPLES OF INTERSEGMENTAL COOPERATION FOCUSED ON MATH EDUCATION
 

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1. **Math for America (MfA)**, run by former AMS president John Ewing and funded by James Simons, has a vision of American mathematics education that places “an inspiring, expert teacher in front of every student in America.” To achieve this vision, MfA works to make high school teaching, “a viable, rewarding, and respected career choice for the best minds in science and mathematics”. MfA teachers train by working together at the Simons Institute and by earning credits in the CUNY system. So, in effect, they are an outreach from the R1 community to the K–12 community via the comprehensive university system. Their core values are:
  - Teaching is a true profession.
  - Great teachers are always learning.
  - Excellence comes out of deep collaboration and ongoing growth.
  - Teacher accountability should be complex.
  - Honoring greatness elevates the profession.



2. **Transforming Post-Secondary Education in Mathematics (TPSE)** offers an example of cooperation among four-year universities that “aims to effect constructive change in mathematics education at community colleges, four-year colleges and research universities.” TPSE’s founders are Eric Friedlander (USC), S. James Gates, Jr. (UMD), Mark Green (UCLA), Phillip Griffiths (IAS), Tara Holm (Cornell), Uri Treisman (UT, Austin), William Kirwan (University System, MD). The organization envisions that “post-secondary mathematics education will enable any student, regardless of his or her chosen program of study, to develop the mathematical knowledge and skills necessary for productive engagement in society and in the workplace”. To achieve its vision, “TPSE Math will facilitate an inclusive movement to strengthen post-secondary education in mathematics by working closely with — and mobilizing when necessary — faculty leaders, university administrations, membership associations, and relevant disciplinary societies in the pursuit of mathematically rich and relevant education for all students, whatever their chosen field of study. TPSE Math will identify innovative practices where they exist, advocate for innovation where they do not, and work with and through partners to implement and scale effective practices.”





3. A successful example of intersegmental cooperation in math education the *Common Vision* group founded by the **Mathematical Association of America (MAA)**. This group believes that, “it is time for collective action to coordinate existing and future efforts [in math education] in such a way that everyone is pulling in the same general direction to leverage the collective power of the whole to accomplish our goal of improved student success, especially in the first two years of college”. They work to bring together “leaders from five professional associations in the mathematical sciences – AMATYC, AMS, ASA, MAA, and SIAM – to collectively consider undergraduate mathematics curricula and ways to improve education in the mathematical sciences”. Their first report appeared in 2015 and is available at <http://www.maa.org/sites/default/files/pdf/CommonVisionFinal.pdf>.

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# WHAT ARE THE NON-COGNITIVE ISSUES AFFECTING STUDENT PERFORMANCE IN MATHEMATICS?

*Based on presentations by Wade Ellis, Rachel Beattie, and Myra Snell.*

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## WHY IS NON-COGNITIVE LEARNING SO IMPORTANT IN DEVELOPMENTAL MATH?

*Based on the presentation of W. Ellis (West Valley College).*

*Available at <https://www.msri.org/workshops/758/schedules/19525>.*

James Stigler studies the cultures of societies that score well on international mathematics exams. He finds that while these cultures are all very different, what they share are teachers who produce environments for their students where productive struggle is expected and encouraged, explicit connections are intentionally made between and among mathematical topics, and deliberate practice is required. For example, students in the cultures studied by Stigler are not doing “1 through 40 odd” as group work, while the teacher moves around the room facilitating. Instead, they struggle in groups, individually, and as a whole class with meaty problems. Deliberate practice does not mean repetitive practice, rather it means increasing variation, complexity, and challenge over time in order to stave off premature automaticity and to encourage deep learning. Top students in the U.S. do productively struggle and make explicit connections, and their practice is deliberate. The problem is that too few U.S. students actually have the opportunity to engage in that rich academic culture.

Many attempts have been made to reform developmental and entry-level mathematics courses and so create rich cultural learning environments in the classroom. Wade Ellis, a leader in the California Community College System, points out that curricula have been revised to include more real world problems, more project-based learning, and more active learning. Methods of instruction have been accelerated, modularized, computerized, and supplemented. Learning habits have been observed, studied, and shared with students. However, Ellis points out that in 2011, the Manpower Demonstration Research Corporation (MDRC) conducted a literature review of the most promising reforms from 1970 to 2010 in the hopes of suggesting profitable directions for future investment and change. The report found that, “even the most promising innovations ... led to only modest improvements in achievement, demonstrating that progress remains to be made in developing and evaluating more robust reforms” [Rutschow and Schneider 2011]. The report concluded by saying “minor modifications in developmental education programs are unlikely to produce dramatic improvements”.

Similarly, Ellis points to the February 2015 issue of *MathAMATYC Educator* (volume 6, number 2, available at <http://www.amatyc.org/?page=EducatorFeb2015>).<sup>1</sup> The issue broke down the impact of different interventions, which Ellis summarizes as follows:

- College algebra redesign resulted in a 10% increase in student success.
- Computer-based instruction resulted in no difference in student success.
- Supplemental instruction resulted in no difference in student success.

So, *why have students not been able to benefit from previous interventions?* Several recent developmental math reform models (see the Appendix, page 54) have produced success by looking closely at exactly this question.

Practitioners in the field observe that students who make it to community college in the U.S. have already demonstrated resourcefulness and perseverance in achieving the dream of entering college. Yet in the face of developmental math, the resourcefulness often seems to dissipate. Many of these students react to an entry-level math course as this Statway® student did:

“I am embarrassed by how stupid I am and suddenly feeling very discouraged ... I can’t even tell which fraction is bigger than another, or where they should fall on the number line. I feel like crying.”

This is the foundation on which a culture of productive struggle, explicit connections and deliberate practice must be built. Observations by teachers as well as social science research (see for example work by Larnell in Section 5) suggest that the issues barring so many students from success in developmental math lie in the space where students’ attitudes, behaviors, and beliefs conflict with those of their instructors, their classrooms, and their academic institutions.

What is needed is a means of building bridges between academic cultures. This requires attention to non-cognitive or “soft skills.” These skills are related to motivation, integrity, and interpersonal interaction. They are the counterpoint to cognitive skills, which are defined as the mechanisms by which we learn, remember, problem-solve, and pay attention. The non-cognitive skills relate to personality, temperament, and attitudes. They impact intellect and knowledge, but less directly or obviously than cognitive skills.

In recent years the U.S. mathematical teaching and learning community has started to explore interesting research on how non-cognitive aspects of learning can be cultivated in students and faculty. Carol Dweck’s concept of “growth mindset” [Dweck 2006] and Jo Boaler’s notion of “brain malleability” [Boaler 2013] suggest that people are not limited to the kind of brain they have at any given point in their lives. Rather, a person’s experiences and choices determine the sort of brain he or she will have in the future. If this is correct, then the mathematical culture that students and teachers live in will heavily determine how their brains will develop. Identifying aspects that can be improved or refocused can potentially unlock access to the rich academic culture currently beyond the reach of so many students and teachers.

<sup>1</sup> Related articles referenced in this volume of *MathAMATYC Educator* include [Dweck 2006], [Flek et al. 2015], [Hopf et al. 2015], [Horton 2015], [Quesnell and Hadley 2015], [Rutschow and Schneider 2011], [Feldon, Sattler and Orrange 2015].

### WHAT PARTS OF NON-COGNITIVE LEARNING CAN WE IMPACT AND HOW?

*Based on the presentation by R. Beattie (Carnegie Foundation for the Advancement of Teaching). Available at <https://www.msri.org/workshops/758/schedules/19533>.*

The Carnegie Foundation for the Advancement of Teaching administers the Statway® Project which created a pathway to and through college statistics for developmental math students. Fundamental to Statway®'s program is its extensive focus on the non-cognitive aspects of learning among community college students. They chose to focus on productive persistence, the idea that students continue to persist when faced with challenges (tenacity) and do so efficiently and effectively (good strategies). The project relates directly to how instructors impact or change students' behavior and beliefs, and its analysis gets at the heart of how students and instructors interact.

The Statway® Project identified four primary drivers of productive persistence:

- Students believe that they can learn.
- Students feel socially tied to peers, faculty, and the course.
- Students believe the course has value.
- Students have skills, habits and know-how to succeed in a college setting.

Statway® students at 19 colleges were surveyed as to whether they felt secure in these drivers of productive persistence. Based on their responses, they were categorized as “high risk (insecure in 4 factors),” “moderate risk (insecure in 2 factors)” or “low risk (secure in all factors).” The risk levels were then compared to measures of participants' academic success in a yearlong Statway® course. High- to moderate-risk students were more likely than low risk students to score below the median on baseline math knowledge, fail the mid-course assessment, and fail the end-of-course assessment. Low-risk students were almost four times more likely to get a B- or better in the Statway® class than were high-risk students. Moreover, the gap between low- and high-risk students widened over the course of the year. This suggests that early and continued intervention targeting these drivers is essential. Having identified some drivers of productive persistence, the Statway® group then focused on determining whether those drivers are changeable, and whether such change leads to student success. They hypothesized that a student's mindset and sense of belonging determine how open he or she is to improving the four drivers of productive persistence. They then designed



*Left to right: Carol Edwards (TODOS: Mathematics for ALL), Linda Gojak (National Council of Teachers of Mathematics) and Brendan Kelly (Harvard University). Back row (left): Umesh Nagarkatte (Medgar Evers College, CUNY).*

specific interventions to move students from a fixed to a growth mindset (to foster a sense of belonging), and then analyzed the impact of these interventions on academic success rates. See Appendix pages 56-57 for more details on Statway®.

### ***Growth mindset***

A mindset is a perception that people hold about themselves. Believing that you are either “intelligent” or “unintelligent” is a simple example of a mindset. People are often unaware of their mindsets even though these can play an important role in academic achievement, skill acquisition, personal relationships, professional success, and other dimensions of life. The concept of a growth mindset was developed by psychologist Carol Dweck and popularized in her book, *Mindset: The New Psychology of Success*. To understand what this means within mathematics, consider the following statement:

Being a ‘math person’ or not is something about you that you really can’t change. Some people are good at math and other people aren’t.

Strongly agreeing with this statement is evidence of an individual having a fixed mindset. Strongly disagreeing suggests a growth mindset. Dweck writes, “In a growth mindset, people believe that their most basic abilities can be developed through dedication and hard work—brains and talent are just the starting point. This view creates a love of learning and a resilience that is essential for great accomplishment.”

A group of Statway® students were asked whether they agreed or disagreed with the statement above at the beginning of the course. Approximately 68% of Statway® students surveyed agreed with the statement. To counter this fixed mindset, Statway® project leaders decided to run a randomized control trial at Santa Monica City College. At the start of the following year, half of the Statway® students spent 30 minutes reading an articles on the neuroscience of learning and on growth mindset. One such reading included the following quote:

“Most people don’t know that when they practice and learn new things, parts of their brain change and get larger, a lot like the muscles do. This is true even for adults. So it’s not true that some people are stuck being ‘not smart’ or ‘not math people.’ You can improve your abilities a lot, as long as you practice and use good strategies.”

The other group read a control article about the brain, but this article did not have the messages about growth mindset. Following the reading and discussion, both groups of students participated in follow up activities:

1. They summarized the article and related it to an experience outside of mathematics where they grew.
2. They wrote a letter to a future student telling that individual about the growth mindset message. This helps students buy into the arguments more by putting them in their own words.
3. They engaged in a rich, challenging mathematical problem immediately after reading the article to give them evidence that they can learn math.

Students in the group that read the growth mindset version of the article, rather than a control version of the article, were half as likely as the control group to withdraw from the class over the course of the year. The Statway® researchers concluded that this was one method to shift a student's mindset and positively impact academic performance.

### *Belonging uncertainty*

People commonly question whether or not they belong in new social and academic settings, especially when they are targeted by stigma and negative stereotypes. This uncertainty makes the meaning of negative social events more ambiguous and more likely to be felt as an aggression (either micro or macro). After each negative event, the individual will ask: "Do I belong here or don't I?" [Walton and Cohen 2007]. Thus a student from an underserved population or an underrepresented group may be more susceptible to the negative consequences of alienation. The following quotes from Statway® students capture that sense of not belonging.

- "I'm embarrassed to be at community college because high school teachers said I would end up at community college because I'm lazy."
- "I don't have any friends here. In between classes, I sit in my car and see everyone talking to others and I wonder: how did everyone else make friends?"
- "I felt that if I stopped coming no one would even notice."

The concerns raised by such reflections caused Statway® researchers to ask all students the following question:

"How often, if ever, do you wonder: 'Maybe I don't belong here?'"

Students' responses were compared to their likelihood of withdrawing from the class after four weeks. The students who often wondered whether they belonged were approximately 3.6 times more likely to withdraw from the class after four weeks than were those who rarely had such concerns.

Statway® combats the concerns around a sense of belonging by guiding students across a year and having students collaborate in their own learning. The Statway® researchers did not directly measure any changes in a sense of belonging among their students, however they believe that the academic success of Statway® students (see Appendix) indicates that such a change may well have occurred.

Thinking positive thoughts is not enough to improve academic results. However, students who have the mindset that they can learn math, that they belong, and that math has value, are more likely to engage in the everyday studying and learning habits that earn higher grades in the class. They are more likely to engage in the process of learning because they understand it is something they can control, and thereby obtain higher grades. These higher grades then provide further evidence of their ability to learn math, offering proof that they do belong, and that math has value, thus initiating an upward spiral.



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## PEDAGOGICAL AND STRUCTURAL CHANGE TO ACHIEVE PRODUCTIVE PERSISTENCE, EXPLICIT CONNECTIONS, AND DELIBERATE PRACTICE

Based on a presentation by W. Ellis ((West Valley College). Available at <https://www.msri.org/workshops/758/schedules/19525>.

In order to scale up interventions on non-cognitive aspects of how teachers teach and students learn, Ellis claims that it helps to view learning as a process and to measure relevant aspects of an individual's performance within that process. For example, a typical classroom may have a teacher assigning many practice problems for homework and providing sample exam problems in lecture and review sessions to prepare students for evaluation of learning. Such a teacher may believe that it is unfair to have students try to solve exam problems with new context. Ellis argues that the likely outcome of this teacher's performance is students who are unable to transfer knowledge from one context to another. These students will believe that, "I need to remember exactly how to solve each of the types of problems shown and practiced in class." Similarly, in a classroom where the teacher controls attendance, participation standards, work product formats, classroom conventions and consequences for non-compliance, students are likely to rebel due to a sense of powerlessness. Such students often appear irresponsible. They may think, "I don't really care what happens with my grade if I can't do what I want to do or do it in the way I want to do it." Students who demonstrate an inability to transfer knowledge or who have an irresponsible attitude are at high risk, according to Horton [2015], and often find it hard to engage in productive persistence and cognitive challenge. Moreover, they may not notice when explicit connections are made.

Ellis is a member of the Process Education community that has established professional and student development programs that apply the Theory of Performance to the learning process [Apple and Ellis 2015]. The Theory of Performance was developed to form frameworks that explain performance as well as performance improvements. According to the theory, an individual's performance of a task can improve, provided that the performer knows:

- His or her identity within the task.
- The skills and knowledge necessary for the task.
- The context of the task.
- The personal factors within and outside of the performer's control.

Ellis's group identifies relevant performance areas for developmental math, including *cognitive complexity*, *method content delivery*, and *control over delivery*. For example, do teachers control the classroom and pitch content low, or do they challenge students mathematically while allowing them control over aspects of classroom norms? For each performance area, they develop concrete methods to train teachers to become proficient in non-cognitive aspects of learning, and also be able to teach those skills to their students. All this is done within the context of developmental mathematics, where both students and teachers struggle to achieve success. In this way they mitigate Horton's risk factors and create a community in which students will learn to persevere while developing an academic mindset, as well as develop various learning strategies and social skills that will open the door to successful academic performance.

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## STRUCTURAL CHANGE

*Based on a presentation by M. Snell (Los Mendanos College), respondent to W. Ellis.*

The non-cognitive issues mentioned above play an important role in student learning, and need to be acknowledged, but Myra Snell, Professor of Mathematics at Los Medanos College in Pittsburg, California, asks whether doing so is a necessary condition for increasing the completion rate of transferable math. Is it sufficient? She makes a convincing case that pedagogy and professional development are not enough. She argues that there are structural changes (also, in a sense non-cognitive) that must also be confronted.

Snell tells a poignant story of what led her to realize that the structure of developmental mathematics had to be radically changed. Her department won a large five-year grant to improve its algebra sequence. The department took the standard “algebra for everybody” approach, and assumed that everyone needs to know algebra, and the grant was dedicated to finding a way of teaching algebra well. On the curriculum side the department contextualized the content and approached algebra from a mathematical modeling perspective. Instruction was activity-based, and instructors wrote exams together and used student performance on exams to hone the classroom activities. To specifically address the affective domain, counseling support was integrated in the classroom. To support faculty with the content, pedagogy, and non-cognitive aspects of the course, the department arranged for faculty to work collaboratively using Stigler’s lesson study model. Instructors met weekly to work on classroom activities together, to observe each other’s classes, and to choose artifacts from the observations to evaluate whether the activity was producing the learning they wanted. To address student preparation, the department added strict placement prerequisites to each course in the algebra sequence. The grant goals were to increase course success rates by 10% in each course, and those goals were met.

It was then that Snell had what she describes as a “stupifany.” She asked her institutional research office to track the cohort of students for three years to see how many of them passed a transferable math class. The answer was a devastating 17%. This led Snell to ask: *What kind of improvements would we have to see in the passage rates of our pipeline in order to improve outcomes?*

A little arithmetic produces the answer. Students in developmental math take between one and four semesters of developmental math, followed, it is hoped, by a transferable course. Imagine the path of students placed into a two-year developmental math sequence followed by a transferable math course. Assuming a 70% pass rate for each of the three courses and a 70% persistence rate from each course to the next (an assumption that exceeds typical pass and persistence rates in developmental math pipelines), only 17% will complete transferable math ( $0.7$  to the 5th power). Assuming phenomenal pass and persistence rates of 80%, the completion rate of transferable math is an unacceptable 33% ( $0.8^5$ ). Snell’s calculation demonstrates that while working on the cognitive and non-cognitive aspects of developmental

mathematics is important, the solution must include *shortening the academic lifespan of developmental math*. There are four main approaches:

1. The **stretch model**, in which students place into transferrable math immediately. They receive additional support and time to complete the transferrable course, while also completing the required developmental material.
2. The **accelerated model**, in which students take one intensive semester of developmental math. In many cases the content is tailored to the subsequent transferable math course to improve alignment between remediation and college-level work.
3. The **boot camp model**, in which students and faculty get intensive and compressed instruction outside of the regular academic schedule.
4. The **co-requisite support model**, in which the student is mainstreamed into the college level course and enrolls concurrently in a remedial support course.

All these models engage students in direct instruction on mathematical content and practice, as well as in non-cognitive issues. They each solve several issues that plague developmental math. All four avoid the attrition issues related to the long sequences of classes mentioned above. Also, all four devalue or eliminate placement tests, which have been shown to be unreliable in validity studies (Bailey, Judith Scott-Clayton). The accelerated and stretch models avoid the practice of requiring developmental math students to become proficient in a generic list of mathematical topics. Often such lists are poorly aligned with the students' transfer-level math courses.



*Mathematical Sciences Research Institute, Berkeley.*

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 COMMENTS BY K. STEVENSON
 

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These approaches to remediation, particularly the first two, are controversial because many of the implementations of the models significantly reduce, and in some cases eliminate, direct instruction on and assessment of algebra skills. This conflicts with university and college entrance requirements that focus predominantly on algebra preparation. The justification for reduced emphasis on algebra rests on the fact that the target populations for these implementations are non-STEM students. The idea is to create alternate “pathways” through developmental math to college math. These pathways are viewed by some as a worrying method of tracking students out of STEM fields early in their academic careers. Pathway proponents point out that each year under the current system 1.2 million U.S. community college students are “tracked out” of AA and BA degrees entirely. So, it would be hard to do any worse than the current system in two-year colleges. These programs have documented dramatic success in getting students to and through college level statistics and quantitative reasoning courses (see Appendix). This indicates that better alignment between the type of quantitative reasoning required and the student’s field of study promotes a more successful learning environment. However, the controversy hinges on *what college-ready in math really means and whether entering college students are prepared to make a sound choice about their future majors and careers*. The lack of consensus on these two questions underscores the importance of developing venues in which intra- and intersegmental discussions occur in a productive and collegial manner to ensure that policies around mathematics remediation are serving all students equitably.

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# EQUITY ISSUES IN DEVELOPMENTAL MATH

*Based on talks by Gregory Larnell (University of Illinois at Chicago), April Lea Go Forth (Resources for Indian Student Education), and James Gray (Community College of Aurora).*

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## WHAT DOES IT FEEL LIKE TO BE A PROBLEM?

*Based on a talk by G. Larnell (University of Illinois at Chicago). Available at <https://www.msri.org/workshops/758/schedules/19533>.*

In *The Souls of Black Folks*, W. E. B DuBois asks, “How does it feel to be a problem?” DuBois refers to the nature of identity and black life in the U.S. in general, but the question is poignant when focused on the identity of black and brown students within the U.S. academic culture. It powerfully demonstrates the ease with which discussions of reform and change place students in the framework of “being a problem”. And framing learners as the problem—particularly black and brown learners—assigns the deficit to *them and absolves the institutional structures of responsibility*. Like DuBois, Dr. Gregory Larnell, Professor of Education at University of Illinois at Chicago, suggests that a more appropriate starting point is to investigate learners’ experience.

- What does it mean to be a black or brown learner in non-credit-bearing remedial/developmental mathematics courses?
- Who are our students becoming as mathematics learners in these courses?
- What psychosocial phenomena are emerging amid their learning experiences?
- Who are the socialization agents that contribute to or detract from the math-learning experience there?
- How do the socialization agents contribute to the construction of the learner’s identity?

**“The case study indicates that the problem appears to be deeply embedded and exists in places beyond the control of the classroom.”**

Larnell’s research draws from three recent studies involving approximately 150 undergraduates at four-year institutions. They are largely qualitative studies of learners’ experiences in the classroom and in the broader institutional environment. The methods included ethnographic observations, video analyses of classroom episodes, and semi-structured interviews with students enrolled in developmental mathematics. Researchers used videos not only as primary material, but also as a means for students to observe classroom situations from the outside and reflect on those observations. Students were interviewed to unpack their math backgrounds, discuss classroom experiences and classroom artifacts, and explore other experiences related to their identity as math-learners. The goal was to give the students a voice in answering Larnell’s variant of DuBois’ question: *What does it feel like to be a problem?* What emerged were two types of obstacles facing these students: identity (stereotype) threat and the “cooling out” phenomenon.

### ***Stereotype Threat***

Stereotype threat is a situational predicament in which people are or feel themselves to be at risk of confirming negative stereotypes about their social group. For example, students’ racial, gendered, and mathematics-specific identities might be primed in ways that cause them to question themselves and their academic trajectories. Students may experience stereotype threat over the course of a semester, a year, or more. Larnell asks: What does it look like? Where does it happen? Who is involved? How do learners react? The following case study helps to paint the picture of stereotype threat in action:

On his first day at the university Cedric walks up and down the hallways looking for his math class. As he does so, he is scoping out the building, noticing who is seated in the classrooms and what is written on the boards in the rooms he passes. He is getting oriented within the context. He does this day after day, noticing that in his classroom there are more black and brown students in the seats, and there is no calculus on the board. By comparison, in the rooms around him there are fewer black and brown faces, and calculus and other more advanced topics on the boards. He recognizes the calculus because he took AP calculus in high school. He had been very successful in high school and had plans to pick up a STEM major. However, somehow he ended up placed in a non-credit bearing math class through the placement exam. So as he is doing his “identity contingency detection work” he is crafting ideas about who can do mathematics, and those ideas are racialized, and in some cases gendered [Larnell 2014].



**“It is in the realm of the university and the student community, where the university staff and student organizations play an important role.”**

Larnell believes that students informally and unconsciously start to categorize math learners and what they look like, indexing them by race and gender. This is how students try to make sense of their own experiences and how they relate to all possible academic trajectories available to them within the institution.

### *Cooling-out Phenomenon*

“Cooling out” was originally defined as a systematic (and perhaps inadvertent) process in which socialization agents cool out students as they take courses and seek advice (see work of Peter Riley Bahr, University of Michigan). Often well-meaning actors slowly discourage students from identifying themselves as mathematicians or STEM students. For example, students in non-credit-bearing mathematics courses may be subjected to socialization messages that inadvertently lead them to disidentify with the domain. Larnell finds that the cooling out phenomenon plays out in ways that are asystematic, as they involve peer networks and home supports, as well as the constellation of support systems around learners. The following case study demonstrates how asystematic cooling-out occurs:

At one university, a support network was developed in which older students worked with new students to help them adjust. The senior students would put on skits for the junior students to try to give them a sense of the college experience. One such skit was about the non-credit-bearing remedial math experience. It was a socialization stage for younger learners to learn from older, more experienced learners: Who was successful, and what they did; who was not successful, and what they did. As an institutional practice, this mechanism ran the risk of slowly teaching students (who had relatively high aspiration for STEM careers, in some cases) that mathematics might not be a good subject for them to pursue. It socialized students to move to other majors where they saw more people that looked and talked and felt more familiar, away from the subject matter that they had originally intended to study.

The case study indicates that the problem appears to be deeply embedded and exists in places beyond the control of the classroom. It is in the realm of the university and the student community, where the university staff and student organizations play an important role.

### NATIVE AMERICANS: A CASE STUDY ON THE IMPACT OF STEREOTYPE THREAT AND COOLING OFF

*Based on a talk by April Lea Go Forth (Resources for Indian Student Education).*

*Available at <https://www.msri.org/workshops/758/schedules/19533>.*

The impact of cooling off and stereotype threat is powerful. April Lea Go Forth, who has deep experience working with Native American students, has observed this firsthand. She notes that all students come into the classroom with hardwired information about what is natural for them. Students whose culture matches that of the classroom generally thrive. Problems arise when a student feels as though her way does not fit with the ways of the classroom. Such a student questions whether she belongs, or whether what is being taught is meaningful. If language issues are thrown into the mix the result is a full-on clash between the student's and the classroom's culture.

The tragic irony is that so many of the cultures whose children feel alienated by the classroom are the very same cultures who sacrificed tremendously (in terms of land, life, and language) in order to guarantee access to education for their people. For example, legal treaty laws with Native American tribes specified educational rights. These rights were paid for at the expense of land. Moreover, Go Forth points out that, "Native people worldwide see science and understanding Mother Earth as part of their culture." It follows then that as mathematics is the language of science, Native students should come into math classrooms with a great deal of social capital. And yet, the opposite is true.

National data show that Native American students do well in first through third grade, where learning is contextualized. Much of the learning takes place in a storytelling environment where students can imagine themselves as the protagonist. Go Forth feels that the informal context may counteract the fact that the students see few, if any, faces like their own. However, as instruction becomes more text-based in fifth through seventh grade, Go Forth reports that students see less and less of themselves in their learning. At this age, data show



*Left to right: April Lea Go Forth (speaking), Ann Edwards, Rachel Beattie, and Gregory Larnell.*

decreases in self-esteem and a sense of belonging. Students start skipping school, and subsequently, there is a drop in performance that worsens as students move to high school and college. Since this fits the expectations of the student and the system, no one is surprised, and thus the downward cycle proceeds unabated. As a result, Native Americans have the highest academic dropout level of any group of U.S. students.

Cooling off and stereotype threat are not well enough understood for us to know what concrete steps need to be taken to mitigate them. However, there are several informal networks that have proven effective, including home networks and informal student groups that band together out of their awareness of the problems. Such informal solutions can enrich the formal professional development of staff, teachers and students. In particular, Go Forth stresses the importance of finding teachers who want to have a connection to the community they teach in and who are capable of attaining it. Once identified, such teachers must be trained in learning and teaching styles that relate to all their students. Moreover, teachers must be provided with, or encouraged to develop, teaching and testing materials that connect to their students' cultures.

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### THERE'S SOMETHING GOING ON HERE: IDENTIFYING AND CORRECTING INEQUITY IN COLLEGES AND UNIVERSITIES

*Based on a talk by J. Gray (Community College of Aurora).*

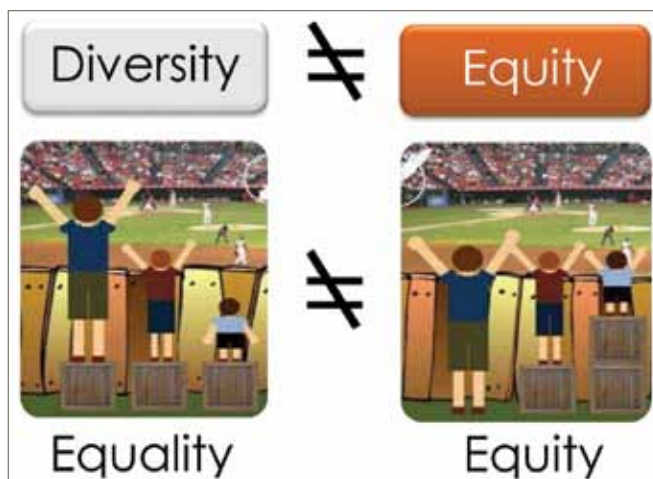
*Available at <https://www.msri.org/workshops/758/schedules/19536>.*

Equity is an abstract concept that holds different meanings for different people. One of the most common misperceptions is that equity is a concept synonymous with equality or diversity. Most institutions measure equity in terms of input and access: Does their student body or faculty reflect the ethnic, racial and cultural mix that surrounds them? Are resources equitably distributed among populations within the university? The Center for Urban Education at the University of Southern California helps academic institutions achieve equity. Its bedrock principle is that equality is not equity. The following vignette helps make the distinction clear.

Imagine a group of twelve people trying to look over a wall. One is tall and can see over the wall, ten are of medium stature and can peek over the wall, and one is too short to see over the wall. Let's assume that this group accurately reflects the diversity of height in their community. If each person is given an identical crate to stand on, is that equitable? Well, if the crate is designed for the majority (i.e. medium stature individuals), it is not equitable. The tall person does not need a crate, and that crate should go to the short person.



*James Gray (Community College of Aurora).*



Slide 6 from James Gray's talk "There's something happening here: Lessons on transformative mathematics education from Colorado's equity in excellence project".

The Community College of Aurora in Colorado was proud that their diversity matched the racial and ethnic breakdown in the community: 40% white, 25% black and 21% Hispanic. Moreover, they worked hard to ensure that all students had equal access to resources. Nevertheless, when they started working with the Center for Urban Education, they discovered that the outcomes were not equitable. The graduation rates did not break down as 40% white, 25% black and 21% Hispanic, and the developmental math results were also inequitable.

Completed Two Levels of Developmental Math	
Female	25.2
Male	22.4
Black	17.2
Hispanic	27.9
White	27.9

Adapted from slide 19 of Gray's talk.

The campus was valuing diversity and equality rather than equity. The Center for Urban Education facilitated the development of mechanisms by which the Community College of Aurora could identify and correct inequities arising from the campus infrastructure and policies, and from its faculty and staff's attitudes, behaviors and beliefs. The creation of such mechanisms is driven by the Center's "equity scorecard," a tool analogous to "balance scorecards" used by businesses to match visions and goals to outcomes. The equity scorecard

guides the process of defining specific and relevant measures of the gaps between goals and outcomes. It identifies and implements practices that should remove or reduce those gaps, and then measures them again. Thus the scorecard acts as a guide to completing and repeating the assessment cycle. It combines tools from the business world's balance scorecards with theories relevant to equity: organizational learning theory, practice theory, and critical race theory.

The Center for Urban Education does not tell institutions what the problems are or how to fix them. Instead, campus "evidence teams" are taught to conduct research for evidence of specific patterns of institutional behavior:

- institutional practices that are supporting African American, Latino, and Native American students;
- practices that are inadvertently contributing to or failing to address inequities;
- how practices could be modified, reconsidered, or replaced to close the equity gap.

This focus on inquiry as a strategy for institutional cultural change immediately engaged the faculty and staff at the Community College of Aurora. Evidence teams were formed and taught to use specific protocols to look into the issues enumerated above. One team was tasked with scrutinizing the campus environment. Team members sat in spaces such as the registration office. They were trained to look for clues about how students of different races felt upon entering that space. Often observers sat for an hour and saw no difference, but after repeated visits, they started noticing something. The collected "somethings" were discussed with a project specialist from the Center for Urban Education, and new directions were suggested. The Center did not tell the college what to find. It told them where and how to look, and then followed up with advice on how to understand what was observed.

According to James Gray, Chair of Mathematics at the Community College of Aurora, this method of inquiry manifested itself importantly in the realm of teaching effectiveness and faculty development. Assume that one has two teachers, Teacher A and Teacher B. If Teacher A has lower passage rates than Teacher B, and Teacher B is asked why this is the case, the discussion is generally causal: "I do this and it has that effect on student learning." By contrast, if Teacher A is asked why, the discussion proves to be more reactionary: "I have high standards," "The students' skill level is low," or "The students lack motivation." The importance of these conversations changed when the Center for Urban Education suggested that Gray break the data down by ethnicity. For example:

Ethnicity	Teacher A	Teacher B
Asian	76%	83%
Black Non-Hispanic	55%	70%
Hispanic	63%	80%
White Non-Hispanic	71%	75%
Total	66%	77%

*Adapted from slide 11 of Gray's talk.*

In general, Gray notes that in the scenario just described Teacher B had no idea why she is outperforming Teacher A by 4 percentage points with white students as compared to 17 percentage points for Hispanic students, yet Teacher A still explains the difference with the same conversation about “high standards,” “student skill level,” and so on. Both conversations are disappointing, but both offer an opportunity for improvement. Teacher B can be observed specifically for her strengths in teaching Hispanic students, and Teacher A has a specific goal to work towards. In theory, teaching techniques can be amended to produce more equitable outcomes, and biases can be remedied.

However, while comparing the two instructors’ outcomes may be useful from a research perspective, it is too confrontational for professional development and would probably result in mutiny. A more comfortable way to begin is to frame the conversation in terms of international students. It is generally accepted that students from certain countries find it difficult to adapt to a classroom culture that expects students to engage with the faculty member by asking questions or working in groups. Such students may feel shy about “interrupting” the teacher and may not feel confident using English to express themselves in a group. Instructors do not in general find it threatening to discuss potential strategies that will help their international students make the cultural adjustment. They usually find it natural that the system would treat international students differently (not equally) so that they can more equitably access the classroom activities.

The conversation is more awkward when it turns to a discussion of students raised in the same city but born with different complexions, where obstacles to change are more challenging to overcome. The Center for Urban Education helps by laying out five principles [Witham et al. 2015] for designing a good environment that fosters equity.

1. Clarity in language, goals, and measures is vital to effective equitable practices.
2. “Equity-mindedness” should be the guiding paradigm for language and action.
3. Equitable practices and policies are designed to accommodate differences in the context of students’ learning—not to treat all students the same.
4. Enacting equity requires a continual process of learning, disaggregating data, and questioning assumptions about relevance and effectiveness.
5. Equity must be enacted as a pervasive institutional- and system-wide principle.

The principles are best understood in context, and below is an example of how they played out in the math department at the Community College of Aurora under the guidance of Chair James Gray.

### ***1. Clarity in language, goals, and measures is vital to effective equitable practices.***

The math department at Aurora observed that although 65.5% of students in the two lowest developmental math courses successfully completed the course, only 24% ultimately completed the subsequent developmental math class. The department set a goal of getting greater numbers of students to complete the developmental math sequence by redesigning developmental math to focus on study skills as well as mathematics, and by accelerating its format. The result over a two-year period was that 45.2% of these students passed the course (21.1% increase). This aggregate result appeared to indicate that the intervention worked. However, at the Center’s suggestion, the math department disaggregated the data by race and ethnicity. These data told a different story.



	Completed Two Levels of Developmental Math	Completed Redesigned Course
Female	25.2%	50%
Male	22.4%	36%
Black	17.2%	29.6%
Hispanic	29.7%	42.2%
White	27.9%	67.4%

*Adapted from slide 19 of Gray's talk.*

The intervention had benefited the white students significantly more than all other groups. It was not an equitable intervention because it gave the biggest boost to those who were already faring better than the others.

Gray also used the Center's inquiry model to look at teacher efficacy and professional development. He shared with each instructor a table like the one below comparing that instructor's passage rates to average rates, with the data disaggregated by race and ethnicity.

Ethnicity	Teacher A	Average	Difference
Asian	55.6%	79.5%	-23.9%
Black Non-Hispanic 43.5%	55.8%	-12.3%	
Hispanic	78.9%	69.9%	9.0%
Native American	25.0%	63.8%	-38.8%
White Non-Hispanic	65.9%	72.7%	-6.8%
Unknown / Not Reported	33.3%	34.2%	-0.9%
Total	56.4%	65.7%	-9.3%

*Adapted from slide 20 of Gray's talk.*

As in the scenario above with Teacher A and Teacher B, most instructors had no idea why they were more successful with one group than they were with others.

These findings led Aurora to make some fundamental changes to its decision-making. On the faculty development side, the department used the equity scorecard's structure and tools to institute equity mentoring for all full-time instructors. Instructors had number-specific goals (e.g., for Teacher A above, an increase of 2 additional black students per class would balance the ratio with the average). On the classroom redesign side, they required learning labs for all students—and faculty—at this level.

## 2. *“Equity-mindedness” should be the guiding paradigm for language and action.*

In Aurora’s math department, they changed the faculty evaluation process to capture equity-minded measures. Classes were observed the first day of school and syllabi were critiqued using equity scorecard measures. Inquiries focused on how instructors dealt with structure and expectations, and how they communicated with students. Typical observations included:

- What instructors said: “The 2 to 1 rule: The scope of the MAT 050 class will cover the equivalent of three years of math in a traditional pre-college classroom. For every hour spent in class, you should expect to spend an average of two hours outside of class preparing, studying and completing assignments. If this is not possible, you should consider making changes to your schedule.” “Students who are struggling should seek help.”
- What syllabi said: “Attendance is mandatory! There is no such thing as an excused absence. Up to 90% of your grade in the course will be based on your participation in activities that take place in class.”
- What students did: They came in late, texted during class, and left the classroom to make phone calls without comment from the instructor.

Looking at these observations from the student’s perspective brings to light some harmful messages that were being (unintentionally) sent: “Doing three years of math in one term” could sound terrifying; a commanding, rigid tone might sound judgmental and distancing; and failing to correct behavior (e.g., texting or lateness) can suggest low expectations and tolerance of such behavior. These characteristic interactions indicate a teacher who is reacting with anger to students’ past behavior, and yet who perhaps also fears correcting their present behavior. The classroom observations were followed up by interviews with the instructors. In her interview Teacher A said:

- “It’s not the students who have the skill level who are struggling in the class. It’s the students who never learned this the first time around who are struggling.”
- “If I take this step for this one student, isn’t that enabling (or lowering standards)? Aren’t I really creating harm for their future?”
- “I teach adults, not school children. I can’t make them come to class on time.”

The equity-mindset approach enjoins teachers to look at their actions, biases, and motivations to try to seek out why they struggle to have students come to class on time, participate well, and study effectively. By observing instructors in their classrooms, Aurora was able to identify and share “day 1” classroom procedures and processes that could improve all students’ behavior, and do so disproportionately for students from the struggling groups. The equity scorecard showed the instructors where to look and gave them actionable targets.

## 3. *Equitable practices and policies are designed to accommodate differences in the students’ learning contexts — not to treat all students the same.*

Differences in learning contexts can be as direct as content and as subtle as culture. Consider an instructor who asks her class to estimate the distance from Austin to Denver. For the international students, the question is considerably harder than it is for the rest of the students.

A subtler version of this kind of content difference came to light when The Community College of Aurora looked at differences between success rates for students from each of their two feeder school districts. More of the students from one district were placed into developmental math than from the other. The department found that students from one district were taught math procedurally while those from the other were taught math conceptually. Since the placement was biased towards procedural knowledge, students from the procedural district had an advantage. The racial equity question came into the mix because the two districts were segregated in terms of their Hispanic population, so using the same placement tool for all students was not racially equitable.

Another subtle impact of policies and practices is seen in the way instructors communicate expectations to their students. Teacher A from Aurora made the following observations about half way through the semester in which she had received her equity training:

“In the past, I had thought of my students as adults, free to choose their own path – I gave every student equal opportunity to pass or fail, but the decision to attend class, complete assignments, study for tests, take advantage of office hours and tutoring was up to them. Who was I to tell them what to do?”

“I came to see that many of my behaviors were white middle-class woman behaviors. While another person who looked like me might be able to understand that my suggestions voiced to the class as a whole were really individual mandates, those black and Hispanic males from 18 to 25 were hearing that it was fine with me if they chose to fail. As with T-shirts, one size does not fit all.”

These comments speak to the importance of instructors’ expectations and how they are communicated. They also demonstrate that such expectations and communication can be changed when focused training and well-defined goals are put in place.

#### ***4. Enacting equity requires a continual process of learning, disaggregating data, and questioning assumptions about relevance and effectiveness.***

The assessment cycle must be a cycle, not a dead end, in order for progress to be made. It is never enough to discover a problem, inquiries must refine practices and the new practices must be assessed. Perhaps the most compelling qualitative sign of change can be seen in Aurora’s Teacher A. At the end of her first semester of equity training she made the following comments.

- “So I tried something different. I got in the faces of these huge boys and told them that they were too smart to fail my class and I was not going to put up with it.”
- “Then the weirdest thing happened. They kept showing up. I would arrive at school to find them camped outside my office door, waiting for help. They moved to seats in the front of the room and started to volunteer to present problems and answer questions. They went to the tutoring center to do their homework after class.”
- “When the student I’d caught cheating on his first test and then sentenced to weekly meetings in the tutoring center and mandatory meetings with me for quiz and test corrections came to me at the end of the semester to thank me for being the first teacher who ever believed in him, I burst into tears.”

Moreover, Teacher A's teaching effectiveness also improved. The table below gives her Fall 2014 disaggregated passage rates (as compared to average rates) results, which are an astonishing improvement over her Spring 2014 results.

Ethnicity	Teacher A	Average	Difference
Asian	NA	77.8%	NA
Black Non-Hispanic	66.7%	50.0%	16.7%
Hispanic	83.3%	69.9%	13.7%
Native American	100 %	83.3%	16.7%
White Non-Hispanic	77.8%	75.3%	2.5%
Unknown / Not Reported	0.0%	14.8%	-14.8%
Total	70.8%	64.7%	6.1%

*Adapted from slide 39 of Gray's talk.*

Teacher A had never had a semester in which her aggregated passage rates were over 70%. Moreover, she improved in all disaggregated groups except the “unknown/not reported” group. Not only is the improvement important for the students' sake, it is also important from the perspective of “buy in.” The instructor engaged in the equity training and saw success. And just like developmental math students who taste their first success after putting in effort, she is now more likely to continue to improve and to help others do so.

We also see dramatic improvements when looking at Aurora's work from a broader perspective. The original course redesign in Spring 2014 disproportionately helped white students, while the subsequent course redesign (in Fall 2014) had a more equitable impact.

	Completed Two Levels of Dev Math	Completed Redesigned Course - Spring 2014	Completed Redesigned Course - Fall 2014
Female	25.2%	50 %	59.1%
Male	22.4%	36 %	54.5%
Black	17.2%	29.6%	42.6%
Hispanic	29.7%	42.2%	60.9%
White	27.9%	67.4	69.4%

*Adapted from slide 40 of Gray's talk.*

From Spring to Fall 2014 every group benefitted, and the groups that benefitted the most, males as well as black and Hispanic students, were those who needed it the most. This change was equitable.

### ***5. Equity must be enacted as a pervasive institutional- and system-wide principle.***

Observing the kinds of professional development interventions enacted in the math department at Aurora, it is clear that such change would be impossible in isolation. The conversations and interventions with faculty were sufficiently sensitive and uncomfortable that if the deans and president had not held the equity project as a priority, the results would in all likelihood have been disastrous for Gray. But Gray points out that one of the most important equity improvements came out of examining his own hidden biases.

As part of the math department's inquiry into its own practices, an inventory was taken of who taught which classes, and where and how the classes were taught. In the case of college algebra, 57.7% of the classes were taught face to face, with 46.2% taught in the high schools and 7.7% online. The vast majority of instructors were part-time instructors. Approximately 42% held master's degrees in math education, and the remaining instructors held MS degrees in mathematics or a related field. Overall 84.6% were white, 7.7% Asian, 3.8% Hispanic or Latino, 3.8% Middle Eastern, and 65.4% men. Not one black man or woman had taught college algebra in thirty years. Since the chair does all the hiring for instructors, these data forced Gray to take a look at his own biases. He began an inquiry on how each person was hired, and this pointed out a bias towards hiring people who are known: high school teachers and instructors from other local community colleges. What he had never noticed was that neither of these groups had any black faces. "Nothing has convinced me more of the pervasiveness of institutional racism than this ... more than taking a look at myself," said Gray. Identifying the biases in hiring made it possible, and even easy, to correct the inequity, and the math department at Aurora now has two African American instructors in college algebra.

The take-away lesson from Aurora is that equity is doable. However, it must be enacted as a pervasive institution- and system-wide principle. A culture must be built in which it is acceptable and commonplace to look at data and observations broken down by ethnicity. This must be done even when it is uncomfortable, keeping in mind that the objective is not to blame but to bring about measurable and equitable change in procedures and processes.

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# CONNECTIONS TO K-12

*Based on talks by H. Hellenbrand, N. Heller, K. Belin, L. Brady, P. Dituri, A. Federman, G. George-Gilkes, N. Lewis, M. Moshos, D. Loewenberg Ball, and W. McCallum.*

## WHAT DID DEVELOPMENTAL MATH STUDENTS DO BEFORE THEY GOT TO COLLEGE?

As discussed in previous sections, a great deal of time has been spent working on improving the content and delivery of development math. The same is true of developmental English programs. However, the biggest obstacle to the success of both programs concerns the systems in which the content and pedagogy are embedded. For example, in California's tiered educational approach (high school, 2-year college, 4-year college), the completion agenda calls for high school graduates to attend college in greater numbers. Practically speaking, this means that large urban universities must sustain developmental programs that inevitably add time and cost for both institutions and students. Developmental programs exist to "correct for the errors" of the previous system. Yet how can high schools know what to prepare students for until and unless they can confer with universities and colleges?

In fact, teachers in all segments ought to confer with each other, but instead, each segment tends to blame the one before, asking, "What the heck did this kid learn in *high school*? In *middle school*? In *elementary school*? At *home*?" The behavior is reminiscent of Richard Buckley and Eric Carle's Greedy Python:

He saw his own tail,  
long and curved,  
and thought that lunch  
was being served.  
He closed his jaws  
on his own rear,  
then swallowed hard...  
...and disappeared!

To avoid the python's fate, master teachers and educational scholars from the K–12 segment participated significantly in the CIME workshop. Math for America (MfA) master high school teachers Audrey Federman and Giselle George-Gilkes used the image of "plugging potholes while building new roads" to describe the hard work of introducing urban high school students from underserved communities to new and engaging mathematics. They find that their students persist with adequate support, mastering complex and subtle skills. Unfortunately those same students often end up labeled as "developmental" in the transition to college. The dynamics of that transition are further complicated by each revolution in content, practice and assessment, Common Core being the most recent. The potholes/road image, coupled with the challenge/opportunity of Common Core, emphasizes the urgent need for each educational segment to understand what goes on in the others, so that students' knowledge can be more easily recognized and built upon.



## INNOVATIVE HIGH SCHOOL CURRICULA

*Based on talks by Math for America (MfA) teachers N. Heller, K. Belin, L. Brady, P. Dituri, A. Federman, G. George-Gilkes, N. Lewis, and M. Moshos. Available at <https://www.msri.org/workshops/758/schedules/19526>.*

Five MfA master teachers shared their creative and innovative approaches to teaching math in New York City public schools. The presenters each had ten or more years of experience teaching and had collectively educated thousands of New Yorkers. The majority of their students start each year below grade level (according to state exams) and come from families of low socio-economic status (qualifying for free/reduced lunch). Dr. Philip Dituri framed the teachers' presentations with the following questions:

1. What kinds of mathematics do students placed into developmental mathematics already know, and what are they lacking?
2. What do standardized state tests and placement exams actually measure, and how reliable is the data we use to make very important decisions about the future of our students?
3. What does being a high school graduate mean from a mathematics perspective?
4. How important are the topics taught in developmental mathematics to the activities in higher-level course work?

The presenters described how students in New York City's most challenged neighborhoods engage deeply in mathematical content, concepts, and practice. The students work actively and independently, demonstrating proficiency in rigorous classroom activities. Yet, standardized state tests and national placement exams, which are used as gateways to graduation from high school or college-level mathematics, do not measure the rich mathematics the students are doing. These high-stakes tests regularly place college-bound students below college-level mathematics courses. The MfA teachers challenged the status quo, asking: What does this contrast say about the scope of high school mathematics measured on college entrance exams? What does it say about the "college-ready" construct?

Lauren Brady teaches a senior year statistics class at a public school in East Harlem. It aligns with a college-level course and includes multiple research projects. In one project, students:

- investigate two quantitative variables,
- design a survey and collect data,
- calculate the Pearson Correlation Coefficient,
- write a formal research paper in APA format,
- orally present/defend results.

They learn to develop hypotheses and draw conclusions by collecting data, writing literature reviews, computing inferential statistics by hand, and identifying bias. This curriculum addresses not only statistical content but also many essential non-cognitive skills. The students achieve outstanding academic results: 97% of the students pass the college-aligned statistics course, and those among this group who take statistics in college usually report success.

Nevertheless, most of the students in the class who do attend college end up in developmental math. Brady asks, “Are these students really not college-ready?”

A different class, led by master teacher Michael Moshos, focuses on exam literacy. Moshos teaches the students to focus not on studying for placement tests, but rather study the tests themselves (potentially redressing their own disadvantages relative to those with access to greater test preparation resources). They categorize problems into major topics, identify the knowledge that a given category of question relies upon, look for patterns in types of questions, and identify key vocabulary. Then students work on fluency by identifying distractors and misconceptions in multiple choice questions (e.g., obviously wrong answers or common errors), recognizing question structure, developing tools for identifying the correct answer and predicting questions based on visual cues and answers. In this class, as in all the M/A classes, students

1. make sense of problems and persevere in solving them;
2. reason abstractly and quantitatively;
3. construct viable arguments and critique the reasoning of others;
4. model with mathematics;
5. use appropriate tools strategically;
6. attend to precision;
7. look for and make use of structure;
8. look for and express regularity in repeated reasoning.

Moshos’s students regularly score 80% or more on the standardized tests they study. This represents an improvement of 50 to 60 percentage points. Are students who have mastered these Common Core Standards of Mathematical Practices, as well as the content of statistics and other topics, ready for college math or not?



*Left to right: Richard Sgarlotti, Robert Megginson, and Hyman Bass.*

## INNOVATIVE ELEMENTARY AND MIDDLE SCHOOL CURRICULA

*Based on the talk by Deborah Loewenberg Ball. Available at <https://www.msri.org/workshops/758/schedules/19530>.*

Dituri's questions regarding the transition from high school to college can be equally well applied to students transitioning from elementary to middle school or from middle to high school. In each case, we see the same themes appear: It is difficult to advance along the road while filling in the pot holes, but it is impossible if we are ignorant of where our students come from, what their culture is, and how that culture ties into the culture of our classroom. The practical steps to improving elementary and secondary teaching are relevant to all segments and resonate with much of what colleges and universities have come to realize are best practices in developmental education.

Deborah Loewenberg Ball is Dean of the School of Education at the University of Michigan and is noted for her work in mathematics instruction and the mathematical preparation of teachers. She defines teaching as deliberately maximizing the quality of the interaction with students in ways that increase the probability of their learning worthwhile content and skills. Successfully teaching complex mathematical knowledge and skill to students of a broad range of abilities and experience requires teachers to pay attention to how they see and hear their students. Jackson, Gibbons and Dunlap (in press, 2016) studied this by asking middle school teachers, "When your students don't learn as expected, what do you find are typically the reasons?" Two sample responses were:

These kids are used to be[ing] spoon-fed and they'll sit there and say, 'I don't get it'... [U]ntil you actually sit down and show them step [by] step how to do that problem, they don't get it. They don't know how to think.

I normally look first at me to see if is there something in the lesson that I didn't emphasize well enough or ... I may talk to the teacher they had last year and say, 'When you went over this was this something that they struggled with?'

The first response exemplifies unproductive reasoning: Even if the teacher were correct, the response suggests no solution that is under the teacher's control. The second response is productive and proposes a road to improvement.

To improve how students are seen and heard, Ball recommends that teachers practice explicit teaching. In explicit teaching, a teacher unpacks practice or knowledge to make it open to learners, but does not do the work for the students. Compare this to a teacher who breaks down practice or knowledge into small constituent parts; presents information, rules, and



*Deborah Loewenberg Ball during her plenary talk (see <https://www.msri.org/workshops/758/schedules/19530>).*

examples; and assigns only uncomplicated tasks that address the information presented. In explicit teaching, the teacher’s role is to make elements visible, provide language, and support learning, rather than demonstrate information for students to follow and then shift to independent work [see Ellis’s distinction between community college teachers with weak versus strong teaching performance in Section 4]. In explicit teaching, instructors observe and listen to their students as the students do the work of learning. Thus, “seeing” and “hearing” students is a natural part of explicit teaching.

### DEVELOPMENTAL MATHEMATICS AND THE COMMON CORE

*Based on the talk by William McCallum.*

*Available at <https://www.msri.org/workshops/758/schedules/19531>.*

Explicit teaching leads naturally to the eight standards of mathematical practice spelled out in the Common Core for K–12 mathematics (below). This type of teaching plays into the Common Core’s holistic view of how mathematical practices are learned, where connections are made between topics, and between grade levels. The following diagram demonstrates how the Common Core is built on streams of topics, called progressions, of mathematics.

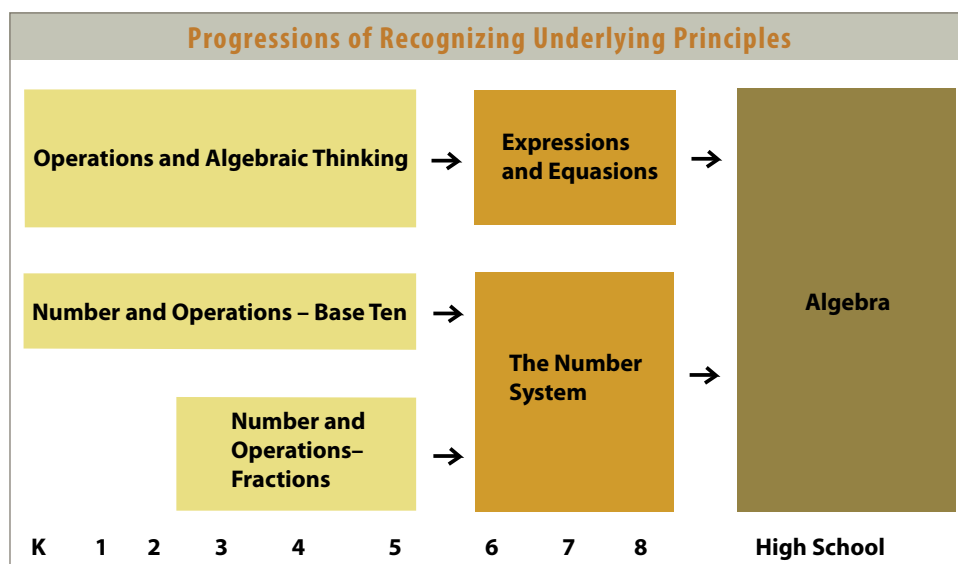
Common Core Stream of Topics: Progressions of Mathematics								
K	1	2	3	4	5	6	7	8
Geometry								
Measurement and Data						Statistics and Probability		
Number and Operations in Base Ten						The Number System		
Operations and Algebraic Thinking						Expressions and Equations		
Counting and Cardinality			Number and Operations – Fractions			Ratios and Proportional Relationships		Functions

*Based on McCallum slide 5.*

Bill McCallum is one of the lead writers of the Common Core Standards. He introduced the Standards to the workshop attendees and took them on a tour of the pathway to algebra from kindergarten to high school. Before starting the tour, he framed the concept of college-ready in algebra in a functional way by asking, “Is algebra a scree of symbols that you do stuff to or is it something that you reason about?” His point was that too many students arrive at college primed for “algebraic action” when often they would be better off thinking strategically. His favorite advice is, “Don’t just do something, stand there,” meaning rather than mindlessly implementing algorithms, students should look for structure and choose

the best operation for the purpose at hand. They must understand the meaning behind algebraic symbols/manipulations and be able to reason about them. This is central to the algebra progression in Common Core.

In general, the Common Core's progressions resist the idea of mathematics as a list of topics because lists quickly become too long for students to keep in their active memories. Rather the progressions invite students to recognize underlying principles. This recognition "shrinks" the mental real estate required for memorization while deepening mathematical understanding.



*Based on McCallum slide 6.*

The algebra progression in the Common Core is the culmination of several progressions that gradually build upon one another. In following these progressions, students see algebraic reasoning as part of a continuum of ideas that grows out of arithmetic. As a result, the whole continuum becomes easier to retain, just as a story is easier to remember than a list of words. The hope is that Common Core will contribute to solving the “developmental math problem” by producing students who are able to think, and in thinking, find that mathematics makes sense and is useful. By engaging with the subject more positively and deeply, learners will become fluid in the what, the why, and the how of algebra in particular, and of mathematics in general.

## REFERENCE

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# REFORMS PROJECTS IN DEVELOPMENTAL MATH

One of the questions around which the 2015 CIME workshop was organized asked: *Armed with a better idea of where our students come from, how do we teach content in ways that acknowledge and leverage each student's prior learning experiences?* The state of developmental math today as described in the previous sections suggests that improvements in teaching and learning must address content, non-cognitive issues, and structure. With this in mind, *how can developmental mathematics instruction blend or bridge college-level and remedial instruction, using synchronous and asynchronous instruction, to achieve maximal efficiency and impact? Can such instruction be scaled up to effect real systemic change?* As mentioned in Section 4, there are four main approaches:

1. *The **stretch model**: students place into transferrable math immediately with additional support and time to complete the transferrable course while also accumulating the necessary prerequisites to do so.*
2. *The **accelerated model**: students take one intensive semester of developmental math. In many cases the content is restricted to only the developmental math necessary for success in the subsequent transferable math course.*
3. *The **boot camp model**: students and faculty get intensive and compact instruction outside of the regular academic schedule.*
4. *The **co-requisite support model**: students are mainstreamed into a college-level course and enroll concurrently in a remedial support course.*

Successfully scaling reforms and curricular improvements involves building robust curricula that encourage local contextualization and reinforce essential structures and faculty engagement. Models that thrive start with firm higher principles, which are broad enough to allow local stakeholders to adapt their use to local faculty and student development culture, content needs, technology resources, and existing non-cognitive support structures. For example, a program that is effective in a large, urban four-year university context will not necessarily be as effective in a different setting—a campus serving an affluent suburban population, say, or an urban community college. However, in order to scale faithfully, models cannot be so finely tailored that each section of each course is marching to a different drummer. Strategies with campus- and system-wide efficacy are urgently needed.

At this workshop, several models were presented and discussed. These are listed below together with some descriptions of the scale and results. Some models are large multi-state initiatives while others are statewide and grassroots projects. What follows is not a comprehensive catalog of all types of interventions; rather, it is a list of those approaches discussed at the workshop.

1. *The stretch model: students place into transferrable math immediately with additional support and time to complete the transferrable course while at the same time accumulating the necessary prerequisites to do so.*

**A. New Mathways Project (NMP):**

The NMP replaces existing remediation models with high-quality, accelerated math pathways that integrate mathematical content, social and psychological supports, and research-based curriculum design. The NMP has the potential to drive long-term systemic changes that dramatically increase the number of students completing math coursework aligned with their chosen program of study and successfully achieving their postsecondary goals.

The NMP is designed for implementation at scale, achieving significant breadth and depth of implementation across and within institutions of higher education over five years. Implementing NMP requires coordinated action at multiple levels of state and local education systems (<http://www.utdanacenter.org/higher-education/higher-education-resources/policy-resources/scaling-modern-undergraduate-mathematics/>).

*Scale:* Each year, the NMP model has steadily increased in scale across Texas. In 2015:

- 40 colleges are engaged in training and coaching — approximately four times as many as there were in 2012.
- 34 colleges are implementing the NMP — approximately three times as many as there were in 2013.
- Three pathways are now supported with NMP curriculum (statistical reasoning, quantitative reasoning, STEM-Prep).

*Organizers:* Uri Treisman, executive director,  
The Charles A. Dana Center at The University of Texas at Austin.

*Design Principles:* The NMP model is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

- Multiple pathways aligned to specific fields of study.
- Acceleration that allows students to complete a college-level math course in one year.
- Intentional use of strategies to help students develop skills as learners directly linked to their courses.
- Curriculum design and pedagogy based on proven practice coupled with a context sensitive improvement strategy.

*Results:* Using the most recent data available from the Texas Higher Education Coordinating Board, only 24 percent of all first-time-in-college (FTIC) developmental education students in Texas complete developmental courses in one year and just 8% of those students complete a college level course in the same amount time. Comparatively, 64% of NMP students in academic year 2014 completed developmental coursework in one semester and 23% of those NMP students also received college-level math credit in one year.



Gateway completion rates were even higher (43%) at schools that encouraged successive semester enrollment in a yearlong pathway.

The Dana Center is currently collaborating with the Manpower Demonstration Research Corporation (MDRC), an education and social policy research organization, to evaluate the implementation of the project. *Laying the Foundations: Early Findings from the New Mathways Project*, analyzes the initial development of the New Mathways Project (NMP) and its first year of implementation at nine community colleges across Texas during the 2013–2014 school year (see <http://www.utdanacenter.org/higher-education/higher-education-resources/new-mathways-project-evaluation-and-research/mdrc-report-new-mathways-project/>).

- B. STATWAY®:** The Carnegie Foundation for the Advancement of Teaching's Statway® Project uses innovative curricula and assessments, an online platform, and a unique pedagogical approach. The pathway promotes collaborative learning, addresses socio-emotional factors that affect student success, and provides embedded language and literacy supports for students. It is focused on statistics, data analysis, and causal reasoning, combining college-level statistics with developmental math. It is designed to teach mathematics skills that are essential for a growing number of occupations and needed for decision-making under conditions of uncertainty. The Foundation also has a quantitative reasoning pathway called Quantway®.

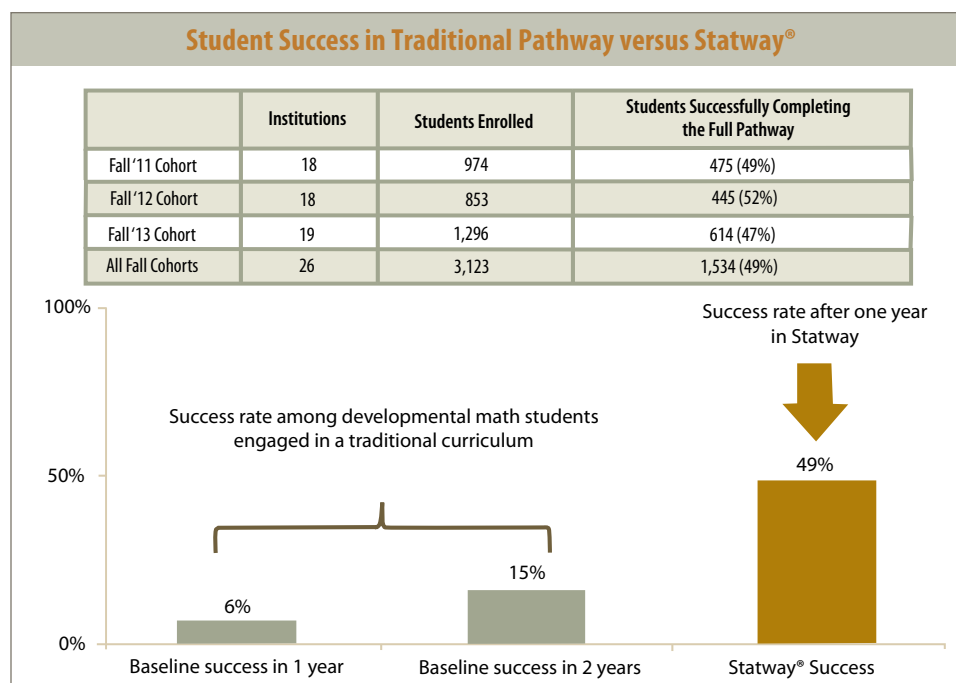
*Scale:* More than 27 higher education institutions across 9 states offer Statway®.

*Organizers:* Pathways Networked Improvement Community facilitated by the Carnegie Foundation for the Advancement of Teaching.

*Design Principles:*

- Engaging relevant content: Pathways promotes active collaborative learning and engages students in relevant mathematics that they can use in their daily lives.
- Explicit connections to concepts: Pathways builds a student's underlying conceptual mathematical understanding, making explicit connections between mathematical or statistical facts, ideas, and procedures to improve both conceptual and procedural understanding.
- Language and literacy supports: Pathways language and literacy supports are interwoven in the instructional materials and classroom activities in order to remove language barriers.
- Productive persistence: Pathways addresses socio-emotional factors that drive student engagement and motivation by incorporating an evidence-based package of student activities and faculty actions.
- Faculty development and support system: Pathways offers rich professional learning opportunities, ongoing evaluation and rapid data analytics, and a community of practitioners and researchers working toward continuous improvement.
- Rigorous on-going evaluation that enables continuous improvement: Pathways's rapid data analytics and common summative assessments across the network highlight variation and inform opportunities for improvement.

*Results:* Of the 1,296 community college students in the Fall 2013 Statway® cohort, 614 (47%) completed the full Pathway with a grade of C or higher and earned college credit.



Based on statistics from Figure 2 of the Pathways Impact Report (2015).

(See <http://www.carnegiefoundation.org/resources/publications/pathways-impact-report-2013-14/>.)

Propensity score matching analysis gives evidence that Statway® accelerates student success in acquiring college-level statistics credit. This propensity score matching technique statistically reduced selection bias and accordingly increased the validity of causal inference. These results were replicated across two Statway® cohorts and across all sub-groups of students, regardless of gender, race, ethnicity, and math placement level. Additionally, studies show that Statway® students tended to accumulate more college credits with a grade of C or higher than their non-Statway® counterparts. (See <http://www.carnegiefoundation.org/resources/publications/pathways-impact-report-2013-14/>.)

**2. The accelerated model:** students take one intensive semester of developmental math. In many cases the content is restricted to only the developmental math necessary for success in the subsequent transferable math course.

**A. California Acceleration Project (CAP):** The California Acceleration Project is working with 61 colleges to implement three high leverage strategies that accelerate students' progress, substantially increase student completion of transferable, college-level English and math courses, and narrow equity gaps. See <http://cap.3csn.org/>.

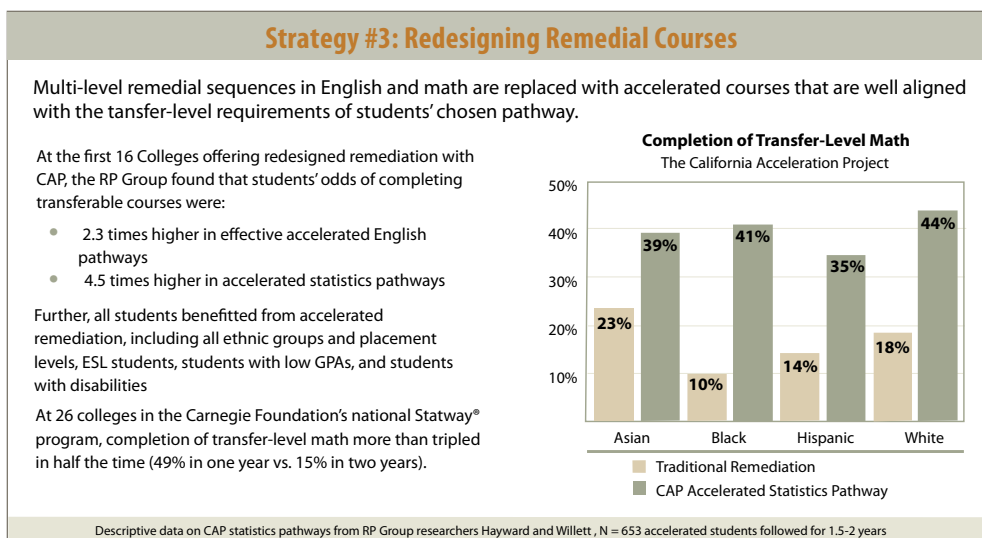
*Scale:* 12,500 students over 4.5 years.

*Organizers:* Myra Snell and Katie Hern.

*Design principles:*

- Design backwards from transfer math.
- A thinking oriented curriculum.
- Low stakes collaborative practice.
- Just in time remediation.
- Attention to the affective domain (identifying, understanding and addressing how people learn).

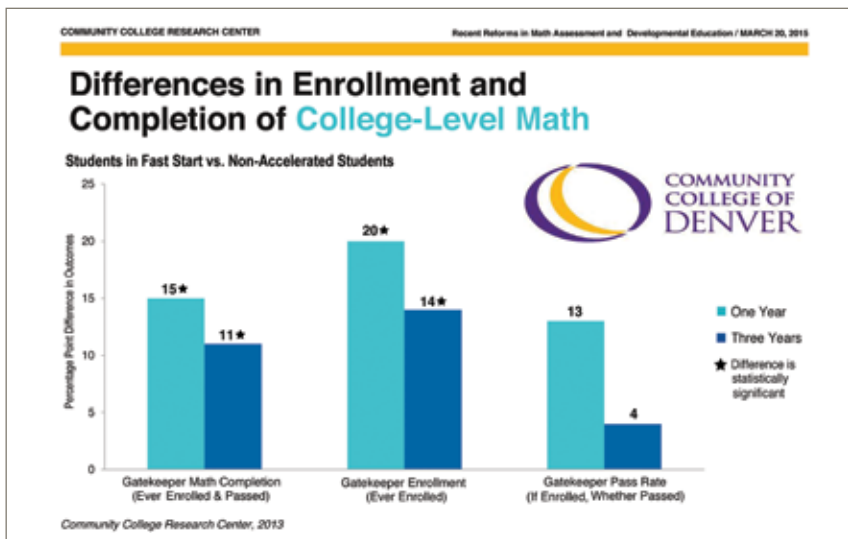
*Results:* The Research and Planning Group provides leadership in research planning and assessment within the California Community College system (rpgroup.org). The group took the first eight campuses with CAP programs that fed into their standard statistics course. They did a statistical regression analysis in order to control for 13 covariates that are usually associated with performance in mathematics (GPA, prior success or failure in the discipline, placement level, financial aid status, race, etc.). Controlling for those covariates attempts to isolate the effect of acceleration. Students in these accelerated statistics pathways were 4.5 times more likely to complete a college level math course than students in traditional remediation. Moreover, the equity impact was phenomenal. African American students were 7 times more likely to complete a college level math course, moving from 7% to 47%, and students who had been placed into the lowest level of remediation were the students whose rates increased the most.



**B. Other Accelerated Programs Mentioned at the Workshop:**

- a. **Fast Start at Community College of Denver (CCD):** "For more than 5 years, professionals at CCD have been experimenting with accelerated curriculum formats, beginning with developmental math and extending their work to developmental English (reading and writing) and, more recently, have been linking developmental courses to entry-level college courses." (See 2010 Follow-Up of the Community College of Denver Fast Start Program, by Debra D. Bragg, Elaine Delott Baker, and Margaret Puryear, available at <http://files.eric.ed.gov/fulltext/ED521421.pdf>)

Results:



Bailey, slide 30 from Thomas Bailey's talk  
 (see <https://www.msri.org/workshops/758/schedules/19539>).

- b. **CUNY Start:** For students bound for the City University of New York (CUNY) whose scores on the CUNY Assessment Tests indicate a need to develop academic skills, CUNY Start offers intensive instruction and college advisement in reading, writing and math on either a full-time or part-time basis. Individuals who complete the program are significantly better prepared for college-level coursework, and in many cases are able to bypass required remedial coursework. (See <http://www.cuny.edu/academics/programs/notable/CATA/cti-cunystart/about.html>).

Results:

Proficiency Gains in Math <sup>2</sup>	CUNY Start Students		Matched Comparison Group <sup>3</sup>		
	N	% Achieving proficiency	N	% Achieving proficiency after One Semester <sup>4</sup>	
All Students Needing Remediation in Math	1,354	53.0	1,354	10.2	-42.8
Initial Need in Math Only	166	62.0	166	12.7	-49.3
Initial Need in Math + Reading Only	99	50.5	99	10.1	-40.4
Initial Need in Math + Writing Only	421	61.3	421	13.5	-47.8
Initial Need in Math + Reading + Writing	668	45.8	668	7.5	-38.3

Bailey, slide 26 from Thomas Bailey's talk  
 (see <https://www.msri.org/workshops/758/schedules/19539>).

3. *The boot camp model: students and faculty get intensive and compressed instruction outside of the regular academic schedule.*

- A. **Pacific Crest Learning to Learn Camps:** During the past 20 years, Learning to Learn Camps have been implemented for diverse environments including honors students, entering first-year students, HBCUs, Research I universities, state colleges, community colleges, and high school students. These camps share a basic framework but were customized to meet the needs of each specific institution and its learner group.

*Scale:* 3000 students over 4 years.

*Organizers:* Dan Apple and Wade Ellis.

*Design Principles:* Learning is a process, all students can improve their process of learning, learning skills are the basis of learning and those skills can be incrementally improved.

- Move students from a fixed to a growth mindset.
- Move students from "I am a memorizer" to "Learning mathematics is understanding, and I think and want to understand it."
- Students are assessed and self-assess to improve learning performance.
- Mastering content is helped significantly by applying students' existing knowledge.
- An intensive residential program from 8 am to 9 pm over 5 days.  
Alternately, the approach can be built into a math course (2 hours per week for a 15-week semester) as supplemental instruction.

*Results:* The results were consistent across most of these institutions with over 90% of the 3,000 students passing the camp. The camps have evolved over the years as the methods of implementing Process Education and Learning to Learn have improved, with the success rate now approaching 95% for the most recent camps. [Apple, Ellis and Hintze 2015] Learning to Learn Camps: Their History and Development. *International Journal of Process Education*. 7.

*Examples:*

- Hinds Community College Utica Campus, Utica, Mississippi.
  - Grand Valley State University Recovery Course.
  - Nursing Recovery Camp, Hinds Community College.
- B. **California State University System's Early Start Math Program:** California State University System (CSU) has 60,000+ entering freshmen each year across 23 campuses. First-year students are placed into developmental math via a common placement test (the Entry Level Math Test or ELM) with a common cut-off score (50 out of 80 points). The exam is administered at the CSU campuses in the spring of the students' senior year of high school, and places the students either out of developmental math or into one or two semesters of developmental math course work. This work must be completed within one year. The Early Start Math Program requires all CSU students who place into developmental math to start their developmental math course work in the summer before they matriculate. There are some guiding principles (see below) but each CSU campus is free to interpret the mandate as best fits their local student and faculty populations. Some do intensive one- or two-week boot camps, while others run full courses in compressed shorter summer terms.

*Scale:* 20,000 students per summer.

*Organizer for CSU:* Eric Forbes.

*Organizer at California State University, Northridge (CSUN):* Katherine Stevenson.

*Design Principles:*

- For all CSUs: 20,000+ students each summer.
  - Free to any student qualifying for financial aid.
  - Flexible: ESM can be taken at any campus (i.e., not necessarily the students' destination campus).
    - Positive: Grading system that encourages maximum participation.
    - Credit = improved fall placement in math.
    - Required Progress = student put in a good faith effort, but did not improve placement.
    - No Credit = student did not put in a good faith effort, and did not improve placement.
- For CSU Northridge: Scale 2400+ students each summer.
  - Peer mentors in the classroom trained in non-cognitive issues.
  - Orientation to the campus integrated into program.
  - Intensive six-week courses offered in a variety of formats to allow for students with different needs and work/school balances.
    - 3 unit face-to-face—4 days per week, 3 hours per day, synchronous instruction.
    - 1 unit face-to-face—1 day per week, 3 hours per day, asynchronous emporium style instruction with possibility of improving placement one to two levels.
    - 1 unit online—asynchronous emporium-style instruction with possibility of improving placement one to two levels.

*Results:*

- CSU: From 2010 to 2014, as a result of Early Start efforts, the number of students beginning their fall term college-ready has increased by 15 percentage points. In fact, CSU Institutional research reports that 3,300 additional students were GE ready in Fall 2014 thanks to Early Start Math. (Ed Sullivan's report to CSU Board of Trustees, March 2015)<sup>1</sup>.
- CSUN: Cal State Northridge has approximately 2400 students participating each year since 2012 (40% of the freshman class). As of 2015 students were evenly distributed across the three formats above with *two thirds of students improving fall math placement*. This lowered developmental math enrollments by approximately 800 students in Fall 2015.

Passage rates:

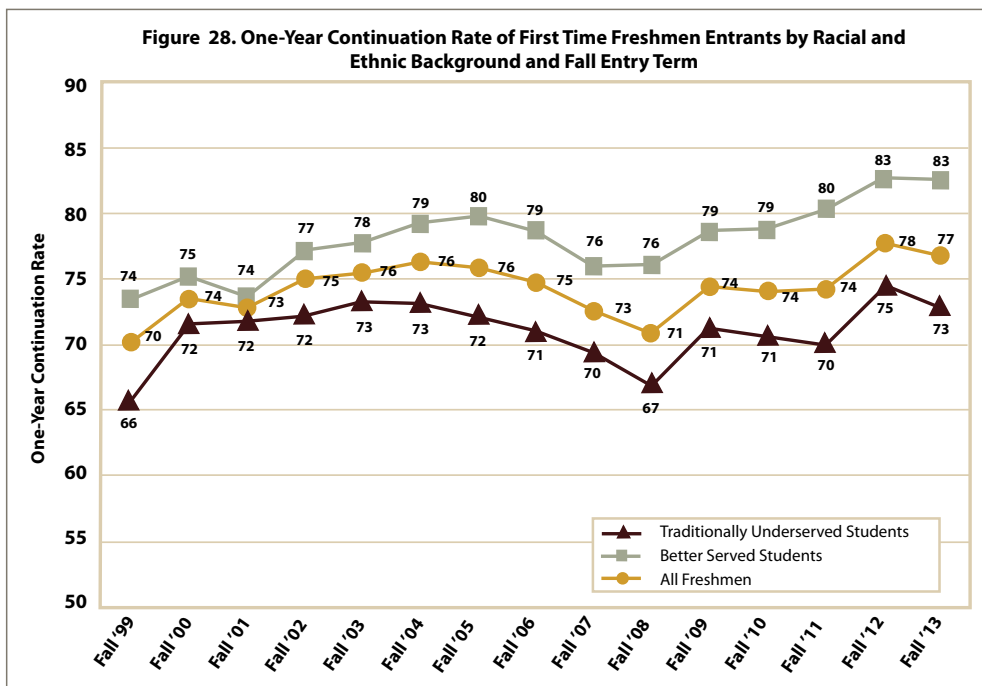
- 3 unit face to face — 84%.
- 1 unit face to face — 84%.
- 1 unit online — 34%.

<sup>1</sup>Available at [http://www.fullerton.edu/analyticalstudies/presentations/CSRDE2013\\_hip\\_moon\\_et\\_al.pdf](http://www.fullerton.edu/analyticalstudies/presentations/CSRDE2013_hip_moon_et_al.pdf).

As a result of Early Start Math, significantly more first time freshman take a GE math class in their first year at college, and the freshman to sophomore retention, which had stagnated for traditionally underserved students, increased 5 percentage points from 70% to 75% in 2012 (the first year of Early Start Math, see chart below). After a dip in 2013, the retention numbers improved again in 2013 and 2014, though official Institutional Research Reports on those data are not yet available. Thus, CSUN’s Early Start Math seems to help shrink the achievement gap.

*Based on figure 28 from the CSUN Office of Institutional Research’s report “Undergraduate persistence at Cal State Northridge from 2000 Onwards”.*

*See <http://www.csun.edu/sites/default/files/Persis2000-13ReportFinal.pdf>.*



**4. The co-requisite support model: the student is mainstreamed into the college level course and enrolls concurrently in a remedial support course.**

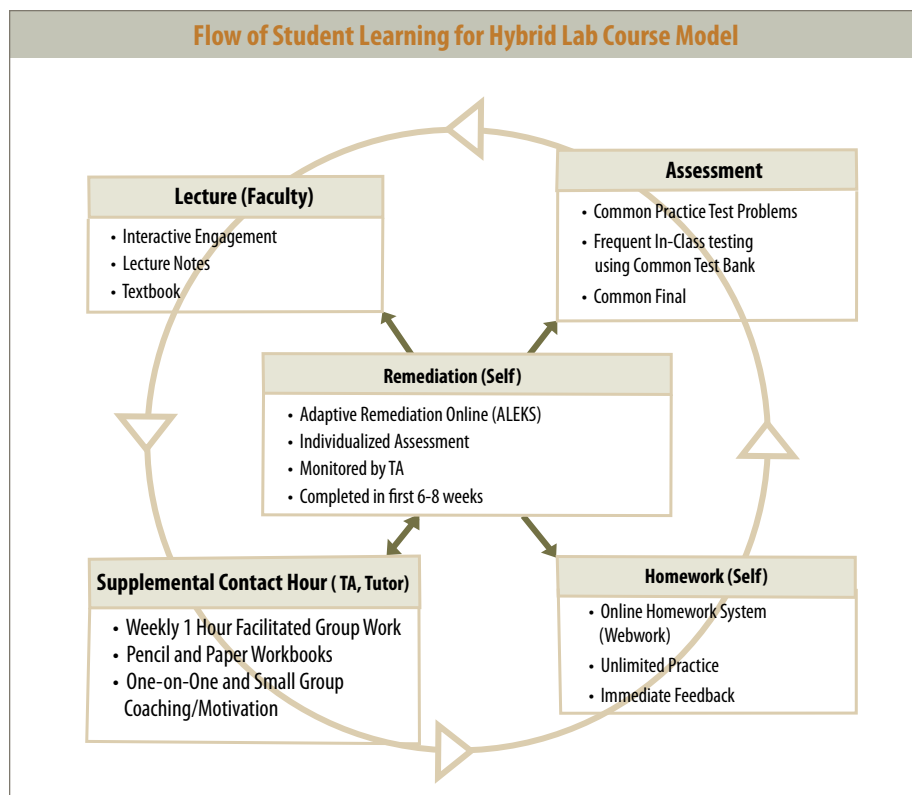
- A. **The CSU-Consortium** includes four community colleges and 10 CSU campuses. It has developed an innovative, technology-enhanced hybrid course model that has significantly improved course completion and content mastery outcomes in entry-level mathematics courses.

*Scale: Over 10,000 students per year in California.*

*Organizer: Katherine Stevenson.*

**Design Principles:** The model relies on five primary components that are carefully articulated to create a reliable “flow of learning” for students. The approach has proven both cost effective and scalable to other courses and institutions. Under the model, supplemental instruction is delivered in a co-requisite lab contact hour while real-time tailored remediation is delivered via adaptive online systems. See figure above and [Stevenson and Zweler 2011]. It targets high failure rate, multi-section, gateway courses in which prerequisite knowledge is a key to success.





### Examples Presented at CIME Workshop

- a. **CSUN Business Math:** Serves over 1800 students per year. Improved C or better rates from 35% to over 67% have been sustained since 2008 (CSU IR data). Improved deeper learning documented both within the course and within subsequent accounting and economics courses.
- b. **CSU Monterey Bay:** Applied the model to precalculus and statistics classes serving. Class size increased from 40 to 55–80, but the pass rate improved from approximately 60% to 75–80%.
- c. **Pasadena City College Fast Track Math:** Takes students who placed into Pre-Algebra but took a pre-algebra boot camp and places them directly into Intermediate Algebra with additional co-requisite support in supplemental instruction. Serving 100 students per year. The program has served approximately 200 students with a completion rate of 60%. For students who follow the traditional three-course prerequisite sequence, the probability of their completing Intermediate Algebra is 7%.

### REFERENCES

- [Apple, Ellis and Hintze 2015] D. Apple, W. Ellis, and D. Hintze, "Learning to learn camps: their history and development", *International Journal of Process Education* 7 (2015), 63–74. <http://www.processeducation.org/ijpe/2015/camps.pdf>.
- [Stevenson and Zweier 2011] K. Stevenson and L. Zweier, "Creating a learning flow: a hybrid course model for high-failure-rate math classes", *EduCause Review* (2011). <http://er.educause.edu/articles/2011/12/creating-a-learning-flow-a-hybrid-course-model-for-highfailure-rate-math-classes>.



**T**HE MATHEMATICAL SCIENCES RESEARCH INSTITUTE (MSRI), located in Berkeley, California, fosters mathematical research by bringing together the foremost mathematical scientists from around the world in an environment that promotes creative and effective collaboration. MSRI's research extends through pure mathematics into computer science, statistics, and applications to other disciplines, including engineering, physics, biology, chemistry, medicine, and finance. Primarily supported by the U.S. National Science Foundation, the Institute is an independent nonprofit corporation that enjoys academic affiliation with more than 100 leading universities as well as support from individuals, corporations, foundations, and other government and private organizations.

MSRI's major programs, its postdoctoral training program, and its workshops draw together the strongest mathematical scientists, with approximately 1,700 visits over the course of a year. At any time, about eighty mathematicians are in residence for extended stays. Public outreach programs and VMath, the largest mathematical streaming video archive in the world, ensure that many others interact with MSRI throughout the year.

MSRI created the Critical Issues in Mathematics Education Workshop Series in 2004. This series of workshops addresses key problems in education today and is designed to engage mathematicians, mathematics education researchers, and K-12 teachers. The workshops provide participants a unique opportunity to learn about research and development efforts in this area. In addition participants develop ideas about methods for working on these problems and get to analyze and evaluate current or proposed programs. These workshops offer a space to make connections and exchange ideas with others concerned with the same issues in their fields.

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*For more information visit [www.msri.org](http://www.msri.org)*




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“**W**e know what to do to improve  
developmental math education;  
the time has come for immediate action.”

— KATHERINE STEVENSON

CALIFORNIA STATE UNIVERSITY, NORTHRIDGE





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“**B**egin at the end.  
to improve education,  
put the learner first.”

— LEE ZIA

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