

Access, Power, and Participation in mathematics classrooms

Using observation as a strategy to
improve teaching and learning

Joi Spencer

Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning

A companion booklet to CIME Workshop 14, March 2017

CIME 2017 Organizing Committee

Michael Driskill, Math for America
Esther Enright, Boise State University
Rochelle Gutierrez, University of Illinois
Jodie Novak, *Lead*, University of Northern Colorado
Miriam Sherin, *Lead*, Northwestern University
Joi Spencer, University of San Diego
Elizabeth van Es, University of California, Irvine

MSRI Educational Advisory Committee

Deborah Ball, *Chair*, University of Michigan
Hélène Barcelo (*ex officio*), MSRI
Hyman Bass, University of Michigan
Herbert Clemens, Ohio State University
Ricardo Cortez, Tulane University
David Eisenbud (*ex officio*), MSRI
Mark Hoover, University of Michigan
Maria Klawe, Harvey Mudd College
Robert Megginson, University of Michigan
Robert Moses, The Algebra Project
Alan Schoenfeld, University of California, Berkeley
Hung-Hsi Wu, University of California, Berkeley



Critical Issues in Mathematics Education (CIME) is supported by the Mathematical Sciences Research Institute, the National Science Foundation (DRL-1261327), and Math for America. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation nor those of the Math for America. This booklet is published in 2019 by the Mathematical Sciences Research Institute, 17 Gauss Way, Berkeley, CA 94720-5070, telephone 510.642.0143; www.msri.org

Author: Joi Spencer, University of San Diego. Booklet coordinator: MSRI Deputy Director Hélène Barcelo.

CONTENTS

Welcome to CIME 2017 — <i>Congresswoman Barbara Lee</i>	4
Foreword: CIME Design and Goals — <i>David Eisenbud</i>	5
Introduction: Our Challenge — <i>Joi Spencer</i>	7

Understanding equity

How the E-Word Helps and Hurts Our Cause in Mathematics Education: The Need to Rehumanize Mathematics — <i>Rochelle Gutiérrez</i>	9
Building an Equity-Focused Inventory to Understand Mathematics Learning Opportunities in Classrooms and Schools — <i>Joi Spencer</i>	19
The Interaction of Power and Cognition in Undergraduate Mathematics — <i>Aditya Adiredja</i>	25
Creating Video Case Studies to Study Equitable Classroom Interactions — <i>Math for America</i>	32

Making meaningful observations

I DO (NOT) Belong: Experiences of Black Women and Girls in Mathematics Education — <i>Nicole Joseph</i>	36
Mathematical Impediments in the Classroom: Microaggressions and Bias — <i>Darryl Yong</i>	45
Observing for Equitable Opportunities for English Language Learners to Participate in Lessons, Activities, and Interactions — <i>Haiwen Chu and Rebecca Perry</i>	49
Seeing Beyond Ourselves: A Conceptualization of Mathematics Teacher Noticing for Equity — <i>Victoria Hands and Elizabeth Van Es</i>	56

Taking action

Turning Conversations into Actions: Addressing Inequities in Mathematics Classrooms — <i>Dorothy White</i>	60
Opening the Gateway to STEM Disciplines: Lessons from the Arlington Emerging Scholars Program in Calculus — <i>James Álvarez</i>	65
Promising Teaching Practices for Advancing the Mathematics Education of Young Emergent Bilinguals — <i>Sylvia Celedón-Pattichis</i>	73

WELCOME TO CIME 2017

I want to welcome you to the 2017 Critical Issues in Mathematics Education workshop. I'd like to thank MSRI director David Eisenbud, as well as Elwyn Berlekamp, the NSF, and Math for America for your work to ensure quality math education for all students. As we know, STEM education is the key to our economic future. We must keep encouraging our students to engage in science, technology, engineering, and math fields. Sadly, many of our children lose confidence in their math skills at an early age. And this is especially common amongst students of color, who have less access to exciting and engaging math education. Now, more than ever, we must address these challenges that prevent students from excelling in STEM fields.

Later this month, I will reintroduce the Computer Science for All Act. This bill would authorize \$250 million to support quality computer science education in K-12 public schools. It is critical that all students, but especially our young girls and children of color have equal access to STEM education. My legislation would do just that. We must act now to ensure our young people have the skills necessary to compete for 21st century jobs. On behalf of the 13th Congressional District, I wish you all a memorable and inspiring event. I hope that today's workshop empowers you to continue working to ensure that every child, regardless of their economic background, has the opportunity to succeed.

— *Barbara Lee at MSRI, March 15, 2017*

FOREWORD: CIME DESIGN AND GOALS

CIME is a program designed to probe deep and complex problems and to engage and connect experts in order to discuss new directions for their solution. The program is central to MSRI's education initiative and is governed by MSRI's Educational Advisory Committee (EAC), currently chaired by Deborah Ball.

The CIME program brings together research mathematicians, mathematics education researchers, K-16 teachers, and other stakeholders and gives them opportunities to:

- Learn about research and development efforts that can enhance their own work and about the contributions they can make in solving the problems of mathematics education.
- Develop ideas about methods for working on these problems and the nature of the evidence used to support different kinds of claims or programs related to the problems.
- Share their own work related to the problems — such as course development, research, teaching, and assessment.
- Make connections with others concerned with these issues who work in related environments.
- Gain respect for the expertise of members of other professional communities concerned with mathematics education and practice engaging in collective, cross-professional work.

The design of the program builds on assumptions that research mathematicians have much to contribute to mathematics education and that productive participation requires two key ingredients: (i) better understanding of critical issues, and (ii) capacity for working across professional communities. Workshops recruit new allies to the improvement of mathematics education, frame critical issues, draw attention to the need for diverse participation and success, and provide images of productive engagement for participants to draw on as they return to their professional enclaves.

Each CIME workshop has been informed by, and has informed, developments in mathematics education nationally. The programs have sharpened key issues for professionals in different roles in mathematics education. They have bridged issues such as the professional education of teachers, undergraduate mathematics education, and the development of standards across institutional boundaries. They have networked individuals interested in learning how to work productively across professional communities to improve mathematics education. Each CIME convening includes thoughtful attention to underserved populations and deeply rooted shortcomings of the mathematics education enterprise.

— *David Eisenbud*

INTRODUCTION: OUR CHALLENGE

The 2017 Critical Issues in Mathematics Education (CIME) workshop occurred just as our nation was grappling with the realities of a new administration in the White House and a couple of short months after the release of the film *Hidden Figures*, which documents the role of African American women in sending the first Americans into space. The workshop's theme — *observing for access, power, and participation in mathematics classrooms as a strategy to improve mathematics teaching and learning* — brought together attendees from across the nation as well as a variety of specialties. As in years past, attendees included learning scientists, mathematicians, teacher educators, K-12 teachers, and even high school students.

Workshop highlights

The plenary sessions, delivered by three leaders in the field — Rochelle Gutiérrez, Nicole Joseph, and Dorothy White — helped to establish the *what* of our gathering: What is meant by access, power, and participation? Each plenary speaker provided attendees with essential definitions, rich context, and compelling research and stories to guide our time together. Gutiérrez asked attendees to consider whether or not the field has a shared vision or language for what constitutes equity in mathematics education. Equity, she argued, will be contingent on our commitments to “rehumanize” mathematics. Joseph’s plenary began with the stories of Black women in mathematics. Her talk highlighted their achievements against a backdrop of discrimination and exclusion. In White’s talk, attendees experienced first-hand what it is like to be placed in the low academic track at school, and were later challenged to consider how we might better prepare mathematics teachers to avoid such practices.

Attendees included learning scientists, mathematicians, teacher educators, K-12 teachers, and even high school students.

This year’s CIME workshop leaders brought with them a wealth of expertise. Darryl Yong took up the issue of microaggressions and provided ways to prevent these in mathematics classrooms. James Álvarez presented the results of the Emerging Scholars Program at the University of Texas, designed to improve the success rates of undergraduate calculus students. Haiwen Chu and Rebecca Perry’s work presented observation data from mathematics classrooms serving students who are learning English. In their work, they found it necessary to make use of numerous observation tools in order to address their research

questions. Victoria Hands and Elizabeth Van Es are interested in what teachers see and notice when they look at video clips of mathematics classrooms. In their talk, they discussed the characteristics of those teachers who, when viewing such clips, pay close attention to issues of equity. Aditya Adiredja's work documents the contexts and stories that undergraduate women of color draw upon as they solve problems in a linear algebra course. The Math for America team's session presented insights into how the mathematics classrooms of master teachers can be used as tools to instruct emerging mathematics teachers. Sylvia Celedón-Pattichis presented research documenting those practices that promote and advance the mathematics learning of emergent bilingual students. And finally, in my own presentation, I described an inventory tool that teachers, principals, and school leaders can use to take stock of how equitable their mathematics classrooms and programs are.

I sincerely hope that you enjoy these studies and that they provoke you towards an even greater commitment to improving the experiences of mathematics learners everywhere.

Organization and voice of the book

I've used the plenary talks as touchstones to group the chapters along three themes: understanding equity, making meaningful observations, and taking action. Drafting the text forced me to make personal choices about which aspects of each presentation to include and emphasize; that said, I've chosen to present the chapters in the voices of the presenters, hoping to capture the enthusiasm and commitment to their work that was so evident at the workshop. Responsibility for the final wording rests with me.

A challenge for us

Taken together, the presentations at CIME 2017 challenged attendees to consider:

- What observation tools do we currently have?
- What can these tools help us to see in relation to access, power and participation?
- What are the limitations of observation and observation tools in relation to access, power and participation?

To these, I add:

- What are we currently not seeing?
- What new tools do we need to build so that we may better see?
- How can our tools move us beyond seeing to doing?

I would also like to extend these challenges to the reader. I sincerely hope that you enjoy these studies and that they provoke you towards an even greater commitment to improving the experiences of mathematics learners everywhere.

— *Joi Spencer*

How the E-Word Helps and Hurts Our Cause in Mathematics Education: The Need to Rehumanize Mathematics. Based on a talk by Gutiérrez, R. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning (Critical Issues in Mathematics Education, vol. 13)*, by Spencer, J. Berkeley: MSRI (2019).

HOW THE E-WORD HELPS AND HURTS OUR CAUSE IN MATHEMATICS EDUCATION: THE NEED TO REHUMANIZE MATHEMATICS

Based on a talk by Rochelle Gutiérrez

The goal of this talk is to provide a shared language and some constructs for thinking about our visions about equity. One of the things that often happens when people gather together and talk about how they want to address equity is that people jump right in to talk about how they want to do this work. But this approach overlooks the opportunity to work together to build theory as well as to work toward general consensus around the meanings and purposes of equity.

Purposes of viewing/coding videos

“I think about the different purposes that we could have for why are we viewing and why we would be coding classroom videos, and I think that first of all we should be focusing on things that can be high leverage. So things are going to be action oriented. There is going to be some low hanging fruit, but I am arguing that we need to be going for the harder stuff.” — Gutiérrez

Videos can describe characteristics across settings, evaluate teacher’s practice, provide formative information to help teachers notice things and move toward action, and they can unearth values and definitions of equity, and open a conversation to discuss equity. For example, if you ask teachers to watch a video and to look for equity in the video, then you inherently discover their working definition of equity.

Our definitions and goals matter. From different perspectives, we may understand equity in different ways. We tend to conflate many things in our work in equity. For example, “ambitious teaching” or “rigorous” teaching does not necessarily mean equitable teaching. Whenever we use the word equity we must consider from whose perspective and towards

what purpose? Is it from the classroom students' point of view? A mathematician's point of view? A mathematics educator's point of view?

After viewing a four-minute classroom video clip, the participants were challenged to identify any examples of equity that they observed in the video. When asked to share what kinds of indications of equity they were looking for in the video, participants offered:

- Gender balance: Who is being called on?
- Participation: Who gets to participate?
- Participation: Who decides what gets to be the topic of discussion?
- Relevance of mathematics to the lives of the students
- Mode of participation (needing to stand in the front of the room to participate, approving voices for people who don't feel comfortable with that, ...)
- Is there equal selection for students based on eagerness to participate? For example, is the teacher calling on people who are eager and not eager? Does everyone get to participate, whether they're eager or not?
- Are the students who are not verbal still seen as participating?
- Degree to which students have the exposure to the process under which knowledge is co-constructed.
- Who is being assessed? (Who understands, who gets feedback?)
- Ownership of ideas/thinking

Evolution of definitions and awareness of equity

National mathematics education organizations and initiatives address equity in different ways. For example, in 2000, the *Principles and Standards for School Mathematics* from the National Council of Teachers of Mathematics (NCTM) offered six principles of effective mathematics instruction, one of which placed an emphasis on equity. In describing this principle, the council was clear to emphasize that equity does not mean that every student should receive identical instruction, but that reasonable accommodations are made to promote access and attainment for all students including those who are learning English and for those with special needs. This framing of equity was not about changing what happened in the classroom as a general approach to teaching, but providing accommodations so that subsets of students had access to the mathematics classroom.

Equity does not mean that every student should receive identical instruction, but that reasonable accommodations are made to promote access and attainment for all students.

In 2008, NCTM moved beyond these accommodations and started to bring in words such as “respect” and “understanding” to accept the different ways to perform mathematics in classrooms. This provided a new view such that equitable practices now involved

“build[ing] a relationship with mathematics that is positive and grounded in [students’] own cultural roots and history” and accepted that there are different ways that people come to understand mathematics.

“We also recognize that the part of this equitable mathematical practice is supporting a greater goal of helping students learn to care about others and treat all human beings with dignity and respect.”
— From the 2008 NCTM Equity Position Statement

In 2010, the Common Core State Standards in Mathematics (CCSSM) were written. The CCSSM are a step back from NCTM’s 2000 Equity Principle and the 2008 Equity Position Statement, and do not specifically mention equity. Furthermore, the standards only address emergent bilinguals in the appendix.

Amongst the eight Practice Standards in the CCSSM, two practice standards in particular contrast with the 2008 Equity Position Statement. These are: “reason abstractly and quantitatively” and “critiquing the reasoning of others.” Reasoning abstractly and quantitatively is one way of doing mathematics. However, reasoning contextually and qualitatively are parallel, but equally valid, ways of reasoning — ways that potentially demonstrate the dignity for other human beings as described in the Equity Position Statement. “Critiquing the reasoning of others” without first trying to *understand and appreciate* what other learners are already thinking dehumanizes the process of learning mathematics. Attempting to convince students of the one way to approach a problem and convincing people to come to the same way of thinking is a form of colonization.

How can mathematics classrooms open up a space for students to see a window into another person’s reasoning?

Dimensions of equity and learning

In 2002, I asserted three aspects of a definition of equity:

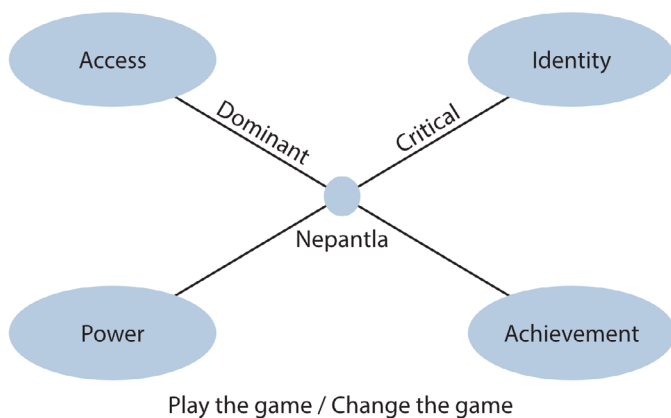
1. *Dominant equity* has a focus on the gatekeeping aspects of mathematics, including achievement and providing marginalized students with access to opportunities and choices for their lives such as jobs and careers
2. *Critical equity* is being able to analyze and critique knowledge and events in the world. It includes recognizing the relationship between mathematics and power, and understanding the origins of mathematics and how the Western view of mathematics has become the dominant one.
3. *Relationship-based equity* moves mathematics beyond what happens in schools. It involves restructuring the way we all do mathematics and altering the relationships between people, mathematics and the planet.

Four dimensions of learning/equity

In 2008, I began writing about four dimensions of learning/equity: access, achievement, identity and power. Access is the opportunity to learn. Do you have quality textbooks? Do you have access to the latest technology or to a teacher with a mathematics degree? Just because you have access doesn't mean you get to achievement. Access is a necessary but insufficient condition for Achievement. Achievement is standardized achievement and participation. Who takes the necessary mathematics classes? Are you being prepared to be part of the STEM pipeline? Achievement and access form the "dominant axis." Most schools and mathematics departments and society already agree on these. The "critical axis" is made up of identity and power. Identity is a precursor to power in the same way that access is a precursor to achievement. Do I have to park my identity at the door when I enter the mathematics classroom? Am I allowed to use my home language and be myself and be whole in the mathematics classroom? Can I be a legitimate participant using algorithms from my parents' or my home country?

Identity is a precursor to power in the same way that access is a precursor to achievement.

However, just because I get to be myself does not mean I am addressing power. Power has two levels. There is micro-level power: Who has power in the classroom? Whose ideas get revoiced? Who gets credit if you are working in a group? Who gets positioned as having had those correct ideas? Who gets to come to the front of the class to present those ideas? At a macro level, Power is about what happens outside the classroom as well: Do I get opportunities to use mathematics as a lens to critique what's going on around the world? Do I get to use mathematics to actually do anything about those injustices? Do I get to practice mathematics in ways that support the needs of my community?



*Dimensions of learning equity
(Guitierrez 2007, 2008, 2009)*

When we bring these two axes together, we give children tools to participate and influence mathematics. The access/achievement axis helps children to "play the game" and improve their opportunities in a world that already exists. The identity/power axis helps children to "change the game," changing both who gets to participate and do mathematics as well as

what mathematics as a practice is. Often our rationale for work in mathematics education is “these people need mathematics” and “we’re going to be good citizens and open the door and let more of these people in.” But it is a two-way street. Mathematics needs people. By asking people who have traditionally not been in the field to be in the field, they are actually going to change the field.

There are going to be times when we need to play the game more than we are changing the game.

Nepantla is in the middle. It is both a space of tension and a way of moving through the world. Nepantla is an Indigenous Nahuatl term that refers to the fact that when you have multiple views on reality and those come together and are intentioned, this is where you birth new realities. If we are going to do this equity work, we are going to have those tensions. There are going to be times when we need to play the game more than we are changing the game. There might be times when we are foregrounding the mathematical rigor and other times when we foreground the identity and the power.

A shift in focus: Rehumanization

Scholars have theorized equity in ways that attend to important issues of identity and power. Even so, because the word equity is bogged down with history, and people address the word equity from different angles, it leads to a failure to promote dialogue or a consensus about the meaning of equity. Thus, we may need a new angle to examine a long-term problem that is ethical, not technical. One tool can be language. Consider the term “rehumanization” instead of equity. With an emphasis on quantifying, categorizing, and reducing complex and multi-layered relationships to mere abstractions, mathematics as a practice seems to be mainly in the service of economics and warfare. I call for a radical re-imagining of mathematics as a human practice, a version that embraces the body, emotions, and harmony.

Where is this “mathematics as a human practice” that is being created by people? For many students, they feel like they are just reproducing what has already come before you. They are not really creating anything new. Rehumanization takes a different approach. It suggests that we have to address politics of teaching and politics of mathematics. We might stop and consider that mathematics operates with unearned and unquestioned privilege in society. When you say to somebody else that you are a mathematician you get confession (“I was never very good in mathematics”) and adulation (“You must be so amazing and so smart.”). Do we challenge that mathematics operates as a proxy for who is smart and who is not smart? We have to acknowledge that mathematics in society can feel dehumanizing and that there are things that happen in classrooms and conversations that can feel dehumanizing. What would it take to rehumanize mathematics? We have to imagine a rehumanized experience in school and create these rehumanized experiences.

I call for a radical re-imagining of mathematics as a human practice, a version that embraces the body, emotions, and harmony.

Examples of dehumanization

Some of the things that students tell me are dehumanizing in the mathematics classroom include:

- We have to just go along with whatever labels and categories we have been placed in
- We just have to buy into a product; we don't get to create anything new
- We have to leave our language and emotions outside of the classroom
- Speed is valued over reflection—the idea that the faster you are, the smarter you are
- You just have to pretend (that the work makes sense that this is real world)

Mathematics as a field dehumanizes as it tends to privilege some things over “the other.” For example, algebra and calculus have privileged status. When the school mathematics curriculum is largely comprised of algebra to calculus, we get rid of everything else because there is no room for it. We rob kids of the opportunity to play with things like topology or spatial reasoning or getting to play in other number bases. The other thing that is dehumanizing is privileging rule-following over rule-breaking. How can we get students to come up with their own imaginative worlds and rules? Another way that it is dehumanizing is that it privileges Western mathematics and the “standard algorithm” over many other possible algorithms. Privileging abstraction over context, mind over body, logic over intuition, and critiquing the reasoning of others over appreciating their reasoning — these are all ways that school mathematics is dehumanizing.

A rehumanizing mindset

One way in for rehumanization is to think about windows and mirrors. Having a window would be having you view things you have never thought about before. The curriculum, as well as other people in a classroom, are examples of windows. But everything cannot always be about looking out through that window. We sometimes need for the curriculum and for other people in the room that you are working with to reflect back on us. It should affirm who you are and what knowledge you bring to the classroom.

In Mayan culture, the greeting “In Lak’ech” provides the meaning that “I am another you” and the other person responds with “you are another me.” The point is not that “I am you” but an emphasis on our shared consciousness. It acknowledges that there is a part of me in you and a part of me in you at all times. So we need to think about how we can make Lak’ech more present in classrooms as a rehumanizing force.

A shift in focus

<i>Mainstream equity</i>	<i>Rehumanizing mathematics</i>
Adhering to pre-determined outcomes	Making sense, being active
Comparing groups	Recognizing learning will never look the same for everyone
Looking for particular behaviors	Helping students (and teachers) be true to themselves
Universal way of thinking about learning	Reconnecting with ourselves and others
Measuring and capturing growth quantitatively	Measuring and capturing growth with various tools

Addressing the idea of misconceptions

One small shift that we can take towards rehumanizing the mathematics classroom is related to the idea of misconceptions. When we frame students as having misconceptions, we step towards being dehumanizing. Students have conceptions that work for them until the conception is no longer generalizable. But if we start to think they have misconceptions, this becomes a form of colonization. We are saying that “you are valid if you come to my way of viewing the world.” Instead, we should consider how we can help to expand and draw out from students that which they know intuitively.

What might we “look for” that reflects rehumanizing mathematics?

<i>Participation/ Positioning</i>	Status, hierarchies in the classroom, legitimate participation (authority shifts from text/teacher to other students, students as meaning makers), teacher aware of positioning
<i>Culture/Histories</i>	Students reconnecting with their own histories or ancestors/roots (funds of knowledge, algorithms from other countries, ethnomathematics)
<i>Windows/Mirrors</i>	Students being able to see themselves in curriculum & in others (appreciation, not just critique), also a new world (standing alongside of peers, seeing new things, new axioms, goal is not a ways consensus); fostering respect/dignity, becoming the best person in their own eyes
<i>Living Practice</i>	Understanding mathematics as something in motion (ethnomathematics, history, debates, highlighting breaking the rules, axioms leading to divergent answers, mathematics for ones own purpose), students thinking of maths as a verb, not noun

<i>Broadening Maths</i>	Decentering of; Algebra/Calculus/Number Sense, symbolic representation, favoring the general case to make room for other forms that allow students to see more qualitatively or other forms that would count as maths
<i>Creation</i>	Students inventing new forms of mathematics not just reproducing what has come before (for example, invented algorithms, breaking rules)
<i>Body/Emotions</i>	Invitations to and examples where draw upon other parts of the self (for example, voice, vision, touch, intuition over logic), the senses matter for any real world problem (can't just pretend)
<i>Ownership</i>	Mathematics as something one does for oneself, not just for others (for example, just for school); questions and answers are useful/reasonable for ones own purposes; desire to “play” or “express oneself” through mathematics

Video and rehumanization

We should be cautious about the limitations of video observations. When we privilege observation tools, we can get stuck with making abstract, speculative high inferences. When we watch a video, we do not know the bigger picture, the meaning that the students got out of the interaction. We do not know the meanings behind individual acts. We also don't know how many instances would qualify for something being consistent. How much of a classroom do we need to see before it is consistent? If we care about what mathematics is, how would we actually know what is changing via video?

Some Limitations in “Observation” (High Inference)

<i>Intentions</i>	How do we know what an individual is intending to do? Might we judge a teacher without having the bigger picture?
<i>Meaning making</i>	How might we know if/when authority shifts from teacher to other students to self? Meaning outside of school or given audience? Seeds planted-good and bad. How do we know what meaning any given student makes of any given interaction in a classroom?
<i>Pattern</i>	How many instances would qualify as consistent” or often” in teaching for it to be a pattern or tendency? How many different times would a teacher need to be followed to adequately capture teaching?
<i>Changing the game</i>	How do we know mathematics as a practice is changing-relationship between living beings, mathematics, the planet? Can it be measured quantitatively? Qualitatively? Over time?
<i>Role of other agents</i>	How do we capture the ways the text/curriculum disciplines students?

Given this caution about video observations, consider what you would want to see in a video of a mathematics classroom that would tell you that it was being rehumanized. Some examples might include connection and acknowledgement of the mathematical ideas that came before in history and students reasoning contextually versus just abstractly. In addition, positioning and looking at status and hierarchies and what counts as participation would be important in the rehumanized mathematics classroom: Does the authority shift from the text to the students? Do students get to connect with their own histories and roots? Are they allowed to appreciate and not just critique? Are students allowed to see mathematics as a living practice/something in motion?

In rehumanized classrooms, the definition of mathematics is broadened and goes beyond algebra and calculus. Students are involved in the creation of algorithms, they are encouraged to break rules. Ultimately, these students have ownership.

About the presenter

Rochelle Gutierrez' scholarship focuses on equity issues in mathematics education, paying particular attention to how race, class, and language affect teaching and learning. Her current research projects focus on: developing in pre-service teachers the knowledge and disposition to teach powerful mathematics to urban students; the roles of uncertainty, tensions, and "Nepantla" in teaching; and the political knowledge that mathematics teachers need to effectively teach in an era of high-stakes education.



References

Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Towards a new equity research agenda. *Mathematical Thinking and Learning* 4(2&3), 145-187.

Gutierrez, R. (2007). (Re)defining equity: The importance of a critical perspective. In Nasir, N. and Cobb, P. (Eds.), *Diversity, equity, and access to mathematical ideas*, pp. 37-50. New York: Teachers College Press.

Gutiérrez, R. (2008). A "gap gazing" fetish in mathematics education? Problematizing research on the achievement gap. *Journal for Research in Mathematics Education* 39(4), 357-364.

Gutiérrez, R. (2009). Embracing the inherent tensions in teaching mathematics from an equity stance. *Democracy and Education* 18(3), 9-16.

Gutiérrez, R. (2012). Embracing "Nepantla:" Rethinking knowledge and its use in teaching. *REDIMAT - Journal of Research in Mathematics Education* 1(1), 29-56.

Further resources

Gutiérrez, R. (2015). Nesting in Nepantla: The importance of maintaining tensions in our work. In Joseph, N. M., Haynes, C. & Cobb, F. (Eds.), *Interrogating Whiteness and relinquishing power: White faculty's commitment to racial consciousness in STEM classrooms*, pp. 253-282. New York: Peter Lang.

Gutiérrez, R. (2013). Why (urban) mathematics teachers need political knowledge. *Journal of Urban Mathematics Education* 6(2), 7-19.

Gutiérrez, R. (2009a). Framing equity: Helping students "play the game" and "change the game." *Teaching for Excellence and Equity in Mathematics* 1(1), 4-8.

Gutiérrez, R. (2007). Context matters: Equity, success, and the future of mathematics education. In Lamberg, T., & Wiest, L. R. (Eds.), *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, pp. 1-18. Stateline (Lake Tahoe), Nevada: University of Nevada, Reno.

National Council of Teachers of Mathematics (2008). "Access and Equity in Mathematics Education: A Position of the National Council of Teachers of Mathematics". [www.nctm.org/Standards-and-Positions/NCTM-Position-Statements/].

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, NJ: NCTM.

National Governors Association Center for Best Practices & Council of Chief State School Officers (2010). *Common Core State Standards for Mathematics*. Washington, DC: Authors.

Style, E. (1996). Curriculum As Window and Mirror. *Social Science Record* Fall 1996. First published in *Listening for All Voices*, Oak Knoll School monograph, Summit, NJ, 1988.

Building an Equity-Focused Inventory to Understand Mathematics Learning Opportunities in Classrooms and Schools. Based on a talk by Spencer, J. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning* (Critical Issues in Mathematics Education, vol. 13), by Spencer, J. Berkeley: MSRI (2019).

BUILDING AN EQUITY-FOCUSED INVENTORY TO UNDERSTAND MATHEMATICS LEARNING OPPORTUNITIES IN CLASSROOMS AND SCHOOLS

Based on a talk given by Joi Spencer

“Inequities in mathematics education manifest themselves in material ways. We can all see them, but they come about through invisible, normative, and subtle means. We don’t know how they come to be.” — Spencer

The goal of this workshop is to think about how might we begin to engage in an inventory — a tool to help us keep track of whether or not we are making the progress we need to in relation to equity in mathematics education. While the mathematics education community often talks about inequity, it is unclear whether we have agreement regarding what these inequities are. Are we all talking about the same thing when we say that there is injustice related to mathematics education? Let us consider three questions:

First: Is the system of mathematics education unjust? If so, in what ways is it unjust?

Some examples of how the system of mathematics education is unjust include:

- A lot of students do not have access to mathematics education. A lot of mathematics materials are inaccessible to them and are disconnected from their skill level, their experiences and their lives.
- At the high school level, we cannot cover the breadth of curriculum necessary to help our students pass the college mathematics placement exam and many of the students get stuck in college remedial mathematics.

- The way we teach mathematics often does not relate to young people’s lives (especially those from diverse backgrounds) and it does not help them to gain a vision of how to use mathematics to improve their lives. We just are preparing them to become engineers. We do not prepare students in general to use mathematics.
- If a child goes to school everyday in 5th grade and cannot add fractions, that is unjust. We have too many kids leaving 5th grade who cannot add fractions.
- Part of the problem is high stakes testing. We are dumbing down the system.
- There are not enough bilingual math teachers. The Common Core is more of a reading exam than a math exam. It is almost impossible for them (English learners) to be successful.

Second: Where is the injustice located? Is it in the instruction? Is it located in society? Do the injustices happen before students reach our classroom?

This second question pushes us to be more specific and directed in our work and to know specifically what injustice we are addressing and how we are defining it. If we do not know where the pivot points are, then our work becomes diffuse. I want to push all of us who are doing this work around equity to be more directed in our work and to know specifically what injustice we are addressing.

And third: Are the injustices observable? Are they things that we can actually see and name?

This third question deals with one of the major themes of this conference — observation. This is a harder question because while we usually see the result of inequity, it is not so clear of how these injustices come to be. Consider, for example, some observable injustices — phenomena that might be seen on a video or observed in other ways.

Observable/tangible injustices

<i>Rates of participation</i>	Who gets called on
<i>Types of participation</i>	The kind of engagement that occurs when a person gets called on
<i>Types of interactions</i>	The kinds of interactions that occur in a classroom, simple structures, the ways in which students and teachers interact; students and students interact; students, teachers and curricular materials interact
<i>Revoicing</i>	Who gets revoiced and and what the nature of the revoicing is
<i>Language</i>	Language spoken/allowed to be spoken in the classroom

<i>Course tracking</i>	What course a student gets placed in
<i>Assessment scores</i>	Standardized or classroom specific

Consider also unobservable injustices — phenomena that cannot easily be captured by video or other means.

Non-observable/intangible injustices

<i>Participation</i>	Why students choose to/choose not to participate
<i>Structural issues</i>	The ways in which institutions structure pathways and resources and access
<i>Engagement</i>	The impact of individual actions on student decisions to engage/not engage — for example, we do not know the impact on a student who gets slighted. One student may be challenged and empowered by this slight, and another may shut down
<i>Identity</i>	The development of student identity — we have measures of identity but don't have good tools to capture the development
<i>Teachers' decision-making</i>	About who gets which course, who gets called on, etc.
<i>Norms</i>	Implicit norms are rarely made clear in the context of a classroom

For those of us who are empiricists, who are interested in observation, we must be clear about what our tools are capable of capturing and what they are not capable of capturing. Can we build better tools to help us get better at examining injustices and inequities in mathematics and to help us to understand the complexity of injustices that may occur?

We must be clear about what our tools are capable of capturing and what they are not capable of capturing.

Inequities in mathematics education manifest themselves in material ways (we can all see them), but they come about through invisible, normative, and subtle means. Let us consider the tools we can use to make sense of the intangible (the invisible, normative, and subtle) means through which inequities come about in mathematics education as well as the tangible. In my own work I have developed an inventory that can be used with schools, department chairs, principals, teachers, and others in the field to help them become conscious of and eventually dismantle these inequities.

Inventories tailored to the users

Using an inventory is like taking stock. We can take inventories of classrooms, of schools, of districts, and even of our math departments. The reason that an inventory is important is that it allows us to define what is valued, to keep track of what is important to us, to

compare where we think we are to where we actually are, and to make the invisible visible. I will share inventories that I have used with teachers, with schools (programs, departments, principals), and with pre-service teachers.

Teacher-focused inventory

Many of the teachers that I work with operate with a color-blind norm. They teach in a diverse city with diverse schools. They operate with a norm of not paying attention to difference. However, the student mathematics outcomes in San Diego (my city) are unjust. The inventory that I use with teachers asks them to begin to pay attention to particular things in their classrooms. The inventory looks a lot like a checklist. Before the inventory is implemented, the teachers and I have a conversation and decide what goes on the inventory. Here is a sampling of items that have appeared on individual inventories:

- Where are the critical junctures in my mathematics course?
- Where do differences in mathematics success and achievement begin to appear?
- Who is successful in my classroom? Who is not?
- Which patterns of success emerge in relation to gender, race, class, and/or language background?
- Where are the “forks in the road” in my mathematics classroom?

School- and program-focused

I also get the opportunity to work with principals. I do not think that we engage with principals enough. A sample of items that have appeared on principal-focused inventories include:

- Where are the critical junctures in our mathematics education program for students?
- Where do differences in mathematics success and achievement begin to appear?
- Who is successful in our mathematics program and who is not?
- Which patterns of success emerge in relation to gender, race, class and/or language background?
- Where are the “forks in the road” in our mathematics education program?

Preservice teacher-focused

Another space that I do this work in is with preservice teachers. It is important to get those who are becoming teachers to think about these same kinds of questions — particularly the intangibles that they do not see. Many times, preservice teachers are asked to conduct

classroom observations. In the past I would ask them to look at constructs such as cognitive depth and mathematical discourse. A few years ago, I added to that an inventory where they now have to think about the following questions related to inequity:

- What opportunities do students have to learn mathematics with understanding?
- How was mathematics understanding and success of traditionally marginalized students supported?
- What examples of differentiation did you see?
- Who spoke in class today? Who listened? Who succeeded? Who struggled?
- How did the lesson draw on the cultural, social, and linguistic experiences of students?

The goal of doing this work — of tailoring an inventory tool — is for educators to consider their particular spaces. This tool does not simply inventory that which is easily observed, but also the intangible. These intangibles are often shaping what the outcomes are in our mathematical spaces.

As Bonilla-Silva writes,

Racism springs not from the hearts of “racists,” but from the fact that dominant actors in a racialized social system receive benefits at all levels (political, economic, social, and even psychological), whereas subordinate actors do not. Racial outcomes then are not the product of individual “racists” but of the crystallization of racial domination into a racial structure: a network of racialized practices and relationships that shapes the life chances of various races at all levels (Bonilla-Silva, Lewis, & Embrick, 2004. p. 558).

I want to encourage us to continue to develop powerful observation tools to promote and achieve equity in mathematics education.

About the presenter

Joi Spencer is Associate Dean and Associate Professor of Mathematics Education at the University of San Diego. Her research has examined the mathematics learning opportunities of students in some of the poorest schools in the country. She teaches mathematics methods courses to prospective secondary mathematics teachers and leads teams of mathematics and STEM teachers in improving their practice through Lesson Study.



References

Paul, F. (2003). Re-Tracking within Algebra One: A structural sieve with powerful effects for low-income, minority, and immigrant students. Presented at the Harvard Civil Rights Project and University of California Conference on Expanding Opportunity in Higher Education: California and the Nation. Sacramento, California, October 2003.

Bonilla-Silva, E., Lewis, A., & Embrick, D.G. (2004). "I did not get that job because of a Black man...": The story lines and testimonies of color-blind racism. *Sociological Forum* 19(4), 555-581.

The Interaction of Power and Cognition in Undergraduate Mathematics. Based on a talk by Adiredja, A. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning (Critical Issues in Mathematics Education, vol. 13)*, by Spencer, J. Berkeley: MSRI (2019).

THE INTERACTION OF POWER AND COGNITION IN UNDERGRADUATE MATHEMATICS

Based on a talk given by Aditya Adiredja

Based on the Conference Board of Mathematics Survey (2010), 29% of full-time math faculty members are women, and 5% of them are women of color including Asian, Black, Latinx, Native Alaskan, Hawaiian, and Native-American Women. It is important to learn about and listen to the stories and experiences of women of color in the field of mathematics teaching. Their stories uncover relations of power in their work and speak to how identity impacts them and their students of color.

This presentation presents frameworks for interpreting such stories and creating counter-narratives of successful mathematical sense-making.

The importance of stories

To start the discussion, consider two quotes:

*“I was like, wow! I guess it’s good to see how other Hispanic people are so good at doing math... **It’s not what people usually think of.** I think it makes me proud that there’s a chunk of us, I’ll put myself in that group, that are willing to do whatever to be good at math or to excel in math.”*
— Vanessa, *Latinx (Oppland-Cordell, 2014, p. 20)*

*“I actually thought, I can do this stuff **instead of just looking at notes from a professor and doing the same thing.** I can actually think of stuff for myself and use the tools that they gave me to accomplish something. /... / So even if you are doing the same basic thing [in your proof], you still have [the feeling that] that’s my work that I can actually say is mine.”* (Hassi & Laursen, 2015, p. 325).

These quotes demonstrate that students are aware of their status (and the ensuing narratives related to that status) in the field of mathematics. The quotes also highlight the sig-

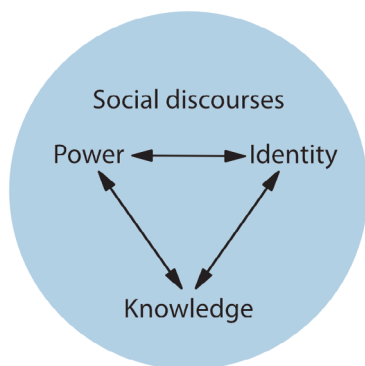
nificant impact on students when they see faculty who share their same background doing strong mathematical work. Such experiences have the power to recalibrate the discourse that places some social identities at the top and others at the bottom of a supposed hierarchy of mathematical intelligence.

Mathematical thinking, learning, and cognition are situated within the learner's identity. Mathematical thinking is not separate from who our students are as social beings. Their lives and stories and experiences are tools for building new mathematical knowledge. The perspectives that mathematics educators (and particularly mathematics faculty) hold about their students' knowledge and reasoning carries weight and power. It is mathematical faculty who decide what successful sense-making looks like. Likewise, they have the power to position students as successful or not.

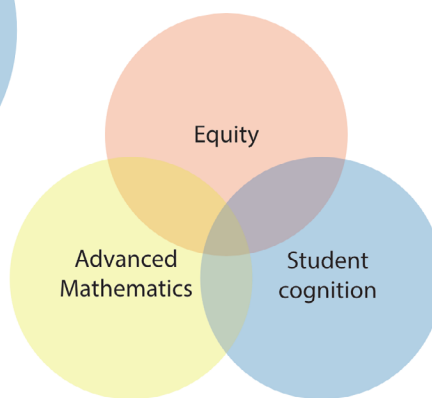
Conceptual framework

"[K]nowledge and power are inextricably linked. That is, because the production of knowledge reflects the society in which it is created, it brings with it the power relations that are part of society. What counts as knowledge, how we come to "know" things, and who is privileged in the process are all part and parcel of issues of power." (Gutiérrez, 2013)

Power, identity, and knowledge are interrelated. This interrelation influences how we as educators position our students and how students negotiate themselves within mathematics. My research lies at the intersection of equity, advanced mathematics, and student cognition. Specifically, my work studies women of color and their sense-making in advanced mathematics.



Sociopolitical lens and the intersection of equity, advanced mathematics, and student cognition



Lately I have been focusing on the role of narratives and thinking about from which students I elicit mathematical thinking? How do I view their knowledge?

A focus on women of color

We often think about women’s experiences with sexism or people of color’s experiences with racism. However, women of color experience both sexism and racism at the same time. This intersectional research is important for our field. This focus on women of color is not about deficits. Like that of many other scholars (Ellington & Frederick, 2011; Martin & McGee, 2009), the focus of this work is on successful students, where success is not just defined by standardized achievement.

In addition to a common deficit view in relation to achievement, many in our field hold deficit views of student mathematical thinking. When we think about student sense-making, we focus too much on misconceptions — on students’ mistakes and lack of understanding. This narrative, which paints students general mathematical thinking as deficient, intersects with the deficit narratives about particular groups of students.

“Knowledge in Pieces” is a theoretical framework established in physics education. These ideas suggest that we as mathematics educators must look at student mathematical misconceptions differently. A paper of Smith, diSessa, and Roschelle (1993), changed how I looked at things. The paper argues that looking at and looking for student misconceptions is the wrong way to go. Instead, we should look at how foundational our students’ prior knowledge is to their learning. Our goal should be to understand a subject’s own way of reasoning about a topic, not solely to assess its correctness with respect to a normative standard.

Teachers usually assume that when students do not follow the same way of thinking as their teachers, then it means that the students do not have the required ways of thinking to do mathematics. This is reflected in achievement gap research, that is, research that focuses on documenting gaps in achievements between different populations of students (men vs. women, White and Asian students vs. Black and Latinx students). When we report on achievement gaps, we need to realize that we center the dominant culture. This suggests that we want everyone to catch up or become more like that dominant culture — the norm. However, a group of scholars, especially black scholars, chose to focus on successful students, where success isn’t just defined by achievement but also by how students are making sense of their experience and navigating racism. Much of this research provides a counter-narrative to what we normally hear about students of color in mathematics (Ellington & Frederick, 2011; McGee & Martin, 2009).

Our goal should be to understand a subject’s own way of reasoning about a topic, not solely to assess its correctness with respect to a normative standard.

Using anti-deficit perspectives to construct counter-narratives

My goal is to create a counter-narrative of women of color (WOC) in mathematics, and to document their successful mathematical sense-making. I am looking for more stories like the movie *Hidden Figures*, which is so popular because it highlights a different story and it

changes the narrative of the role of Black women in the advancement of science in this nation. Here are two examples of counter-narratives from my research that examine the mathematical sense-making of women of color in undergraduate mathematics. The first shows how one student successfully integrated her prior knowledge into her new understanding of the epsilon-delta definition of a limit. The second shows a group of students using of a wealth of everyday contexts to make sense of the idea of basis in linear algebra.

Epsilon-delta definition of a limit

In my research, I have examined the mathematical sense-making of women of color in undergraduate mathematics. The first study that I present here is from my dissertation work, where I analyzed one Chicana student's sense making of the formal definition of a limit using an instructional tool called the Pancake Story. The Pancake Story is designed to tap into students' intuitive understanding of quality control (in this case in a scenario about making pancakes to strict specifications) and help them leverage that intuition to build an understanding of the formal epsilon-delta definition of a limit.

In the case that I am presenting here, by using Knowledge in Pieces as a framework to analyze the student Adriana's sense-making, I established a story about successful-sense making by a woman of color. Sensitivity of the analysis detected awareness of productive mathematical ideas despite uses of non-normative language. The student did not simply accept ideas from the story, but took time to align those ideas with her prior knowledge. The result was accurate mathematical knowledge integrated in her prior knowledge utilizing resources from the story.

On the other hand, if we were to take a deficit perspective on the student's sense-making, we would determine that she fit with other students from the literature who struggled with the formal definition. It seems that there was a misconception in her use of functional dependence, and such misconception persisted after the intervention. Even with support, Adriana still struggled. Any changes in her explanation would not transfer to real abstract mathematical contexts. The problem with this interpretation is that it produces inaccurate claims about the student and her sense-making. It refuses to acknowledge her contribution in making sense of the mathematics. It accepts non-normative language as an indicator of lack of knowledge, and prioritizes immediate change as evidence for learning. Lastly, it undermines the progress made using non-formal mathematical resources, thereby continuing to solely privilege formal mathematics in learning.

The problem with a deficit-perspective interpretation is that it produces inaccurate claims about the student and her sense-making

Basis in linear algebra

In a second study, in collaboration with Michelle Zandieh from Arizona State University, I considered a group of women of color who were enrolled in linear algebra. This was a collaboration with Michelle Zandieh from Arizona State University. Specifically, I looked

at how they described the idea of basis, and the everyday contexts that they used to explain these ideas. I shared some of the explanations the group of women provided with the attendees of this meeting and had them analyze what worked and did not work in the students' explanations. The goal was to get them to try to look at students' work from an anti-deficit perspective, and to see if they could build from these students' explanations.

The group of women used contexts such as friendships, driving, storage room, fashion, recipes, even religious teachings to explain basis. These examples provided a window into their sense-making and intuitive ideas that are related to the concept of basis. Often when we think about basis, we think of span and independence; but our analysis revealed there are other aspects and nuances of basis including: the notions of minimal, essential, representation, non-redundant, and different/sameness. These words are closely related, yet they entail different aspects of basis for the students. The creativity and the breadth of the everyday contexts the students used to describe basis was deeply informative. Likewise, the degree to which the contexts that they generated to describe basis fit with the formal definition of basis was eye opening. Their everyday explanations provided a window into how students make sense and contextualize basis and as such contribute to local theory-building about the learning of basis. This study also repositions women of color as potential resources instead of a population that needs extra support.

This study repositions women of color as potential resources instead of a population that needs extra support.

The value of the work

Mathematical thinking and learning are situated within dominant narratives about people in society. Our perspectives as mathematics educators on students' knowledge and reasoning carry weight and power. We decide what successful sense-making looks like, and who can be positioned as successful in mathematics. It is not about optimism versus pessimism or half full versus half empty, but accuracy. If we are more accurate in our assessment of students' understanding we can contribute to the effort of humanizing mathematics and mathematics education.

“Racism is not merely a simplistic hatred. It is, more often, broad sympathy towards some and broader skepticism towards others.”— Ta-Nehisi Coates

About the presenter

Aditya Adiredja is Assistant Professor in the Department of Mathematics at the University of Arizona. He is an active member of the Equity Perspectives in Undergraduate Mathematics Education Working Group at the Conference for Research in Undergraduate Mathematics Education (CRUME).



References

Ellington, R. M., & Frederick, R. (2010). Black high achieving undergraduate mathematics majors discuss success and persistence in mathematics. *The Negro Educational Review* 61(1-4), 61-84.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education* 44(1), 37-68.

Hassi, M. L., & Laursen, S. L. (2015). Transformative learning: Personal empowerment in learning mathematics. *Journal of Transformative Education* 13(4), 316-340.

Martin, D.B., & McGee, E.O. (2009). Mathematics literacy for liberation: Knowledge construction in African American context. In Greer, B., Mukhopadhyay, S., Nelson-Barber, S., & Powell, A. (Eds.), *Culturally responsive mathematics education*, pp. 146-158. Mahwah, NJ: Lawrence Erlbaum Associates.

Oppland-Cordell, S. (2014). Urban Latina/o undergraduate students' negotiations of identities and participation in an emerging scholars calculus I workshop. *Journal of Urban Mathematics Education* 7(1), 19-54.

Smith, J.P., diSessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: A Constructivist analysis of knowledge in transition. *Journal of the Learning Sciences* 3(2), 115-163.

Valero, P. (2004). Socio-political perspectives on mathematics education. In Valero, P. & Zevenbergen, R. (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology*, pp. 5-24. Norwell: Kluwer.

Further Resources

Adiredja, A. P. (2017). Anti-deficit Narratives: Politics of Mathematical Sense Making. Manuscript in preparation.

Adiredja, A. P. & Andrews-Larson, C. (2017). Taking the sociopolitical turn in postsecondary mathematics education research. *International Journal for Research in Undergraduate Mathematics Education* 3(3), 444-465.

Adiredja, A.P. & Zandieh, M. (2017). Using Women of Color's Intuitive Examples to Reveal Nuances about Basis. Paper presented at the 20th Annual Conference on Research in Undergraduate Mathematics Education, San Diego, California.

Berry III, R., Thunder, K., & McClain, O. (2011). Counter narratives: Examining the mathematics and racial identities of Black boys who are successful with school mathematics. *Journal of African American Males in Education* 2(1), 10-23.

Blair, R., Kirkman, E.E., & Maxwell, J.W. (2010). *Conference Board of Mathematical Sciences (CBMS) 2010 Survey Report*. American Mathematical Society.

diSessa, A. A. (1993). Toward an epistemology of physics. *Cognition and Instruction* 10 (2-3), 105-225; responses to commentary, 261-280. (Cognition and Instruction, Monograph No. 1.) The monograph includes: an introduction by Editor Lauren Resnick; commentary by F. Marton, N. Ueno, M. Chi and J. Slotta; and my responses (Between brain and behavior: Response to Ference Marton; The pot calls the kettle unsituated: Response to Naoki Ueno; Ontologies in pieces: Response to M. Chi and J. Slotta).

diSessa, A., Sherin, B., and Levin, M. (2016). Knowledge Analysis: An Introduction. In diSessa, A., Levin, M., & Brown, N. (Eds.), *Knowledge and Interaction*, pp. 30-71. New York and London: Routledge.

Gutiérrez, R., & Dixon-Román, E. (2011). Beyond gap gazing: How can thinking about education comprehensively help us (re) envision mathematics education? In Atweh, B., Graven, M., Secada, W., & Valero, P. (Eds.), *Mapping Equity and Quality in Mathematics Education*, pp. 21–34. Springer Netherlands.

Harper, S. R. (2010). An anti-deficit achievement framework for research on students of color in STEM. In Harper, S.R. & Newman, C.B. (Eds.), *Students of color in STEM: Engineering a new research agenda. New directions for institutional research*, pp. 63-74. San Francisco: Jossey-Bass.

Hsu, E., Murphy, T.J., & Treisman, U. (2008). Supporting High Achievement In Introductory Mathematics Courses: What We Have Learned From 30 Years of the Emerging Scholars Program. In Carlson, M. & Rasmussen, C. (Eds.), *Making the Connection: Research and Teaching in Undergraduate Mathematics*. Washington, DC: Mathematical Association of America.

Larnell, G. V., Boston, D., & Bragelman, J. (2014). The stuff of stereotypes: Toward unpacking identity threats amid African American students' learning experiences. *Journal of Education* 194(1), 49-57.

Nasir, N. S., & Shah, N. (2011). On defense: African American males making sense of racialized narratives in mathematics education. *Journal of African American Males in Education* 2(1), 24-45.

Nasir, N. S., Snyder, C. R., Shah, N., & Ross, K. M. (2013). Racial storylines and implications for learning. *Human Development* 55(5&6), 285-301.

Ong, M., Wright, C., Espinosa, L. L., & Orfield, G. (2011). Inside the double bind: A synthesis of empirical research on undergraduate and graduate women of color in science, technology, engineering, and mathematics. *Harvard Educational Review* 81(2), 172-209.

Creating Video Case Studies to Study Equitable Classroom Interactions. Based on a talk by Math for America. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning (Critical Issues in Mathematics Education, vol. 13)*, by Spencer, J. Berkeley: MSRI (2019).

CREATING VIDEO CASE STUDIES TO STUDY EQUITABLE CLASSROOM INTERACTIONS

Based on a talk given by Math for America (MfA), Victoria Bonaccorso (Montclair State University), Lauren Brady (Park East High School, NYSMTP, MfA), Michael Driskill (MfA), Dave Henry (SUNY Buffalo State, NYSMTP), Elizabeth Kent (Lafayette High School, NYSMTP), Eileen Murray (Montclair State University), David Wilson (SUNY Buffalo State, NYSMTP), Nilam Yagielski (Sweet Home High School, NYSMTP)

It is important to open master teachers' classrooms up to preservice teachers. To help provide this access, we created video case studies that serve as curated records of teaching that can be used to teach and support preservice teachers. We collected videos and materials from lessons taught by master teachers and then employed Schoenfeld's Teaching for Robust Understanding (TRU) Framework to bundle video excerpts and lesson materials as case studies. We then used the resulting case studies to study teaching with preservice teachers.

As many scholars have acknowledged in this conference, TRU is quite important in math education. Teachers should focus on teaching through student learning and use their understanding of students' thinking to improve their teaching. TRU is a framework for characterizing powerful learning environments in crisp and actionable ways, providing a way of thinking about what happens in classrooms, and capturing what we know to be good practice (Schoenfeld, 2016). Classrooms that do a good job with incorporating the following five dimensions will produce students who are powerful thinkers.

TRU is practical to use in math education because it is comprehensive and grounded in the literature on effective mathematics instruction. It includes only five dimensions, allowing for ease of use. The framework can be used to view classrooms and is therefore a tool of professional growth. Rather than being an evaluative tool, it creates a common language for professionals to discuss mathematics classrooms.

Five dimensions for powerful thinkers

<i>The mathematics</i>	In these classrooms, students have opportunities to learn important mathematical content and practices with a focus on connections between procedures and concepts.
<i>Cognitive demand</i>	These classrooms allow students to engage in productive struggle.
<i>Equitable access</i>	In these classrooms, every student has the ability to engage with the mathematical content.
<i>Agency, ownership, and identity</i>	Students in these classrooms talk mathematics and have their ideas refined and built upon by each other.
<i>Formative assessment</i>	These classrooms respond to students by engaging with them and they provide opportunities for students to refine their understanding.

The TRU framework addresses the importance of all students becoming “powerful mathematical thinkers.” While one of the dimensions focuses squarely on equitable access, equity is embedded throughout each dimension. The agency, ownership, and identity dimension emphasizes that all students should be developing their mathematical selves through lessons with strong content, and high levels of cognitive demand, where misunderstandings are addressed through formative assessments.

The use of TRU provides a common lens through which participants can view lesson plans and classroom videos as part of the case studies. It allows participants to view one lesson through multiple dimensions. During video watching, a conversational guide can be used to help facilitators engage in productive dialogue with participants. The questions in the conversational guide support educators with different experiences, different expertise, and different strengths to work together to develop a common vision, common priorities, and a common language. TRU provides a tool with which to collaboratively improve instruction and better support students to develop robust understandings.

About the presenters

Victoria Bonaccorso is currently a PhD student in Mathematics Education at Montclair State University. She earned a Bachelor of Science in Mathematics from Northeastern University in 2006 and a Master of Arts in Educational Leadership from Montclair State University in 2012. In the fall of 2017, she will be starting her ninth year as a secondary mathematics teacher at Pequannock Township High School teaching all courses from algebra through calculus.



Lauren Brady is Head of the Math Department, Programmer, co-facilitator of the Professional Learning Committee, and works with under-credited students through advisory at Park East High School in New York City. She is also a Math for America Master Teacher and part of the New York State Master Teacher Program.



Michael Driskill is the Director of Advocacy for Math for America. He spent ten years in the classroom, teaching each grade from 6–12 and courses ranging from introductory algebra to linear algebra. His teaching experience includes both public and independent schools, where he has held positions as a classroom teacher, teacher researcher, instructional specialist, and dean.



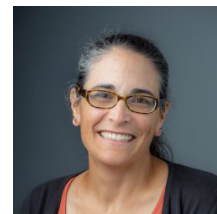
Dave Henry is an Associate Professor in Elementary Education, Literacy, and Educational Leadership at Buffalo State College, State University of New York. His research interests include mental models of electric circuits, mental models in science education, physics education research, and collaborative action research. He is also co-director of the Western NY Physics Teacher Alliance.



Elizabeth Kent has been teaching in the Buffalo Public Schools system for 13 years and at Lafayette High School for seven years. She currently teaches Algebra 2 but has taught all levels of mathematics from grade 7 through pre-calculus. She is a member of NCTM, AMTNYS, and is admitted to the New York State Bar Association.



Eileen Murray is Assistant Professor of Mathematical Sciences at Montclair State University. The core of her research interests is the desire to understand how to best prepare and support secondary mathematics teachers along their professional continuum.



David Wilson is Associate Professor at SUNY Buffalo. He has more than a decade of teaching high school mathematics and another decade-plus teaching mathematics and mathematics education courses at the undergraduate and graduate levels. His research interests include problem-based teaching and learning with a particular focus on geometry and statistics. Most recently, his research has focused on Chinese mathematics teaching, curriculum, and professional development.



Nilam Yagielski currently teaches calculus, a Niagara University Senior Term Enrichment Program course, and pre-algebra at Sweet Home Senior High School in Amherst, NY. With more than 17 years of experience in the classroom, she is also an instructional coach for high school math teachers. She serves on the Building Leadership Team, is the advisor for the National Honor Society, and is very involved with professional development programs for secondary math teachers.



References

Baldinger, E. M., Louie, N., & the Algebra Teaching Study and Mathematics Assessment Project. (2014). *The TRU Math conversation guide: A tool for teacher learning and growth*. Berkeley, CA, and E. Lansing, MI: Graduate School of Education, University of California, Berkeley, and College of Education, Michigan State University. Available at [truframework.org/tru-conversation-guide/].

Schoenfeld, A.H. (2016). The Practice And Use Of Observation In Powerful Professional Development: The Teaching-For-Robust-Understanding (TRU) Framework. Presentation.

Schoenfeld, A.H. (2017). The Role of Video in Understanding and Improving Mathematical Thinking and Teaching. *Journal of Mathematics Teacher Education* 20(5), 415-432.

I Do (Not) Belong: Experiences of Black Women and Girls in Mathematics Education. Based on a talk by Joseph, N. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning (Critical Issues in Mathematics Education, vol. 13)*, by Spencer, J. Berkeley: MSRI (2019).

I DO (NOT) BELONG: EXPERIENCES OF BLACK WOMEN AND GIRLS IN MATHEMATICS EDUCATION

Based talk on a by Nicole Joseph

I would like to frame my presentation around this question: What is the state of Black women and girls in mathematics? I am interested in the experiences of Black women and girls in mathematics because I want to understand how they engage in racial and mathematics identity processes. What does it mean to be a Black woman or girl who is learning mathematics? Overall, I aim to address two issues in my work: one is underrepresentation and the other is social justice. Interrogating systems of oppression that act as barriers to access, retention, and empowerment are important to address when you are trying to get at social justice, which is different from equity.

Before coming to the academy, I was a mathematics teacher in upper elementary, middle school, and high school. I spent nine years teaching in Seattle Public Schools and Federal Way Public Schools. I have also coached math teachers for another five years. Then, I came into higher education. I have been thinking about these issues at the micro and macro level for a very long time. And I still have a lot to learn.

To investigate the state of Black women and girls in mathematics, we will start with two learning community exercises: First, we will consider a statistical story, and then we will look at some lived experiences of Black women in different studies in order to deconstruct what that looks like. Then, I am going to talk about mathematics as embedded in white supremacy. And finally, I will end with a discussion of the affordances of intersectionality — a framework and methodological approach that I use.

Community Exercises

#1: A statistical story

In this exercise, we look at the most recent numbers that we have from the National Science Foundation (NSF) regarding those who are earning PhDs in mathematics and statistics in our nation. The top line of the table shows the total number of doctorates awarded to US citizens and permanent residents each year. The table then breaks out the doctorates awarded to women by race and ethnicity. And towards the bottom, you will see the doctorates awarded to men as well as the fraction of those men who are white. Finally, the total is repeated, this time also showing the total percentage of doctorates going to all women and to white men alone.

Doctorates awarded to US students: Females compared to white males

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Total	508	540	583	645	671	788	863	849	852	912	948
<i>Women</i>	153	152	151	193	218	235	245	230	224	242	254
Black or African American	1	9	5	5	11	16	9	9	10	6	9
Hispanic or Latino	5	6	11	4	5	12	8	9	10	6	7
American Indian or Alaska Native	0	0	0	1	2	3	2	5	0	3	1
White	118	101	102	132	161	154	168	155	163	170	179
Asian	n/a	n/a	n/a	n/a	n/a	n/a	n/a	38	22	34	32
Asian or Pacific Islander	48	54	63	79	53	78	84	n/a	n/a	n/a	n/a
Native Hawaiian or Other Pacific Islander	n/a	n/a	n/a	n/a	n/a	n/a	n/a	1	2	0	1
Other or unknown	15	10	13	22	17	25	21	18	15	22	22
Two or More Races	n/a	n/a	n/a	n/a	n/a	n/a	n/a	1	4	3	4
<i>Men</i>	355	388	432	452	453	553	618	619	628	670	694
White	270	297	326	326	329	405	466	472	473	482	529
% White	76%	77%	75%	72%	73%	73%	75%	76%	75%	72%	76%
Total	508	540	583	645	671	788	863	849	852	912	948
% Women	30%	28%	26%	30%	32%	30%	28%	27%	26%	27%	27%
% White men	53%	55%	56%	51%	49%	51%	54%	56%	56%	53%	56%

Mathematics and statistics doctorates awarded to US citizens and permanent residents. *Source:* Women, Minorities, and Persons with Disabilities in Science and Engineering: 2017 (NSF 17-310), National Science Foundation, Table 7-7 (January 2017).

Consider these two questions: What do you notice about the numbers? What story do you think is being told in our math communities about these numbers?

Here's a sample of things that audience members noticed:

- Native American growth is lowest of all. Black/Hispanic growth is also low.

- Institutions tend to frame diversity as targeting international students without worrying about what’s happening domestically.
- There is an increase in total PhDs being earned by US students and the only increase in women is coming from white women.
- The number of men receiving degrees has also increased so the overall percentage has not changed.
- Between 2008 and 2010 there was an uptick, especially for Black/Latino/a students but that that has come back down.
- These numbers are so incredibly small, so it doesn’t seem like there’s enough data to call it an uptick.
- In the early 1990s we thought we saw a brief uptick and took our eye off the ball and then the problem rose up and bit us.
- This is going to be who is populating the professoriate for the next half century.
- This is a story of inattention. Our departments have this trove of international graduate students so they have to spend no attention on getting graduate students from the US.
- The number of black women graduating with Math PhDs is an unacceptable travesty.
- There was a big jump in 2008: What happened in 2004 to push more women to enter PhD programs? What happened in 2000 when they started their undergraduate studies?

#2: Experiences of Black women in mathematics

In our next community exercise we look at the experiences of Black women in mathematics as shared through quotes from a variety of studies conducted by my colleagues and collaborators. These quotes range from high school through the doctorate level. As you read the quotes, consider the following discussion questions:

1. What do you notice about the lived experiences of Black women?
2. How are they experiencing the mathematics classroom?
3. What do we notice about how they are treated by some math faculty?
4. What do we notice about how they are treated by their peers?
5. Why are math departments so competitive?
6. How does this competitive environment affect Black women and girls?

Quotes of experiences of Black women in mathematics

“Don’t worry, Tamara. Even if you score low, you will get in because of affirmative action. There are hardly no Black students at Princeton.”

It was a summer class that met at 7:00 in the morning in which I was the only Black and the only woman.

For nine weeks, 30 or 40 white men ignored me completely.

I never initiated any conversations as there was no encouragement to do so.

It seemed to me that conversations before class on mathematics between classmates quickly terminated if it appeared that I was listening

This was my first experience attending school in a vacuum.

My mathematics isolation was complete. I was not acquainted with any Blacks who had interest in these subjects.

I just had to sit down and say to myself, “Okay I know I wanted to get my PhD from here, but in some respects a PhD is conferred — so if they don’t want you to get it you won’t get it.”

I think it’s all sort of attributed to this one guy who would say horrible things about not being able to succeed in mathematics. Pull you into his office like you shouldn’t be here, and stuff like that.

It turns out that they had too many students and needed to try to get rid of some students. They did not have enough professors to conduct the research with all of the students. This was part of the strategy.

The faculty did not care whether I finished or not. They were fast paced and the students were competitive. In a sense they didn’t want to share their knowledge.

One of my study partners, I remember him saying on one of our exams: “We did okay. Not as good as any of us had wanted to.” I got a higher B than he’d got and he says, “you know, I thought I’d done at least as good as you.”

Debriefing the community learning experiences

These exercises highlight the need for both representation as well as social justice. Black women are just not represented in mathematics in the same way as their white peers. However, numerical representation is not enough. Mathematics departments could, “with some intentional efforts,” find Black women who could do their masters or a PhD in mathematics. The question for me is also about social justice. Some of the experiences highlighted in the quotes suggest a need for a disruption or a complete dismantling of deep-seated structures in math communities. These are our programs, departments, professional organizations, conferences. That’s our community as mathematicians and mathematics educators.

Further experiences: The first five PhDs

In a 2017 study, I looked at the first five African American women to get mathematics PhDs in the US. They did so during the Jim Crow Era. In my study of these women, I found that they experienced a sense of pride in their segregated communities — their elementary schools, their secondary and undergraduate schools, in part because high expectations were just normal. Rarely were limitations placed on these students. I also found that they engaged in transformative logic in the face of racism and sexism particularly during their graduate programs. They used cognition and logic and their intuitiveness and feelings to navigate very racist spaces. They drew upon their faith in themselves and their identities as mathematicians to navigate these spaces. Finally, I found that these women also contributed to productive processes as mathematicians. Plato talked about how women were only going to contribute to reproductive systems rather than productive systems. They definitely defied his understanding. Although their collective scholarship can be viewed as menial, they did produce research, they got external funding from NSF and other funders, they were chairs for their math departments, they worked in industry, and they did all of that while also serving in their local communities.

This study took a historical, philosophical, and epistemological perspective to demonstrate a very complex analysis of Plato’s theories about women and to illustrate the social construction of mathematics as embedded in white supremacy and to complicate contemporary issues around access, participation and power.

White supremacy

White supremacy is a term that we should not be afraid of. We need to clearly understand what it is so that we can work together as a community of scholars and practitioners to dismantle it. When people hear the word white supremacy, they think of the most overt experience of racism like the KKK. You should know that white supremacy is embedded in the everyday forms of racism committed by well-intentioned white people. Chala Haynes writes that, “White supremacy has both material and ideological attributes that maintain white privilege, whiteness as normal, white innocence and white advantage, permitting

You should know that white supremacy is embedded in the everyday forms of racism committed by well-intentioned white people.

white people to maintain racial group superiority and advantage” (Haynes 2013; Haynes in-press).

In her recent book, *Inventing the Mathematician: Gender, Race, and Our Cultural Understanding of Mathematics*, Sarah Hottinger uses cultural studies in very creative ways to deconstruct mainstream ideas about mathematics and about who can engage in mathematical knowledge. She argues that this limits the ability of all women (and I am arguing Black women in particular and other women of color) to succeed in mathematics as well as limits their access to full subjectivity. Normative mathematical subjectivity does this because the cultural narrative for mathematics is that mathematics is separate from human concern. And that same subjectivity called mathematical knowledge is somehow value free. It is truth and beauty and light. She invokes Foucault to demonstrate how the history of mathematics is characterized as mainly internalist rather than externalist. Internalist histories of mathematics see mathematics as both universal and eternal. Mathematical knowledge and truth are understood as in the Platonic sense — essentially outside of ourselves (of human concern and experience). She also uses portraiture to argue that internalist histories of mathematics play a role in constructing this normative mathematical subjectivity.

Burton’s text features portraits of 45 mathematicians. Two of them are women.

Using David Burton’s textbook *The History of Mathematics: An Introduction*, Hottinger illustrates the idea of visualizing rationality. Burton’s text features portraits of 45 mathematicians. Two of them are women. Since the portraits are accompanied by images of the mathematics that is being discussed, what we learn is that the portraits contribute to the discursive construction of mathematical subjectivity. The choice of portraits in Burton’s book perpetuates two ideas: (1) the association of mathematics as normative, European masculinity, and (2) white supremacy. When we connect back to why these ideas are important for Black women, what we find is that normative mathematical subjectivity prevents many women of color (Black women in particular) from seeing themselves as mathematicians. Burton’s book clearly communicates who is a mathematician, and Black women are nowhere to be found.

When there is no name for a problem, we cannot see a problem. If we cannot see a problem, we cannot address the problem

Educator Akila Anika Naila explains that “We have all been taught by white people, who have been taught by white people, who have been taught by white people, who have been taught by white people using textbooks written by and about white people.” So if we are not challenging as a community this monolithic racial hegemony, what we are doing is reinforcing it.

Intersectionality

The mathematics community has difficulty including and incorporating Black women because as Kimberlé Crenshaw points out, we have no frames for including them. K-12

math teachers and post-secondary math faculty do not usually discuss Black women in our classrooms. We have already seen through our textbooks that we do not see them there. Crenshaw argues that when there is no name for a problem, we cannot see a problem. If we cannot see a problem, we cannot address the problem. We have a predicament of framing. Black women do belong in mathematics. But what framing do we have to help produce counter narratives of success, achievement, and belonging?

Intersectionality is in part a theory about how individuals can face multiple threats of discrimination when a person's multiple identities overlap. The promise of intersectionality is that it can help bring light to the oppressions Black women experience both as a person who is Black and as a woman in math spaces — which include all kinds of spaces such as offices, classrooms, and conferences we attend.

Beginning Intersectional Approaches

We can begin to address critical concerns using intersectional approaches. One example might be for a mathematics department to interview the women of color in their programs to learn what kinds of experiences they are facing in these departments, and then do the activities we have done today to deconstruct their experiences. Not everyone is going to care about this in the math department. But start with the people who do care, who do want to change.

Not everyone is going to care about this in the math department.

But start with the people who do care, who do want to change.

Another intersectional approach is for the mathematics community to ask what interventions do we have in place for Black women? Do we have interventions that address race? There is CAARMS, the Conference for African American Researchers in the Mathematical Sciences, whose aim is to address race; however, the assumption here is that women and men of color have the same needs. There is the Association of Women in Mathematics that aims to get at gender; the assumption here is, “Is racialized sexism real?” These types of interventions can be considered single-axis. They think about one idea when there are multiple identities that are relevant.

Research and practice priorities

I propose four broad research and practice priorities that can push the discourse about mathematics education policy and improve academic outcomes for Black women and girls.

1. *Research: Increase asset-based research studies.* This means looking at the women who have made it through the mathematics pipeline. Study these women to discover what some of their coping strategies were and how they navigated the system. Rather than looking at the terrible numbers that we publish every year that show all of the black students at the bottom on standardized exams, we should study these successes. This approach will shed light on effective ways to support Black women and girls.

2. *Research: Cross-departmental/cross-institution collaboration.* I've been getting lots of invitations from different STEM departments. Having these types of cross collaborations is important. Math education should collaborate with math departments and history, learning sciences, and anthropology because these problems are hard and complex and it takes more than what we know in our individual literature bases to be successful. We need to figure this out together — mathematicians and mathematics educators. The work is hard, but it is work worth doing.
3. *Practice: Curriculum reform in teacher education.* Teachers need to understand what it means to be a Black woman or other woman of color within the context of learning mathematics. Teacher education preparation courses can reengineer diversity courses that are already on the books to include some of these issues.
4. *Practice: Curriculum reform in graduate mathematics programs.* We could include discussion seminars about these issues in our graduate programs. Likewise, high profile mathematics organizations that many doctoral students are a part of (MAA, AWM) can prioritize these issues. They can provide workshops and conferences with social justice tracks.

The overarching importance of intersectionality

In conclusion, I'd like to emphasize that our culture constructs race, gender, and mathematics in ways that ensure that Black women have a difficult time understanding themselves as mathematicians. We can work as hard as we want on ways mathematics can address teaching and learning issues, but we will never operate outside of racism. It's just a fact. Knowing this, I urge our community to create and innovate around intersectionality. Alternative intersectional approaches can give some insight into how we can challenge the construction of mathematics as a hegemonic space. In these intersectional projects we must both deconstruct the hegemony and create multiple realities to ensure that Black women see themselves as capable and insightful mathematical knowers.

About the presenter

Nicole M. Joseph is Assistant Professor of Mathematics and Science Education in the Department of Teaching and Learning at Vanderbilt University. Dr. Joseph was a 2014 National Academy of Education/Spencer Postdoctoral Fellow, and her research interests include mathematics education and equity from an intersectional perspective.



References

Borum, V.O. (2010). *Building a model of success: Examining black women with doctoral degrees in mathematics*. Dissertation Thesis. Columbia University.

Borum, V., & Walker, E. (2012). What makes the difference? Black women's undergraduate and graduate experiences in mathematics. *The Journal of Negro Education* 81(4), 366-378.

Hottinger, S. N. (2016). *Inventing the Mathematician: Gender, Race, and Our Cultural Understanding of Mathematics*. Albany: State University of New York Press.

Joseph, N. M. (2017). What Plato took for granted: Examining the biographies of the first five African American female mathematicians and what that says about resistance to the western epistemological cannon. In Polnick, B., Irby, B. & Ballenger, J. (Eds.), *Women of Color in STEM: Navigating the Workforce*, pp. 3-38. Charlotte, NC: Information Age Publishing Inc.

Joseph, N.M., Hailu, M., & Boston, D. (2107). Black Women's and Girls' Persistence in the P-20 Mathematics Pipeline: Two Decades of Children, Youth, and Adult Education Research. *Review of Research in Education* 41(1), 203-227.

Joseph, N. M., Haynes, C., & Cobb, F. (Eds.) (2016). *Interrogating Whiteness and relinquishing power: White faculty's commitment to racial consciousness in STEM classrooms*. New York: Peter Lang.

McGee, E. (2013). Young, Black, mathematically gifted, and stereotyped. *The High School Journal* 96(3), 253-263.

National Science Foundation (2017). *Women, Minorities, and Persons with Disabilities in Science and Engineering: 2017* (NSF 17-310), Table 7-7.

Further resources

The Anna Julia Cooper Center of Wake Forest University (ajccenter.com) is particularly astute at putting together cross-departmental and intersectional collaborations.

Mathematical Impediments in the Classroom: Microaggressions and Bias. Based on a talk by Yong, D. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning (Critical Issues in Mathematics Education, vol. 13)*, by Spencer, J. Berkeley: MSRI (2019).

MATHEMATICAL IMPEDIMENTS IN THE CLASSROOM: MICROAGGRESSIONS AND BIAS

Based on a talk given by Darryl Yong

Students may experience mathematical microaggressions in diverse learning contexts. Sue, et al. (2007) defined microaggressions as “brief and commonplace daily verbal, behavioral, or environmental indignities, whether intentional or unintentional, that communicate hostile, derogatory, or negative racial slights and insults toward people of color”(p. 273). In mathematical classes, some actions and words from teachers, between teachers and students, and among students may dehumanize, marginalize, or create self-doubt in students.

Some microaggressions happen when teachers are being too obvious with favorite students, presenting examples that are not representative for all students, expressing dismay or frustration at wrong answers and surprise at right answers, not recognizing the partial correctness of students’ work, keeping a discussion within a narrow circle of students, and not giving all students the same wait time.

Students may experience microaggressions in the words from their teachers, such as “you should have learned this in previous years”; “these problems should be easy for you”; or “I would not have expected you to be a science major.” And from fellow students as well, who may have good intentions to help someone figure out an answer by saying, “You will not get it. Just copy my answer.”

“I would not have expected you to be a science major...”

All the examples above happen in classrooms and can demotivate, demoralize, or marginalize students, even though these words and actions are not intentional or conscious. Educators should be aware that an unintentional behavior may cause huge impact on students, and good intentions with inappropriate actions may also cause negative impacts on students.

Support students

The following are supportive words and actions that teachers could use in classrooms:

- Value, name, and allow students to demonstrate a broader range of mathematical competencies.
- Affirm students' accomplishments. Praise tenacity and efforts.
- Provide authentic experience for students to do science/math to combat their self-doubt.
- Be more aware of interactions. "Would I talk/react/ behave in this way if it were a different student? Why?"
- Be more transparent about instructional choices so students do not question if they are being singled out.
- Apologize when you mess up.

Address cognitive errors

Another impediment that may affect the quality and equity of math education is cognitive errors. Cognitive errors are tendencies to think in certain ways that can lead to systematic deviations from a standard of rationality or good judgment. These cognitive errors can lead to disproportionate and unintended effects for people historically excluded from the mathematical sciences. A few cognitive errors include:

Cognitive errors

<i>Anchoring bias</i>	The tendency to rely too heavily, or "anchor," on one trait or piece of information when making decisions.
<i>Confirmation bias</i>	The tendency to search for, interpret, focus on and remember information in a way that confirms one's preconceptions.
<i>Distinction bias</i>	The tendency to view two options as more dissimilar when evaluating them simultaneously than when evaluating them separately.
<i>Mere exposure effect</i>	The tendency to express undue liking for things merely because of familiarity with them.
<i>Negativity bias</i>	Psychological phenomenon by which humans have a greater recall of unpleasant memories compared with positive memories.

Here are a few ways that teachers can combat cognitive errors:

- Slow down. Avoid working while tired or stressed.
- Focus.

- Do not assume that numerical grading systems are automatically objective.
- Engage in counter-stereotyping.
- Routinely consider alternative explanations.
- Ask trusted colleagues to review your decision making.

In conclusion, please remember that microaggressions and bias are just two small pieces of how racism, sexism, and other isms manifest themselves in the classroom. I've focused on these two pieces so as to work toward creating more inclusive and welcoming classrooms, but there are many other things that we also need to pay attention to. There are also institutional and societal practices and structures that are inequitable and need to be dismantled. Addressing microaggressions and bias does not absolve us from working towards greater equity in other realms.

About the presenter

Darryl Yong is the director of the Center for Learning and Teaching for the Claremont Colleges and Professor of mathematics at Harvey Mudd College. His scholarly work is centered on how secondary school mathematics teachers deepen their content knowledge for teaching, gain leadership skills, and learn new pedagogical strategies.



References

- Aguilar, L., Walton, G., & Wieman, C. (2014). Psychological insights for improved physics teaching. *Physics Today* 67(5), 43-49.
- Kay, K. & Shipman, C. (2014). The confidence gap. *The Atlantic* May 2014, 1-18.
- Miller, J., & Garran, A. M. (2007). The web of institutional racism. *Smith College Studies in Social Work* 77(1), 33-67.
- National Center for State Courts (2012). Strategies to Reduce the Influence of Implicit Bias. Formerly available at [www.ncsc.org/ibstrategies], but not currently available; a new report is due in Spring 2019.
- Su, F. (2015). Mathematical Microaggressions. *MAA Focus* October-November 2015, 36-37.
- Sue, D.W. (Ed.). (2010). *Microaggressions and marginality: Manifestation, dynamics, and impact*. Hoboken, NJ: John Wiley & Sons Inc.
- Sue, D.W., Capodilupo, C. M., Torino, G. C., Bucceri, J. M., Holder, A., Nadal, K.L., & Esquilin, M. (2007). Racial microaggressions in everyday life: implications for clinical practice. *American Psychologist* 62(4), 271.

Tversky, A., & Kahneman, D. (1975). Judgment under Uncertainty: Heuristics and Biases. In Wendt D., Vlek C. (Eds.), *Utility, Probability, and Human Decision Making Theory and Decision Library (An International Series in the Philosophy and Methodology of the Social and Behavioral Sciences)*, vol. 11, pp. 141-162. Springer, Dordrecht.

Observing for Equitable Opportunities for English Language Learners to Participate in Lessons, Activities, and Interactions. Based on a talk by Chu, H. and Perry, R. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning (Critical Issues in Mathematics Education, vol. 13)*, by Spencer, J. Berkeley: MSRI (2019).

OBSERVING FOR EQUITABLE OPPORTUNITIES FOR ENGLISH LANGUAGE LEARNERS TO PARTICIPATE IN LESSONS, ACTIVITIES, AND INTERACTIONS

Based on a talk given by Haiwen Chu and Rebecca Perry

The focus of this talk is to share tools developed for classroom observations to improve mathematics lessons via research, evaluation, and coaching in conjunction with two education initiatives: Quality Teaching for English Learners and Math in Common.

We begin with three important questions in relation to equitable instructional practices. First, *For what are we observing?*; secondly, *At what scales — where and when — are we observing?*; and finally, *How can we use this observation to influence practice?*

Quality teaching for English learners

The Quality Teaching for English Learners (QTEL) initiative provides discipline-based, whole-school professional development integrated with instructional coaching for teachers. What this entails is a vision of quality teaching. We will share tools and give specific examples from an elementary school in the Pacific Northwest with six teachers (grades 3–5), one ESL specialist, and two mathematics coaches.

QTEL operates on five principles regarding what it means to teach English Learners with quality. These principles are largely derived from the field of English language arts. Summed up, these principles ask teachers to

- Sustain academic rigor
- Engage in quality interactions

- Sustain a language focus
- Balance high challenge with high support instruction
- Provide quality curriculum

When we work with teachers, we talk about the different instructional scales that we can plan for. These scales include the unit, which encompasses lessons; the lesson, which subsumes moments; moments, which contain tasks; and individual tasks, which contain interactions. Interactions are the back and forth communication that occurs between individuals. This framing allows us to be more effective in our instructional coaching with teachers and provides us with greater focus for planning, observing, and debriefing their instruction with them.

At the lesson level, we provide teachers with a three-moments architecture for thinking about ways of designing instruction that flows naturally. Our three-moments architecture is concept driven: What mathematics do we want students to understand? What experiences will they need to help them interact with these mathematical concepts and develop connections? What will they need to activate what they already know? And finally, how can they extend that understanding into novel settings?

In QTEL, a task is something a student has to do with language to construct meaning. Tasks are focused on constructing meaning rather than grammatical form or linguistic display. A task requires both conceptual and linguistic processes to complete and also has a clearly defined communicative outcome. Each task consists of turns that individual students make as they grapple with making sense and developing meaning.

Observing for quality interactions

At what level do we observe for quality interactions? In our view, quality interactions must contain sustained and reciprocal talk, and a co-construction of knowledge mediated by language. By sustained talk, we mean that turns must be long, must have ideas, must be complex, and must connect with each other. Sustained and reciprocal talk happens between students and teachers and is not dominated by one party. Quality interactions allow students to co-construct disciplinary understandings through the use of language. Talk is about the subject matter and encourages reasoning, applying ideas, arguing, generalizing, and posing questions. This is what quality interactions sound like and what they hope to achieve.

Sustained and reciprocal talk happens between students and teachers and is not dominated by one party.

In relation to our observations, it is important to ask what are we looking at and where are we looking? In the case of QTEL, the “what” is quality interactions, and the “where” is within the turn by turn interactions between students as they engage in mathematics.

How does this actually play out in an instructional coaching cycle? We use a walkthrough observation protocol. Because QTEL prefers sustained observations, we stay from the beginning to the end of the trajectory of a lesson in order to fully understand how the pieces of a lesson fit together. It is important to note that an underlying goal of QTEL is to get teachers to be task engineers. We would like teachers to come away from this work understanding how to set up interactions that require students to communicate with one another in order to achieve a given mathematical goal.

Our work in action

A Remainder of One, by Elinor J. Pinczes, tells the story of 25 bugs who are trying to march in a parade in even lines. But, of course, because there are 25 of them, it does not quite work when they first try with two lines, and then three, and then...

Used in the classroom, this problem illustrated in the book can serve as the basis for a mathematics lesson. In most classrooms, the teacher would start with a read-aloud of this book. If the teacher is reading the story out loud in one voice, then you can anticipate the limited student interactions that would result.

We would like teachers to understand how to set up interactions that require students to communicate with one another in order to achieve a given mathematical goal.

Instead of having the teacher read aloud, QTEL sets up a “reading in four voices.” We recreate the book using four different styles of font: plain, **bold**, *italic*, and underlined. Each font belongs to a different student and is a signal that it is time for a different student to read. We have the students read through the book the first time for meaning. The second time, we use the transitions and the pauses in between the different parts that the students are reading as a time for students to pause and to figure out whether or not they understand what is happening in the story.

As one might guess, with this augmented format, the classroom is transformed into a space of discussion. Students communicate with one another about what the story says. They try to construct and show the different aspects of the story. In short, they engage in sense-making. There is evidence from our observations that the students are engaging in sustained and reciprocal talk.

Three challenges and possibilities of the QTEL work

The details of how this lesson is implemented matter a great deal. For example, should students be given blank sheets of paper or paper with gridlines when working on the math problem presented by *A Remainder of One*? While the official lesson guide stated blank paper, several teachers — wanting to give students a little more support — gave paper with grids. Each particular instructional choice like this one makes a big difference in how students take up tasks and the mathematics that they are actually able to do.

To conclude, there are three challenges and possibilities that confront this work:

1. *Discipline of observation.* We found that teachers needed support and experience with observation. Observers need to observe. Teachers serving as observers wanted to intervene and jump in and support students instead of simply observe.
2. *Focus on coherence vs. individual interactions.* Mathematical ideas build across tasks. We need to work with teachers across tasks in order to build coherent ideas about mathematics.
3. *Putting language to work.* This is still one of the most enduring challenges. Even though this is a language rich task, we have not reached a shared understanding of what language is and what it is not and how to put it to work for designing more quality experiences in teaching.

Math In Common Initiative

Math in Common is an initiative comprised of ten districts up and down the state of California. Together, with two additional partners, we work to help them think about how they can support Common Core implementation in mathematics. The hope is that these funded districts will learn something with WestEd's support that they can share with other districts in California. WestEd provides both the formative evaluation and the technical support on the Math in Common initiative.

Among its other questions, the evaluation team is interested in teachers' instructional shifts. What do these shifts look like? How can we make sense of these shifts? Is instruction moving towards the Common Core, or is it staying the same that it was previously? To address this question, a team from WestEd observes classroom instruction in the participating districts. We collect a small sample of instruction that gives us a taste of what instruction is going on.

What do instructional shifts look like? How can we make sense of these shifts?

We began our observations by using the Mathematical Quality of Instruction (MQI) rating system. The MQI rubric uses a low, medium, high scale. We found that the MQI was missing a few elements, so added a few items from Schoenfeld's Teaching for Robust Understanding (TRU) framework to help us think a little bit more about the mathematics and to help us think more about student agency, identity and authority. The TRU framework uses a 1–3 (novice–expert) scale. The districts we work with are familiar with the TRU framework and have been thinking about how to observe instruction on their own. TRU allows us to have a common discussion with the districts.

Impact across districts

Here, we present findings related to Common Core implementation across the districts. We saw an increase in the use of linking representations in classrooms. In 30% of the lessons we observed last year, there was good use of student sense-making and use of explanations

and mathematical reasoning. Thirty percent of the lessons that we observed received an “expert” rating. However, when we look at student agency, 30% of the lessons received a rating of novice. This may represent that there are limited opportunities for students to do the mathematics themselves. Here is a summary of further findings:

- *Linking between representations.* Spring 2015, 53% of lessons “not present”; Spring 2016, 36% of lessons “not present”
- *Good use of student sense-making, student explanations, math reasoning.* Fall 2015, 30% of lessons rated “high”; Spring 2016, 30% of lessons rated “high”
- *Mathematics.* Fall 2015 and Spring 2016, 30% of lessons rated “expert”
- *Agency, authority and identity.* Fall 2015, 22-23% of lessons rated “novice” and Spring 2016, 22-23% of lessons rated “novice.”

Composite lesson scripts

We were very interested in understanding the details of these lessons. What specifically is going on in lessons rated high and in those rated low? In order to analyze this, we developed a composite script of some of the high-rated lessons and some of the low-rated lessons and we compared how time was used across the high- and low-rated lesson scripts.

What we see is that *within* each one of the composite scripts, there is a lot of variation: no two lessons look alike. What is interesting is that there are clear differences *between* the high- and the low-rated lesson scripts. One difference is the number of lesson segments we saw across the high composite set compared with the number of segments across the low. What we saw in the high is that there was often a warm-up or number talk activity. What we still want to investigate is the extent to which those activities are related to the central mathematical point of the lesson.

We also see that the launch of a lesson is significantly shorter than what is happening in the low lessons. It seems like teachers in high lessons are getting the kids going and sending them on their way to let them have more time to do independent work and to think about the mathematics for themselves. Whole class discussion is also slightly different in lessons rated high compared to those rated low. While “show-and-tell” often accompanies whole class discussions in low-rated lessons (that is, a student is asked to come to the front of the classroom and show what she or he did that day), the high lessons in our study operated differently. Rather than show-and-tell practices, we saw more summarization of the mathematical point (that is, more closure) in these high lessons. As such, in these high lessons, there is more of an opportunity for students to say, “a-ha” and “this is what I learned in math class today.”

We saw more summarization of the mathematical point (that is, more closure) in the high lessons.

Color coding lessons — Why might high scores have happened?

We use a color-coding system (see the table below) to help us gain a clearer picture of what is happening in these lessons. What are the mathematical opportunities provided for students in the lessons? The color codes provide hints of what is happening in each individual lesson. One code is related to when we come to observe a lesson. Does observing at the beginning or end of a unit matter in terms of the rating that a lesson receives? Another code relates to the number of mathematical activities that students are asked to do during a given lesson. In some instances, students are overloaded with activities, but they make little connections between them. In other instances, there are fewer activities. Does this matter in relation to student opportunity to learn the mathematics? Another code relates to the representations that are used in the mathematics class and how these are used to support the mathematics being studied in class. Another code relates to the kinds of interactions happening in the classroom. We know that interactions between students, and between students and teachers really matter in classrooms and this code will help us understand that more. The final code relates to student thinking and how teachers may use students to guide the learning trajectory towards the mathematical goals of the lesson.

We still have a lot of work to do and a lot to investigate. We routinely share this information with the district leaders, including during debriefing discussions with the district where we share observation findings. These discussions are opportunities for us all to learn together. Most importantly, they are opportunities for districts to consider how they can help their teachers structure lessons to create more equitable mathematical opportunities for students.

Color coding to understand the details of lessons

<i>Blue</i>	Placement of the lesson in the unit. When we observe really impacts the observation. Does it matter when we come into to observe the lesson? Does it make a difference if you come at the start of a mathematical unit versus at the end of the unit? Does that impact the rating?
<i>Red</i>	Number of mathematical activities being asked to do during the lesson — you can overload on activities, especially if they aren't connected.
<i>Green</i>	What different representations are being used during the math class? How are they used to support the math in the class?
<i>Purple</i>	Interactions between students, interactions between students/teachers. Looking at kinds of interactions happening in the classroom.
<i>Yellow/ orange</i>	How are the teachers using student's thinking to guide the lesson in the classroom?

Challenges and possibilities of using common observation tools

The idea of using a common standard to understand what good mathematics looks like across districts, classrooms, teachers, and students is something WestEd wants to pro-

mote. A benefit of this work is that it provides an opportunity for conversations around a common instrument. A common observation tool may not “fit” easily with districts’ local purposes; however, the tool can engender a worthwhile discussion. Finally, using the instrument allows for opportunities to “scale up” good mathematics practices. We see promise in using observation as a professional development tool, instead of serving solely the purpose of evaluation.

About the presenters

Haiwen Chu is a Program Associate in the Teacher Professional Development Program at WestEd. He works on the Quality Teaching for English Learners initiative where he specializes in secondary mathematics education.

Rebecca Perry is a Senior Program Associate with WestEd’s Innovation Studies Program. She primarily works with the Math in Common project, a five-year initiative drawing together ten California school districts as they implement the K–8 Common Core State Standards in mathematics.



References

Hill, H.C., Litke, E., & Lynch, K. (2018). Learning lessons from instruction: Descriptive results from an observational study of urban elementary classrooms. *Teachers College Record* 120(12), 1-46.

Mathematical Quality of Instruction (MQI) observational rubric: [cepr.harvard.edu/mqi].

Pinczes, E. (2002). *A remainder of one*. New York, NY: Houghton Mifflin.

Schoenfeld’s Teaching for Robust Understanding (TRU) framework [map.mathshell.org/trumath.php].

Further resources

For additional information about Quality Teaching for English Learners (QTEL): [qtel.wested.org].

For additional information about Math in Common: [www.wested.org/project/math-in-common-evaluation/].

Seeing Beyond Ourselves: A Conceptualization of Mathematics Teacher Noticing for Equity. Based on a talk by Hands, V. and Van Es, E. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning (Critical Issues in Mathematics Education, vol. 13)*, by Spencer, J. Berkeley: MSRI (2019).

SEEING BEYOND OURSELVES: A CONCEPTUALIZATION OF MATHEMATICS TEACHER NOTICING FOR EQUITY

Based on a talk given by Victoria Hand and Elizabeth van Es

To notice, then, is to draw a distinction and create a boundary between things (Mason, 2002) — or to “name” phenomenon — and to interpret that even as an instance of something (Shulman, 1992).

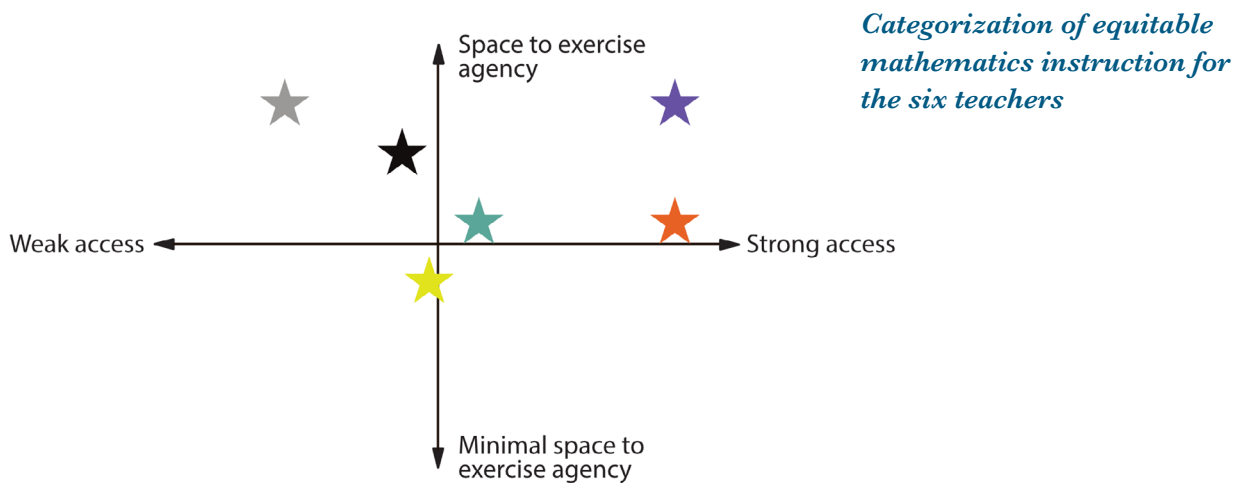
The distinctions and names we use are tied to particular category systems, which are given meaning within broader social, cultural and political discourses, and are recognized, interpreted and valued differently (Gee, 2012).

The research we are presenting seeks to characterize six teachers from two school districts in Colorado and California who were nominated by district officials as exemplifying those who successfully engage a broad range of students in rigorous mathematical activity. In this talk, we will focus specifically on the professional noticing interview we conducted with each teacher and what insights the interviews suggest about teachers who notice for equity.

For the overall study, we collected several sources of data, including

- Baseline interviews with teachers (for example, background, goals, vision for equity)
- Classroom observations
- Videotape of classrooms to develop clips for future noticing interviews
- Three noticing interviews with each teacher (Prompt: “What were you noticing?”)
- Professional noticing interview
- Video club meetings (small-group meetings of teachers where they watch and then share what they notice about classroom videos)

Subsequent to this data collection, the instruction of each of the six teachers was categorized with respect to access and agency. The stars on the graph below represent a characterization of the teacher’s mathematical instruction in relation to access and agency. For example, the gray star represents the instruction of one of the six participating teachers that emphasized student agency alongside a fairly weak focus on access.



Professional Noticing Interview

To conduct a professional noticing interview, we asked teachers as well as two experts on equitable instructional practice to view a set of three clips of classroom activity from the participating teachers’ classrooms. The three clips met the following criteria:

- Lessons included whole class discussions in which teachers were soliciting students’ ideas
- Classrooms were either racially/ethnically diverse or hyper-segregated (with students from less-dominant racial, ethnic, and linguistic backgrounds)
- The lesson made use of a reform mathematics curriculum

The teachers and experts participating in the professional noticing interviews were asked to share both what stood out to them in the clips and how they made sense of what they saw.

Ambitious instruction provides opportunities for students to grapple with mathematical ideas and likewise makes the mathematical norms and expectations explicit to students.

Two kinds of noticing emerged from viewing the selected clips — noticing related to ambitious instruction, and noticing related to culturally sustaining pedagogies. Ambitious instruction provides opportunities for students to grapple with mathematical ideas and likewise makes the mathematical norms and expectations explicit to students. Participants in the professional noticing interviews also gave attention to the presence or lack of culturally sustaining pedagogies in the video clips. Culturally

sustaining pedagogies promote opportunities for teachers to connect with students, honor students' individual strengths, and make the system of mathematics education explicit.

Interpretation of the interview

First, overall, the teachers' noticings of the clips were aligned with the findings of other research studies.

Overall impression of teaching by clip

	<i>Positive</i>	<i>Negative</i>
<i>Clip 1</i>	7	2
<i>Clip 2</i>	4	21
<i>Clip 3</i>	38	9

Opportunities for learn per classroom based on research study

	<i>Access</i>	<i>Agency</i>
<i>Clip 1</i>	medium	medium
<i>Clip 2</i>	low	low
<i>Clip 3</i>	high	high

Second, teachers noticed more of the moment-to-moment discursive moves (by the teacher and students) and how these related to students' opportunities to learn in Clip 3.

Discourse and positioning: # Instances of noticing

	<i>Teacher discourse/ orchestration</i>	<i>Student discourse/ positioning</i>
<i>Clip 1</i>	14	31
<i>Clip 2</i>	28	33
<i>Clip 3</i>	53	61

Third, from the table below, there appears to be a strong relationship between attending to discourse and positioning and noticing for equity. Also, we can identify specific areas of growth for particular teachers.

Attending to discourse vs. Positioning and noticing for equity

(Results for clip 3)

	Green	Grey	Orange	Purple	Red	Pink
Student discourse/ positioning	9	25	0	8	8	11
Teacher discourse/ orchestration	3	21	4	5	9	11
Total	12	46	4	13	17	22

Other themes that emerged is that the teachers who notice for equity tend to have a deep awareness of the informational/interpersonal space; they attend to subtle clues and signals in interactions; they look for ways to signal to students that they “have their backs” even if the teacher is uncomfortable, unsure, or incompetent; and they approach teaching with humility, curiosity, and a willingness to walk on unsteady ground.

References

Gee, J.P. (2012). *Social Linguistics and Literacies: Ideology in Discourses, Critical Perspectives on Literacy and Education*, 4th ed. New York, NY: Routledge.

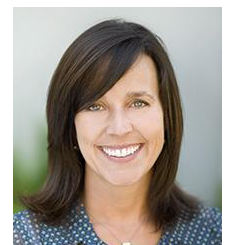
Mason, John (2002). Researching your own practice: The discipline of noticing. *Journal of Mathematics Teacher Education* 6(1),77-91.

About the presenters

Victoria Hand is Associate Professor, Curriculum and Instruction, Math Education, in the School of Education at the University of Colorado Boulder. She focuses on issues of culture, learning, identity, and equity in the mathematics classroom. Her research examines the development of opportunities to learn in mathematics classrooms, and how these opportunities are negotiated differently by groups of students from various ethnic, racial, linguistic, and social backgrounds.



Elizabeth A. van Es is Associate Professor and Associate Dean in the School of Education at the University California, Irvine. Her research is primarily concerned with how to support teachers’ developing their noticing practices for ambitious and responsive instructional practice. Much of her work uses video to help teachers develop a vision of ambitious instructional practice and to learn to attend to student thinking during instruction.



Turning Conversations into Actions: Addressing Inequities in Mathematics Classrooms. Based on a talk by White, D. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning (Critical Issues in Mathematics Education, vol. 13)*, by Spencer, J. Berkeley: MSRI (2019).

TURNING CONVERSATIONS INTO ACTIONS: ADDRESSING INEQUITIES IN MATHEMATICS CLASSROOMS

Based on a talk by Dorothy White

This talk will discuss some of the inequities introduced by tracking and how to address these inequities through courageous conversations. I will lead a group tracking activity that will give you a good sense of how it feels to be tracked and then lead you through the experience of my own tracking dilemma to help introduce specific actions, centered around courageous conversations, to get started in seeking genuine equity.

Origins of tracking

Tracking by ability group is “the practice of evaluating and sorting students into categories for the purpose of providing differential instruction within or across classrooms” (Worthy, 2010, p. 273; Goodlad, 1985; Lucas 1999; LeTendre et al. 2003). There is tracking that takes place across classrooms and there are also in-class trackings.

Tracking is popular in most countries. This is not something that is unique to the US. In the US, it usually takes the form of between-class groupings and we usually group high, average, and low-ability classes. We also have gifted classrooms and special education classrooms that are becoming problematic for certain students in our population. Tracking came into our education system around the 1920s when northern urban cities had an influx of poor, uneducated and unskilled immigrants from Eastern and Southern Europe. We also had migrations of African Americans from rural areas to the north. Later we had Puerto Ricans and other people looking for citizenship. And they would often go where the jobs are, which were urban cities. There was a need to educate these unprecedented numbers of students from diverse backgrounds. What resulted were these two types of tracks of schools. The comprehensive school mainly for white middle class students and the vocational track. The problem is that the children of immigrants, the poor, and students of color were most likely the ones in the vocational track.

How it feels to be tracked

Workshop attendees were divided into three groups. They then sat in different places in the auditorium based on their designated group and were then instructed to study the materials that they were given because they would soon take a test about the information. What participants did not know is that they received different sets of materials — some with lots of rich information that included photos, contextual notes, and other details that would be very helpful for taking the exam. Participants in other groups received sparse study materials with lists of facts and fill-in-the-blank activities. Others received even less than this. After a period of “studying,” these participants/students were given the exact same exam. As the activity unfolded, the three groups began to experience tracking first-hand.

Tracking dilemma: Do you see what I see?

I am an educator of mathematics teachers, preparing teachers in the elementary and middle grades program. There is a particular situation that I want to highlight. I call the scenario, “Do you see what I see,” because it highlights the challenges that I face in getting my preservice teachers to see issues and challenges related to equity that seem quite evident to me.

Our college of education partnered with a local school district and created a professional development school. Through the partnership, we are trying to build an alignment between what we are doing in the university and what schools are dealing with. Our hope is to bring together these two different knowledge bases: the university and the K-12 school. Our professional development partnership school was diverse. It served 575 students in grades 6–8. Fifty three percent of the students were Black, 30% white and 9% were Hispanic. Sixty percent of the students were on free and reduced lunch.

As part of the partnership, I was a professor of residence at the school and taught each of my math methods courses for secondary pre-service teachers on-site at the school. I also provided professional development to the school’s teachers. In one methods course in particular, I wanted the preservice teachers to examine different ways that teachers organize their mathematics classrooms. For ten weeks, my students rotated through the 6th, 7th and 8th grade classrooms at the professional development school. During the rotations, I wanted them to listen to the middle school students’ mathematical thinking.

The enrolled students are not aware that they are receiving different experiences. Tracking is often not an official program, but it is real nonetheless.

The particular study that I highlight comes from the preservice teachers’ rotation in two 7th grade classrooms: those of Ms. Bond and of Ms. Miller. While there is no official designation given to Ms. Bond’s and Ms. Miller’s courses that would mark them as tracking, the two classes are indeed separate tracks. But the enrolled students are not aware that they are receiving different experiences. Tracking is often not an official program, but it is real nonetheless.

Ms. Bond's classroom

Ms. Bond has a class of 14 students. Eighty percent are white and 20% are Black. The desks are arranged in clusters of four. The lesson begins with a Power Point presentation. Although the class is teacher-led, it is clear that Ms. Bond wants the kids to engage in ideas. She uses manipulatives, and the children feel that they can talk together by virtue of how they sit and the tasks that they are assigned.

Ms. Miller's classroom

Ms. Miller has 30 students in her class. Ninety percent of the students are black, 10% are white. Students' desks are paired, but they are in rows. Ms. Miller was lively and started each class with four or five warm-up problems on the board. She asked the kids to solve the problems individually before she started class. As she walked around, Ms. Miller told every student, "I love you," and, she expected them to say it back to her. Ms. Miller's walk-around took about 10 minutes out of each 50-minute class period. When Ms. Miller went over the warm-up problems, she praised those who did it correctly and helped those who responded incorrectly. She didn't call on other kids, she just worked with that particular kid to help them correct their mistake. If a student got the correct answer, she would give them a treat that tended to be candy. After the warm up, the students were then given a worksheet of math problems.

In both classrooms, my preservice teachers attributed student behaviors to characteristics and traits of the students themselves...

My preservice teachers would come into Ms. Miller's classroom and work with clusters of students. However, there was barely space in Ms. Miller's room for them to work in. There were some special education students in this classroom; however, there were no apparent accommodations being made for these students. Many of the students in Ms. Miller's class did not hear directions because of noise. To calm them down, Ms. Miller would turn off the light. As a result, the students were often in a crowded, dark classroom.

Preservice teacher reflections about the two classrooms

At the end of the methods course, I required students to write a final reflection that compared two classrooms that they had spent time in. Most of the students chose to compare the two 7th grade teachers — Ms. Bond and Ms. Miller.

In their reflections, Ms. Miller was described as caring and energetic. They found that the desk arrangement was a way that all students could see the board. Students wrote, "Telling each student she loved them was a good way to establish a relationship with them." They liked how Ms. Miller accepted all student's answers and praised them after they answered questions. Some students described Ms. Miller's class as chaotic and rushed. They noted that the worksheets were not "group-worthy tasks" and that there were no classroom norms. Most noteworthy, the preservice teachers directed many of their reflections to Ms. Miller's students, stating that the students talked too much about non-math things, that

... My students did not see that the enrolled students were receiving two very different mathematics learning opportunities.

they did not pay attention, lacked interest and/or motivation, and that they had behavior problems.

In contrast, the students in Ms. Bond's room were described as motivated, smart, calm, and capable of working well together.

In both classrooms, my preservice teachers attributed student behaviors to characteristics and traits of the students themselves. In other words, my students did not see that these classes represented two separate mathematics tracks and that the enrolled students were receiving two very different mathematics learning opportunities. Furthermore, my students could not understand the role of the teacher in the students' behavior. They could not see what I saw: that just as with the tracking activity at the beginning of my presentation, Ms. Miller's noisy, "chaotic" classroom provided different learning expectations and opportunities for her students.

This then is the dilemma: getting preservice teachers to go beyond connecting student behaviors solely to the students themselves, and moving to recognize the role of the teacher in the students' behavior. As a mathematics teacher educator, I have struggled to help secondary preservice teachers examine how the different ways teachers organize their classrooms, assign mathematical tasks, and allocate class time and space result in differential learning opportunities for students across race and perceived academic abilities.

Courageous conversations in mathematics education

We as educators of mathematics teacher need to be able to talk about situations that we face related to equity. However, people do not know how or where to start with addressing these issues, including those like the dilemma I faced in the methods course.

One way to start is to facilitate and engage in courageous conversations related to the inequities that occur in mathematics classrooms and the equity-related dilemmas that educators face. The book *Cases for Mathematics Teacher Educators* was created with just this strategy in mind. There are 87 authors in this book, which includes cases from mathematics methods, content, and professional development courses. Each chapter contains a case of an equity-related dilemma that an educator has faced. Readers are provided with the context for each case, a discussion of the dilemma and how that educator addressed the dilemma. Three commentators then respond to each case.

One way to start is to facilitate and engage in courageous conversations related to the inequities that occur in mathematics classrooms.

The book *Cases for Mathematics Teacher Educators* is one place to start. This casebook is about facilitating courageous conversations related to the inequities that occur in mathematics classrooms. There are 87 authors in this book, which includes cases from mathematics methods, content, and professional development courses. Each chapter contains a case of an equity-related dilemma that an educator has faced. Readers are provided with the-

context for each case, a discussion of the dilemma and how that educator addressed the dilemma. Three commentators then respond to each case.

Singleton and Hays (2008) define a courageous conversation as a “strategy for breaking down racial tensions and raising racism as a topic of discussion that allows those who possess knowledge on particular topics to have the opportunity to share it, and those who do not have the knowledge to learn and grow from the experience” (p. 18). According to Singleton and Hays, engaging in these courageous conversations requires the following four agreements (p. 22):

1. Each person must stay engaged. Silence buys us nothing. We do not know if your silence signals consent, surrender, or that you have left the building.
2. Expect to experience discomfort. That is part of learning.
3. Each person has to speak their truth. Don’t just tell me what you think I want to hear.
4. Expect and accept a lack of closure.

Facing iniquities is a prerequisite for change

In conclusion, it is important that we understand the difference between equality and equity. Equality means that everyone gets a pair of shoes. Equity means that everyone gets shoes that fit. Equality is quantitative. Equity is about fairness and justice. We can begin to address equity in mathematics classrooms by intentionally noticing the inequities that are present. James Baldwin wrote, “Not everything that is faced can be changed, but nothing can be changed until it is faced.”

About the presenter

Dorothy Y. White is Associate Professor of Mathematics Education in the College of Education at the University of Georgia. Her research focuses on equity and culture in mathematics education by examining ways to prepare mathematics teachers of diverse student populations.



Reference

White, D. Y., Crespo, S., & Civil, M. (Eds.) (2016). *Cases for mathematics teacher educators: Facilitating conversations about inequities in mathematics classrooms*. Charlotte, NC: Information Age Publishing Inc.

Opening the Gateway to STEM Disciplines: Lessons from the Arlington Emerging Scholars Program in Calculus. Based on a talk by Álvarez, J. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning (Critical Issues in Mathematics Education, vol. 13)*, by Spencer, J. Berkeley: MSRI (2019).

OPENING THE GATEWAY TO STEM DISCIPLINES: LESSONS FROM THE ARLINGTON EMERGING SCHOLARS PROGRAM IN CALCULUS

Based on a talk given by James Álvarez

Arlington Undergraduate Research-based Achievement for STEM

This talk summarizes the impact of the University of Texas at Arlington Emerging Scholars Program in calculus from Spring 2010 through 2014. The presentation begins with a review of the Treisman-style emerging scholars program that was developed in the 1970s and focuses specifically on the adapted model used by the University of Texas at Arlington. Results of the Arlington, Emerging Scholars Program (including student evaluation and long-term analysis of the student population) showed higher grades, and no significant difference in STEM retention and STEM graduation.

The goal of the Arlington Undergraduate Research-based Achievement for STEM (AURAS) program was to increase the pipeline for STEM careers for US citizens and permanent residents who are first-time, first-semester university freshmen intending to major in physics, mathematics, chemistry, and engineering. A major component of AURAS involved implementation of the Arlington Emerging Scholars Program (A-ESP) in precalculus, calculus, and chemistry.

Origin of emerging scholars programs

AURAS was an emerging scholars program grounded in Uri Treisman's original research from the 1970s at UC Berkeley. Treisman was interested in understanding the differences in mathematical achievement between two groups of students in calculus at Berkeley. He conducted a study of African American (who had disproportionate low achievement) and Mandarin-speaking Chinese American students (who had disproportionate high achievement) (Treisman, 1985).

This ethnographic study hypothesized that the differences in achievement could be explained by:

- Differences in motivation — certain groups of students are “super” motivated and others that are not
- Inadequate preparation — some students go to schools that don’t have solid training in mathematics
- Lack of family support for or understanding of higher education
- Differences in socio-economic status

However, Treisman found that none of these were true when comparing the two groups (Treisman, 1992). Instead, he found that the differences in performance between the two groups rested in their beliefs about what their idea of “studying math” meant. The emerging scholars program (known as the PDP program at Berkeley) was then begun as a way to address these beliefs.

Two versions of “studying math”

Treisman identified two key barriers to success for the African American calculus students who had an overrepresentation of low achievement: They worked in isolation, and they compartmentalized daily life into “social” and “academic” components.

On the other hand, those students who had an over representation of success had integrated academic and social life, and spent time working both by themselves *and* with others.

The differences in performance between the two groups rested in their beliefs about what their idea of “studying math” meant.

Treisman-style emerging scholars programs are a response to these barriers to achievement. Students in these programs work collaboratively on challenging mathematics. Typical features of emerging scholars programs (ESPs) include

- An immediate and central goal of increasing the number of students who excel in calculus or precalculus
- Intensive class sections that meet for a total of four to six hours per week (whereas non-ESP discussion sections meet for a total of two hours per week)
- Class sections that meet for two hours at a time (rather than one hour), allowing students to work on complex mathematical tasks that require perseverance

A-ESP at the University of Texas at Arlington

The University of Texas at Arlington ranks fifth nationally in undergraduate ethnic diversity (2017, US News and World Report). It is a large, urban university serving 41,000

students. Arlington is an Hispanic serving institution (HSI) with a student body that is 25% Hispanic, 15% African American, 10% Asian American, and 12% international.

When the A-ESP Program began, precalculus was a 3-hour (per week), lecture-based course that met across a 15-week semester. Calculus I and II were 4-hour courses that included three hours per week of large lecture (80–110 students), a one-hour recitation (40–55 students) each week led by a graduate student, and a one-hour problem solving lab (40–55 students) co-facilitated by a graduate student and lecturer. These courses met for a traditional 15-week semester. Both precalculus and calculus were identified as high-loss courses with rates of grades of D, F, or withdrawal above 50%.

The A-ESP component of the AURAS program transformed the precalculus and calculus experiences at Arlington. Students were enrolled in Fall Precalculus A-ESP followed by Spring Calculus I A-ESP, or they were enrolled in Fall Calculus I A-ESP followed by Spring Calculus II A-ESP. Structurally, A-ESP differed from Arlington’s traditional precalculus and calculus courses. A-ESP included

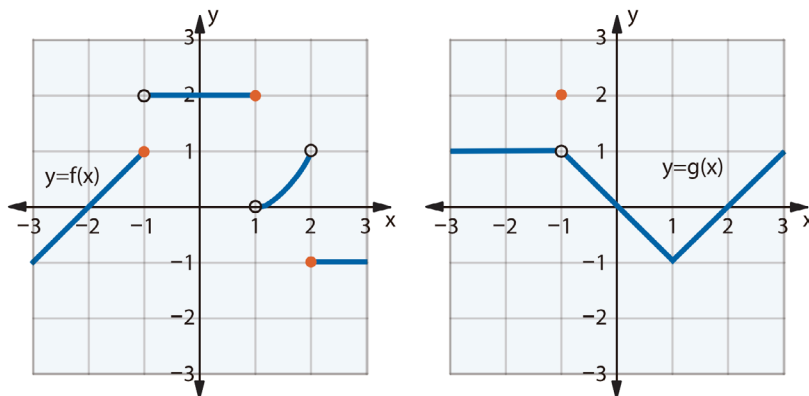
- One additional two-hour workshop per week
- A graduate student A-ESP instructor with 2 smaller sections (25 students)
- Challenging mathematical tasks, a community, and a collaborative learning environment
- Targets towards students broadly at risk
- Targeted boardwork in classrooms with plenty of boards to work — class started with groups working on the board

Students spent time in class explaining to one another, writing out their work, and discussing other aspects of problems. Course TAs received training and learned how to adapt the problems for their specific setting. TAs were also given samples of student responses. Smith and Stein’s *Five Practices for Orchestrating Productive Mathematical Tasks* was used as a tool for training the TAs.

One example of an A-ESP task is given below.

Example ESP task

For the functions f and g shown, list at least five mathematical questions about f , g , or both. At least two of your questions should involve recent ideas from class about limits and continuity.



Next, as a group, answer the questions you posed.

List any surprises or interesting mathematical outcomes you encountered in answering the questions you posed and explain why you were surprised or found the result interesting.

Student-reported benefits

An anonymous questionnaire was given to A-ESP students to understand how A-ESP benefited them, and specifically why they joined, what aspects of the program were most valuable, and what aspects had the most or least impact on their learning.

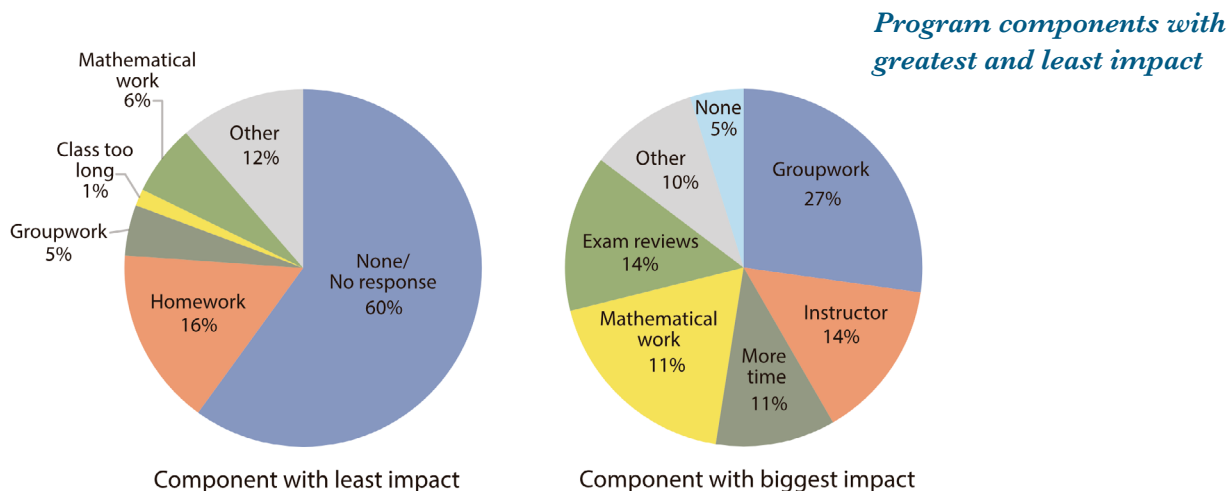
When asked their primary reasons for joining A-ESP, we found that

- 50% of respondents mentioned one or more of the following reasons: to receive extra help in problems, to receive more practice on material, to have more of a challenge and opportunity to better understand concepts
- 14% of respondents mentioned the chance to earn higher grades, gain an advantage, or pass the course

What did students value most about A-ESP?

<i>Aspects Rated >7 (on a 0–10 scale)</i>	<i>% of students rating the category 8,9, or 10</i>
Personal relationships	67
Extra time in workshop	72
Problems on worksheets	65
Practice exam problems	79

Regarding which components of A-ESP had the greatest impact on their learning experience in the mathematics course, the opportunity to engage in group work was identified as the most impactful component of the program. When asked about the least impactful aspects of the program, 60% of participants identified none. As one student remarked of A-ESP, “It made it easier by improving my study habits.” Another stated, “... it gave me time to learn math with others ... everyone had a chance to teach and learn.”



Student performance and STEM retention

What do we know about the effects of A-ESP on student performance and STEM retention?

Higher grades

A-ESP students earned statistically higher grades. Fall A-ESP Calc I students had higher success rates (grades of A, B, or C) than those not in ESP. Students in the Spring A-ESP Calc I also had higher success rates (grades of A, B, or C) than those not in ESP. These differences were all statistically significant.

Did A-ESP recruit stronger students?

This is a natural question, so to try to address it the study looked at AP calculus scores. Based upon reported AP calculus scores, students reporting a score of 3 or greater on the AP calculus exam were less likely to be in the A-ESP at an $\alpha=0.05$ level. So, based on this measure, the study did not “skim the top.”

STEM retention

STEM retention was sampled for all students similar to the A-ESP target group by looking at their current majors as of Spring 2015 and comparing these to their original intend-

ed majors. (This does not include Fall 14/Spring 15 cohort.) Although we only target a selected subset of STEM fields for recruitment into A-ESP, students were considered to have been retained in STEM even if their STEM field as of Spring 2015 was outside our original targets. So a physics/math major that was a biology major in 2015 was considered retained.

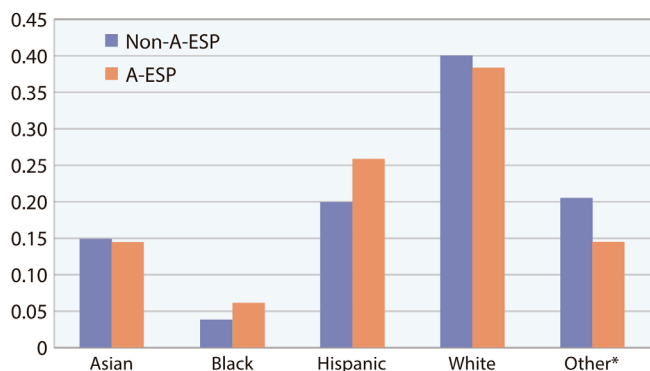
The study found no significant difference in retention between the A-ESP and non-A-ESP groups ($\alpha=0.05$ level). Since A-ESP increases chances of succeeding in the gateway courses, this may be interpreted as a leveling effect possibly attributable to A-ESP participation. Linking the effects of a program in the freshman year to graduation rates (as opposed to retention) is problematic. There is some evidence that having a good freshman year is good but it's too far in the future to attribute that effect.

Impact of gender

- Of all the Calculus I students meeting our target population profile, 20% were female and 80% were male. The A-ESP Calculus I students were 21% female and 79% male.
- Gender had no impact ($\alpha=0.05$ level) on grade earned.

Impact of ethnicity/race

- For our entire target population in Calculus I, students who identified as Asian had an advantage ($\alpha=0.05$ level).
- In A-ESP, no conclusive evidence that ethnicity plays a role in success in the course.
- For African-American students, A-ESP participation was double that of non A-ESP participation.



Percentage of each ethnicity/race for A-ESP vs non-A-ESP

*Refers to biracial, other races not listed, and anyone who left the item blank.

Benefit for first-generation college students

First generation college students were defined as those students for whom neither of their parents graduated from college. While first-generation status was a disadvantage for non-A-ESP students ($\alpha=0.05$ level), it had no statistically significant status for A-ESP students. Being in A-ESP erased the possible disadvantage.

Lessons learned, equity connections

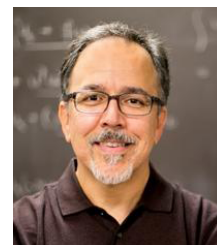
The ESP model has been used widely over the last 40 years and various studies have shown that ESP participants earn higher grades (Bonsangue, 1994; Moreno, Muller, Asera, Wyatt & Epperson, 1999; Duncan and Dick, 2000; Drane, Smith, Light, Pinto, and Swarat, 2005). Our data support the idea that the model may be appropriate for at-risk students and can be effective with as little as two additional hours per week. More studies like these need to be conducted to provide evidence that investments in programs like A-ESP, while seemingly costly per student, actually cost less by increasing pass rates (compare with Bonsangue, 1994).

A-ESP may indeed work to rehumanize mathematics (Gutierrez, 2017). Rehumanizing aspects of this program included collaboration (the aspect that students valued most), community building, and opportunities to communicate with one another about the given mathematical challenges. Student opinions support these rehumanizing aspects as the most effective or productive aspects of A-ESP. Student achievement data support the effectiveness of A-ESP for all groups of students. We need to think more closely about how we might leverage programs like A-ESP to increase the pipeline for historically underrepresented groups.

Note: AURAS was partially funded by the National Science Foundation Science Talent Expansion Program (DUE #0856796).

About the presenter

James Álvarez is Professor of Mathematics and Distinguished Teaching Professor at the University of Texas at Arlington. His research and professional interests include mathematical problem solving, mathematics-specific technology, increasing access to mathematics through improved preparation of mathematics teachers, and program and curriculum development.



References

Bonsangue, M. (1994). An efficacy study of the calculus workshop model. In Dubinsky, E., Schoenfeld, A.H., Kaput, J., & Dick, T. (Eds.), *Research in Collegiate Mathematics Education. I (CBMS Issues in Mathematics Education: Volume 4)*, pp. 130-151. Providence, RI: American Mathematical Society.

Moreno, S.E., Muller, C., Asera, R., Wyatt, L., & Epperson, J. (1999). Supporting minority mathematics achievement: The Emerging Scholars Program at the University of Texas at Austin, *Journal of Women and Minorities in Science and Engineering* 5(1), 53-66.

Treisman, P.M. (Uri) (1985). *A study of the mathematics performance of black students at the University of California, Berkeley*. Doctoral dissertation, University of California, Berkeley.

Treisman, U. (1992). Studying students studying calculus: A look at the lives of minority mathematics students in college. *College Mathematics Journal* 23(5), 362-372.

Smith, M.S. & Stein, M.K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA: National Council of Teachers of Mathematics.

Promising Teaching Practices for Advancing the Mathematics Education of Young Emergent Bilinguals. Based on a talk by Celedón-Pattichis, S. In *Access, Power, and Participation in Mathematics Classrooms: Using Observation as a Strategy to Improve Teaching and Learning* (Critical Issues in Mathematics Education, vol. 13), by Spencer, J. Berkeley: MSRI (2019).

PROMISING TEACHING PRACTICES FOR ADVANCING THE MATHEMATICS EDUCATION OF YOUNG EMERGENT BILINGUALS

Based on a talk given by Sylvia Celedón-Pattichis

A personal story of one emergent bilingual student

Tracking is very persistent, especially with special education students and with ESL students... It's very hard to get rid of those labels along the way: Unless there are teachers who intervene along the way, that doesn't happen for students...

It was really a white English teacher who made the shift for me. I was tracked with that label [English Language Learner] for a really long time. That English teacher saw the potential in myself to do well. Once I got changed to college prep (English), then by default all of my courses became college prep.

I focus on linguistic and cultural influences on the teaching and learning of mathematics. I use the term “emergent bilingual students” (instead of English Language Learner) because I want to take a stance that it is not only about learning English as a second language, but it is also about maintaining my own Spanish, and being able to expand my linguistic repertoire.

— Sylvia Celedón-Pattichis

Research study

The focus of my study was to investigate learning of Latina/o kindergarten students in classrooms that focus on solving and discussing problems in students' native language, Spanish. In the state of New Mexico it is part of the constitution that Spanish be used as a medium of instruction, so we have dual-language schools from elementary to high school.

In my work, I documented teaching practices that supported students' success, with a particular attention to cultural and linguistic resources that the teachers in the study drew upon during their instruction.

The study draws on sociocultural perspectives, sees discourse as an important component and mechanism of learning mathematics, acknowledges that participation and identity build mathematics understanding, and sees the cultural and language backgrounds of students as intellectual resources.

Cognitively guided instruction

Cognitively guided instruction (CGI) is a framework to understand children's mathematical thinking. It is a student-centered approach that emphasizes students' strategies, explanations, and justifications. Also, CGI uses children's mathematical thinking to advance their mathematical understanding. In other words, it builds from what students already know. CGI was an important tool for my study.

Young children and problem solving

NCTM sees problem solving as integral to mathematics. There is often an underestimated problem solving capacity of young children. Carpenter's 1999 and 2014 studies have shown this. Also, there is a lack of research on linguistically and culturally diverse settings, especially where Spanish is used as a language to teach mathematics.

Context of the school

Latina/o students represent the fastest growing group in public schools. Nearly half (45%) are English Language Learners. There is a persistent opportunity gap between Latina/o students and their white and Asian counterparts.

My study was conducted in an urban elementary bilingual school in New Mexico. The school promoted bilingualism and biliteracy and made use of reform-based mathematics curriculum. Mathematics instruction was given in Spanish. Eighty-six percent of the students at the school were Latina/o (mostly Mexican immigrants) and 100% of the students were eligible for free or reduced meals.

Teachers

The seven teachers who participated in the study were interested in integrating problem solving as part of the curriculum. Here, the focus is on two of them: Ms. Arenas, an experienced kindergarten teacher from Guatemala, and Ms. Carrera, a novice kindergarten teacher originally from Mexico. (Pseudonyms have been used instead of their real names.) Both teachers had attended summer institutes put on by the National Science Foundation's Center for the Mathematics Education of Latinos/as (CEMELA), and each had in-class support.

This study was a way for her to get out of her comfort zone and to try some new things with kids.

This was the first year that a lot of these teachers were taking up this work of engaging their students in problem solving, encouraging different solutions from students, and highlighting those different ways of thinking. Ms. Arenas was very scared of letting go of the classroom and letting go in terms of having the kids come up and explain and share their solutions. She is an experienced teacher, but she is used to teaching very traditionally. This study was a way for her to get out of her comfort zone and to try some new things with kids.

Methods

Problem solving lessons were conducted 1–2 times per week in both of the classrooms. Students worked in whole and small group configurations during these sessions and they had access to a range of tools.

As part of this longitudinal qualitative research study, the data collected included classroom observations, video of classroom lessons, pre- and post-clinical interview assessments with students, and teacher interviews.

Multiple mathematics lens approach for video

Video can be examined from numerous lenses. The multiple mathematics lens approach provides four lenses: task, learning, teaching, and power and participation. The task lens has a focus on what makes this a good and/or problematic task and how it can be improved. The learning lens asks, what specific math understandings and/or confusions are indicated in students’ work, talk, and/or behavior? The teaching lens is concerned with how the teacher elicits and then responds to students’ thinking. The power and participation lens is concerned with who participates, and how/whether the classroom culture encourages participation from all students.

Lenses for observing lessons

<i>Task Lens</i>	What makes this a good and/or problematic task? How could it be improved?
<i>Learning</i>	What specific math understandings and/or confusions are indicated in students’ work, talk, and/or behavior?
<i>Teaching</i>	How does the teacher elicit students’ thinking and respond? (For example, moves, questions, responses to students’ correct answers/mistakes/partial solutions, decisions.)
<i>Power and Participation</i>	Who participates? Does the classroom culture value and encourage most students to speak, only a few, or only the teacher?

Source: Multiple Mathematics Lens Approach, Aguirre et. al, 2012.

Video example

Here, we examine a three-minute video clip of Ms. Arenas' lesson on toy horses using these varied lenses. Ms. Arenas' task was: "I bought 4 toy horses at the store. I put them all in a bag to take home. How many feet were in the bag?"

From the *task lens*, you might notice that there are multiple entry points to the problem. The students are familiar with the context of the story. It seemed pretty clear that the students knew what a horse was and that they knew how many feet a horse had.

From the *learning lens*, students seemed to have knowledge of horses, of counting and of skip counting. They seemed to demonstrate some understanding of repeated addition and multiplication. Also, they all knew that each horse had four feet.

From the *teaching lens*, the teacher stopped Eduardo (the student called to the board to solve the problem) when he made a mistake and redirected that. As students in the class called out varying incorrect solutions, the teacher kept repeating the context. She kept saying, "four horses!" So, she reiterated the context.

From the *power and participation lens*, it seemed that the power in the class rested with the teacher. There was participation from Eduardo and there was choral recitation of the counting.

Results of study

We identified two practices that participating teachers engaged in as they endeavored to move to more student-centered, problem-solving-based instruction. Teaching in a student-centered, problem-focused manner was new for both Ms. Arenas and Ms. Carrera. As they participated in this study, they adopted a number of practices that supported the mathematics learning of their students. In one practice, the teachers generated problems through authentic storytelling and conversations. Storytelling became a way for teachers to frame problem solving and to share student stories. Students and the teachers co-constructed many of these stories. Students were invited to share their experiences. This practice of storytelling drew upon students' familiar ways of talking and negotiating meaning within their own families.

Teachers positioned students as competent problem-solvers by having them explain their thinking to other peers...

Teachers built upon the storying practices of students to engage them in the mathematics. For example, as soon as Ms. Arenas said, "Fijense (amorcitos) pues, les voy a contar una historia" ("Listen my little loved ones, I am going to share a story with you"), students knew that it was time to engage in problem solving. This familiar narrative structure scaffolded students' understanding. Teachers also used a narrative frame to guide students as they

learned to explain their thinking. Stories helped students learn to represent mathematical ideas and connect multiple representations. Teachers continually referred to the stories to help their students make sense of their representations.

A second practice that was evident in the study’s findings was the use of comparing and sharing problem solving strategies. Teachers positioned students as competent problem-solvers by having them explain their thinking to other peers. Likewise, teachers validated students’ explanations and had students test their ideas out — whether they were correct or incorrect.

... and had students test their ideas out — whether they were correct or incorrect.

While enacting CGI, these teachers scaffolded their students’ oral and pictorial representations using the students’ native language. They placed emphasis on having students retell the given story/problem, asking and allowing students to share and compare their problem-solving strategies, and discussing efficient strategies.

Impact of the Study on Student Learning

The results of the study demonstrate that rich problem solving is not out of the reach of emergent-bilingual kindergarten students. Students need access to the problems in their own language and in contexts aligned with their experiences and practices. Further, they need ample time to work on problem solving.

Post assessment results

<i>Problem type</i>	<i>% Correct</i>	<i>% Valid strategy</i>
Join, result unknown (6+6)	80	91
Separate, result unknown (13-5)	73	86
Join, change unknown (7+__= 11)	56	69
Multiplication (6×3)	49	64
Partitive division (15/3)	42	49
Measurement division (10/2)	40	53
Measurement division w/remainder	27	33
Multi-step (2×4)-3	44	49

(The sample size was $n = 45$. The problem types in table were borrowed from work done by —redacted for blind review— (2016) on types of pedagogical questions in mathematics teaching.)

Concluding thoughts

This study responds to Boaler’s (2002) call to provide examples of teaching practices that reduce inequalities in the field of mathematics education. The study documents the teaching practices that support successful participation of Latina/o kindergarteners, many of whom are English Language Learners and/or come from low socioeconomic status (SES)

backgrounds, in reform oriented mathematics classrooms. These promising teaching practices support students to solve challenging mathematics problems and engage in mathematical thinking *before mastering basic mathematics facts*. We can engage Latina/o students from low SES backgrounds in rich mathematics, using their context for storytelling and bringing their backgrounds into problem solving.

About the presenter

Sylvia Celedón-Pattichis studies linguistic and cultural influences on the teaching and learning of mathematics. She is Professor in the Department of Language, Literacy, & Sociocultural Studies at the University of New Mexico.



References

Aguirre, J. M., Turner, E. E., Bartell, T.G., Kalinec-Craig, C., Foote, M. Q., Roth, McDuffie, A.R., & Drake, C. (2012). Making connections in practice: How prospective elementary teachers connect to children’s mathematics thinking and community funds of knowledge in mathematics instruction. *Journal of Teacher Education* 64(2), 178–192.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education* 33(4), 239– 258.