Critical Issues in Mathematics Education Series

THE Mathematical Education of Teachers

Workshop 8 • May 2011

Common Core State Standards
The Interdisciplinary Mathematics Community

By Julie Rehmeyer

MSRI
Mathematical Sciences Research Institute

Published by the Mathematical Sciences Research Institute • 2014 • Berkeley, California

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Preface

In 2004, the Mathematical Sciences Research Institute (MSRI) launched a workshop series, *Critical Issues in Mathematics Education,* to provide opportunities for mathematicians to work with experts from other communities on the improvement of mathematics teaching and learning. In designing and hosting these conferences, MSRI seeks to encourage such cooperation and to lend support for interdisciplinary progress on critical issues in mathematics education.

The main goals of these workshops are to:

- Bring together people from different disciplines and practices to investigate and work on fundamental problems of education.
- Engage mathematicians productively in problems of education.
- Contribute resources for tackling challenging problems in mathematics education.
- Shape a research and development agenda.

This booklet documents the eighth workshop in the series, *Mathematical Education of Teachers,* held at MSRI on May 11 to 13, 2011. Two previous workshops have focused on issues related to educating teachers of mathematics. The second workshop addressed the mathematical knowledge needed for teaching, and the fourth workshop emphasized teaching teachers mathematics. The Critical Issues series returns to the topic of educating teachers of mathematics because:

- The Common Core State Standards, which have been adopted by most states, present both a challenge to ensure that the nation's teachers are prepared to teach to high standards and an opportunity to seek common standards for educating the next generation of teachers.
- It is appropriate to examine what has been learned from ongoing initiatives and research. For example, both the NSF and the U.S. Department of Education have made a substantial and sustained investment in Math Science Partnerships.
- The Conference Board of the Mathematical Sciences has launched an initiative to revisit and update their publication, *The Mathematical Education of Teachers.*
- Now more than ever, there is a need for an active, vibrant, interdisciplinary community that will drive a cycle of improvement in both the teaching of mathematics at all levels (elementary school to collegiate education) and knowledge about mathematics teaching.

These questions guided the workshop design:

1. What are implications of the Common Core State Standards in Mathematics for the mathematical education of teachers?
2. What has been learned about the mathematical education of teachers, both future teachers and current teachers, over the past decade?
3. How can we encourage, develop and sustain an interdisciplinary community of mathematics educators and scholars, including teachers, mathematicians, mathematics educators, and education researchers, in such a way that different communities communicate with and learn from each other, and, in so doing, drive a cycle of improvement in the teaching of mathematics at all levels?

The workshop speakers were chosen for their ability to articulate widely-held perspectives on mathematics education, but this choice is not meant as an endorsement of those perspectives. The content of this booklet is not intended to represent the views of the organizing committee, the Mathematical Sciences Research Institute, or the sponsors of the workshop.

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CIME *Critical Issues in Mathematics Education*

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Workshop Organizers

Sybilla Beckmann, *Chair*, University of Georgia

David Auckly, Kansas State University

William McCallum, University of Arizona

Jim Lewis, University of Nebraska – Lincoln

MSRI Educational Advisory Committee

Michèle Artigue
Université Paris

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Robert Megginson
University of Michigan

Robert Moses
The Algebra Project Inc.

Alan Schoenfeld
University of California,
Berkeley

Katherine Socha
Math for America

Hung-Hsi Wu
University of California
Berkeley

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Introduction

The mathematics research community has a strength and vibrancy that the mathematics teaching community hasn't yet achieved.

Math researchers meet regularly to discuss and build on one another's work. Quality is judged by the recognition and admiration of one's peers, and mathematicians work hard to achieve this. Excellent math researchers lead the community from within its ranks. Math researchers have at least some time to reflect on their own research. And a very high level of education and accomplishment is required to even enter the math research community.

By contrast, math teachers tend to work in isolation, with few opportunities for deep discussions or learning from peers. No repositories of shared, vetted knowledge about mathematics teaching exist. The only evaluation comes from students or outsiders, not from peers. Stellar teachers aren't the leaders of the math teaching community, and the community doesn't share a vision of what constitutes excellence in mathematics teaching. Math teachers have little time to reflect on their teaching practice. And the standards that have to be met to teach mathematics are quite low, with many teachers having very weak preparation.

The result is that the current math teaching environment doesn't cultivate excellence. It doesn't foster autonomy, competence, or relatedness, is often externally controlling, and doesn't promote deliberate practice towards expertise. Many teachers may never improve. Unlike mathematics research, the working conditions of mathematics teaching in the U.S. are currently not designed to foster excellence.

And yet, some teachers do improve. Some schools manage to create much healthier communities. Some teacher education programs produce consistently well-prepared future teachers. Solutions and innovation are springing up here and there.

The Critical Issues in Mathematics Education series of conferences are an effort to spread these innovative ideas and form a community by offering the mathematics community an opportunity to gather, share ideas, and work together to find new solutions in mathematics teaching.

The 2011 CIME occurred with the backdrop of a major change that offers leverage to make fundamental improvements to the math teaching community as a whole: a common curriculum for K-12 education that the great majority of states have agreed to. This new common curriculum both provides a uniting force across the teaching community and requires changes in math teacher education and practice that, if done well, could lead to dramatic improvements.

Three major pathways for improvement emerged during the meeting. A method for peer review and collaboration called lesson study has become a powerful way of creating better lessons, a greater understanding of how students learn, and a professional culture among teachers. The common core curriculum in K-12 math education also creates the opportunity to revamp math teacher education, creating a common core for it as well. And finally, research findings reveal more about what works and what doesn't.

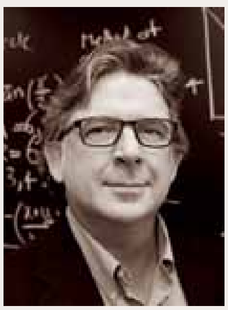
The Mathematics Teaching Community

https://mathematicsteachingcommunity.math.uga.edu/ is an online community for everyone who has taught mathematics at any level, from pre-kindergarten through college. Members can post submissions about mathematics teaching, including activities, questions, or links to useful resources. Members tag their submissions and anyone (members and non-members) can use tags to search for topics of interest. Further information about the site can be found in the FAQ and in postings with the "meta" tag.

The Common Core State Standards

Currently, 45 states have adopted a common set of standards for mathematics education. Until the development of these standards, math standards varied enormously from state to state. This made it difficult for students to get a coherent education, created inequities between different regions, and allowed a third-grader in Tennessee to have a very different set of mathematical skills from one in, say, California.

Part of the reason for this variation is that states have individual control of their own education systems, not the federal government, and the principle of local control of education is highly valued by many people. The Common Core State Standards aren't mandated by the federal government; instead, states have individually, voluntarily adopted them after they were developed by teachers, school administrators, mathematics and math education experts and state leaders. The effort was organized through the Council of Chief State School Officers and the National Governors Association Center for Best Practices. The standards were released in June 2010, so a remarkably large number of states have signed on remarkably quickly.



William McCallum of the University of Arizona was one of the architects of the Common Core State Standards and presented a talk on them at the conference.

The starting point for the development of the standards in mathematics was research comparing math education in high-performing countries that strongly suggested that math curricula in the United States needed to become both more focused and more coherent. Curricula in the U.S. tended to be "a mile wide and an inch deep," exposing students to many different topics but not giving those topics the careful development needed for students to gain a deep understanding of them. Furthermore, often, different groups developed standards on different topics, leading to a stapled-together hodgepodge of tasks that didn't build on one another coherently.

So the Common Core State Standards spend more time on fewer things. In the early grades, for example, the principal focus is on number and operations. Students don't begin learning multiplication and division until third grade, and students in the early grades have little exposure to working with data or recognizing patterns. But the narrower focus allows students to develop a deep mastery of the most fundamental concepts and a solid foundation to tackle algebra in 8^{th} or 9^{th} grade.

The advent of the Common Core State Standards also represent a significant opportunity for the education of math teachers. Existing teachers need continuing education to learn the new standards, so that they can see, for example, how the standards' focus on number and operations helps students develop a unified understanding of the concept of number in middle school, with fractions, decimals and whole numbers simply different versions of the central concept of number. They also need help learning to handle the high cognitive demands of the standards and their emphasis on conceptual understanding. And the greater coordination the standards provide offers the opportunity for collaboration and informal research by teachers, particularly through the method known as lesson study.

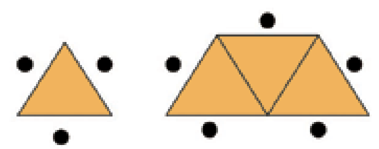
The standards offer an even bigger opportunity for preservice teachers. For the first time, math educators will know what their student teachers will end up teaching. This presents the opportunity that teacher education itself could now have a common curriculum, creating a coherent, nationwide system for training teachers that guarantees that any teacher entering a classroom for the first time will have the essential skills the students will need.

Lesson Study

Teaching is hard. It's simply not obvious what the most effective way of opening a student to a new concept or skill is, even to an experienced and sensitive teacher. Finding out takes careful, methodical research: trying an approach, analyzing the results, using the information to figure out something that might work better, and repeating the cycle. But ordinarily teachers don't have the time and support to do that, and furthermore, it can be difficult to analyze one's own teaching. Lesson study is a powerful method for teachers, teacher educators and mathematicians to do that work in collaboration.

The Lesson Study Process

The work of a group of teachers in San Mateo and Foster City in California will illustrate how this process works.[1] California had recently mandated that algebra be taught a year earlier, so these teachers wanted to figure out how to best prepare students for algebra in the earlier years. They focused on how to build students' ability to recognize and mathematically represent patterns, and they chose a problem from the research-based curriculum *Navigations,* as shown here.



How can patterns help to find how many seats fit around any number of triangles arranged in a row?

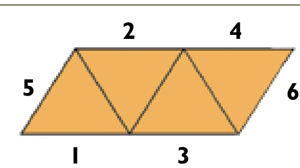
They started by working the problem themselves. Because it was very different from the kinds of problems they had worked as students, they occasionally stumbled. Furthermore, different teachers used different methods of solving the problem. They then worked to identify where the students were likely to have difficulty and what the common misconceptions were likely to be.

The pattern they wanted the students to identify was that there were two more seats than the number of tables, a "plus two" pattern. The principle confusion they identified was that adding a single table added one seat, so the "plus two" pattern had to be distinguished from this "plus one" pattern.

One of the teachers then taught the lesson to fourth-graders from a nearby year-round school and it was videotaped for later analysis. Initially, the teachers decided to give the students a worksheet to fill out, with the number of equilateral triangle tables in one column and the number of chairs in another. They also asked the students to describe in words the pattern they found. Although all 22 students filled the worksheet out correctly, only five of the students correctly identified the "plus two" pattern.

The teachers reconvened and studied the videotape. They aimed to understand student learning, not to analyze the moves of the teacher. They concluded that the worksheet (which the textbook had provided) had proven to be a hindrance rather than a help, giving students a straightforward task that distracted them from truly grappling with the problem.

One teacher observed that the method of counting the tables made a big difference in how easy it was to see the pattern. Most students counted around the tables (as the teacher had herself). But one student had counted one seat on top, then one seat on bottom, then the next seat on top, etc., and finally counted the two seats at the end. This counting method made the reason for the "plus two" pattern very clear.



Using a counting method to develop the ability to mathematically represent patterns

They then retaught the lesson to a different group of fourth graders. This second time, they decided to remove the worksheet entirely, and instead, they gave each student group a unique number of tables to study. They also asked students to share their counting methods with one another and to discuss the patterns with one another. This approach was far more successful.

→ *see page 8*

[1] This example of lesson study comes from the DVD "How Many Seats?" which is available from www.lessonresearch.net. It is also described in the book *Lesson Study Step by Step* by Lewis and Hurd, from Heinemann Publishers.

Contexts for Lesson Study

Lesson study has been used in a wide variety of contexts, including preservice teaching, across a school, across a district, across a region, and even across an entire country (Japan).

Lesson study is a part of preservice teacher training at both Mills College and Stanford University. At both schools, lesson study comes at the end of the program, and it serves to integrate prior experiences. The lesson study process gets the student teachers thinking closely about student learning, and it greatly increases the bonds and sense of collaboration among the students. When the student teachers subsequently begin their teaching careers, reported both Aki Murata from Stanford and Ruth Cossey and Elizabeth Baker from Mills, they often form lesson study groups of their own.

Aki Murata (left) and Ruth Cossey (above), with Bindu Pothen (not shown) have integrated lesson study into the curriculum at Stanford. Elizabeth Baker (not shown) has done the same at Mills College.

→ *from page 7*

The teachers reconvened for a final time. They discussed why a worksheet might undermine or short-circuit students' thinking, not just in this lesson but also in general. One teacher said that the experience made her conclude that "the students need to do the work, not the teacher." Another teacher went on to experiment further in her own classroom, giving half her students a worksheet and half none. A third commented that the lesson study process made her feel like a researcher, performing experiments and interpreting the results. Another teacher said, with wonder and a kind of glee, "It's kind of fun to think about all the different things you can tweak and look and watch what they do. Gee, I guess that's called lesson study!"

Rebecca Perry (at left) and Catherine Lewis (below) of Mills College have been strong advocates for lesson study and presented a talk describing the mechanics of how lesson study works at the conference.

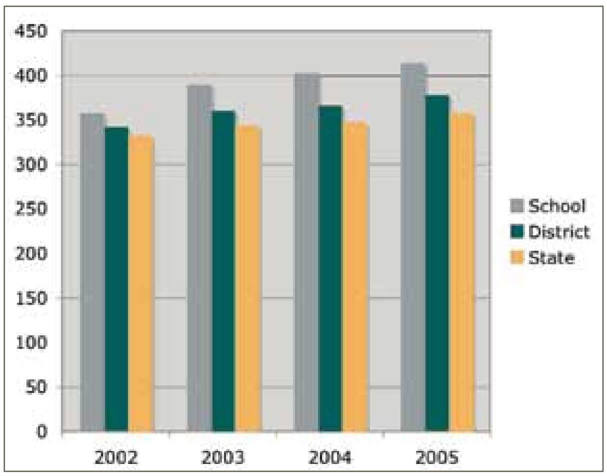
School-wide Lesson Study

Jackie Hurd described her experience with lesson study at Highlands School in the San Mateo-Foster City school district, where it had an enormous effect on the culture of the school as a whole. It began with a small group of pioneers who wanted to try lesson study. Excitement about the work grew over the year, and when the pioneers opened it to the rest of the staff the following year, all but three teachers signed on. In the third year, every teacher participated.

They formed teams of three to six teachers across grade levels, and teams often turned to resources outside the group as well, such as books or teachers at other schools. At the beginning, the work was funded by outside stipends, but over the years, they were able to move the meetings to times like professional development days and decrease their reliance on external funding.

At the beginning of the year, the teachers chose a school-wide research theme, usually driven by a mandate from the district (for example, closing the achievement gap). Then each team would decide how they wanted to address the theme. Teams met at least monthly, and other teachers often joined them, offering additional pairs of eyes. Over the course of the year, the teams each did two or three research lessons, and in April, all the teams would meet and share their results.

Lesson study affected far more than the particular lessons that were researched, instead changing the culture at the school as a whole. It led the teachers to see themselves as a community of learners. They became less cautious and defensive and more ready to ask for help. Teachers became invested in all the students, not just their own. Students and parents were affected as well. Students would ask, "Why are all these teachers in my room watching me learn?," and the teachers would explain that they were learning how to teach them even better. Parents would ask about the results of research lessons. Teachers took a lot of pride in the work. And achievement data improved steadily.



During the period in which lesson study was begun at Highlands School, the school's scores on the California Standards Test in Mathematics improved more than three times as much as scores in the district as a whole.

Jackie Hurd observed as lesson study transformed the culture of the Highlands School in the San Mateo-Foster City school district.

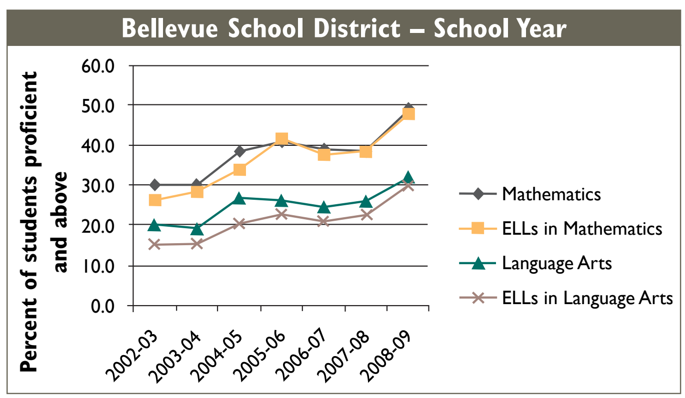
[2] A study of the first ten years of this projects was done by SRI International: *Evaluation of the California Subject Matter Project: California Mathematics Project and Bellevue UED.* H. Alix Gallagher and Teresa McCaffrey, February 2011.

District-wide Lesson Study

The Bellevue Union School District started doing lesson study in 2001 as a grassroots effort, and it snowballed In the first year, 16% of teachers at the four elementary schools in the district participated. By the fifth year, 67% of teachers did. Ben Ford, a professor at Sonoma State University who advised the lesson study process, says that lesson study is most effective when a large enough percentage of teachers in a school participate that conversations in the staff room are altered. Then, the culture as a whole changes.

The results were striking. Sonoma County, where the Bellevue Union School District is located, had a large population of students who weren't native English speakers. The math performance of these students tended to be far weaker, and the county was trying desperately to shrink this gap. The Bellevue district was the only one that achieved this, shrinking the gap consistently over five years.

In 2007, a new superintendent was hired, and so Ford and his colleagues compiled data showing this to persuade him of the value of lesson study.[2] The superintendent did indeed become a big supporter of lesson study, saying, "We believe in the [lesson study] process as a district. Culturally it's highly valued. Teachers and administrators understand how important it is for teachers to gather together to develop the common shared understanding of what it means for students to be proficient…other than just relying on a CST [California Standard Test] score once a year."



Ben Ford advised the development of lesson study in the Bellevue Union School District. The district was the only one in the county that succeeded in narrowing the achievement gap between native English speakers and those just learning the language.

Region-wide Lesson Study

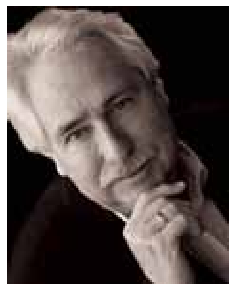
In 2000, a project began to take lesson study to a much larger scale, across an entire region. The Silicon Valley Mathematics Initiative works with about 25 different districts across the San Francisco Bay area, providing small grants and extensive support to teachers to participate in lesson study.

Over the summer, SVMI holds a five-day institute to introduce teachers new to the program to lesson study. Teachers learn the nuts and bolts of the lesson study process, and they are particularly taught that the point is to understand student learning rather than to try to improve instructional methods. Then everyone involved comes to a fall orientation. Teams work together over the fall, teaching a lesson, analyzing it, redesigning it, and teaching it again. Teams are paired up so that one team can watch a lesson given by the other. In January, everyone attends an open house, with many public lessons and a celebration of the work.

Teachers report that the work makes them feel more like professionals. They value the collaboration and the lessening of isolation, and they say that it gives them a deeper understanding of mathematics and of how students learn.

Data from 50 classrooms with 2000 students showed that the students of teachers who participated in the program had significantly higher test scores than teachers who didn't, even though the teachers who didn't participate in lesson study did other professional development activities.

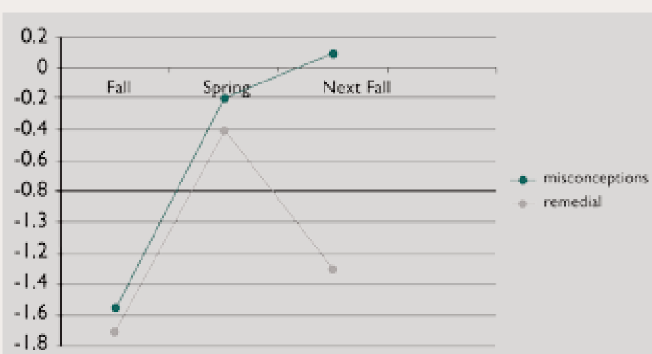
As a result, lesson study has become the highest form of professional development throughout the San Francisco Bay region.



David Foster is the executive director of the Silicon Valley Mathematics Initiative, and Tracy Sola is a teacher at the Belmont-Redwood Shores School District who has been involved with the project. They jointly presented on the Silicon Valley Mathematics Initiative.

Re-engagement

Research has shown clearly that when teachers emphasize conceptual understanding, students learn better. For example, a study by Malcolm Swan and Alan Bell showed that below-grade-level students who did remedial work focused on practicing procedures lost most of their improvement over the summer, while those whose teachers addressed common misconceptions retained their knowledge and even improved after the summer. While this is a powerful finding, it raises a basic question: How can teachers most effectively create this conceptual understanding that is so essential?



Over a series of lesson studies over several years, teachers with the Silicon Valley Mathematics Initiative have developed a powerful technique they call "re-engagement," which is now spreading across the country.

The teachers started by considering a basic conundrum they faced over and over. At the end of a unit, they'd give a quiz and find that some students scored well and some scored poorly, with most in between.

→ *see page 12*

Country-wide Lesson Study

In Japan, lesson study is a formal part of the school system. Japan has a common curriculum, and whenever the curriculum is changed, teachers around the country engage in lesson study on the new material. For example, some years ago a new unit on solar energy was introduced. Schools all over Japan applied to be designated research schools. One hundred schools were chosen, and each received $5,000 to prepare a public research lesson and make all records of the process available. They often worked in collaboration with a local community college or university. People then came to watch these lessons and ask questions about why they had approached it as they had and their experiences with what worked and what didn't.

This process gives Japanese teachers a way to acquire the content knowledge they'll need. At least one teacher from each school takes part, and a typical Japanese teacher sees about ten research lessons a year. A few "boundary-crossing" individuals (often university-based mathematicians or math educators, or K-12 teachers with a particular interest in math) carry knowledge between individual schools and national venues. University lab schools, which work with many different schools, facilitate this process.

To learn more

- www.essonresearch.net describes the lesson study-related activities at Mills College and contains links to many resources.
- The Silicon Valley Mathematics Initiative has an open house in late January every year with public lessons (www.svmimac.org)
- The DePaul University Chicago Lesson Study Group holds two conferences a year (www.lessonstudygroup.net).
- The Greenwich Japanese School opens its doors for lesson study and hosts a conference in November of each year.
- The Sonoma County Office of Education has an annual lesson study conference (www.scoe.org/pub/htdocs/lesson-study.html).

→ *from page 11*

The question was, what next? They seemed to have two choices: Reteaching the lesson, leaving the kids who'd done well bored, or move on, leaving the kids who'd done poorly behind. Neither seemed desirable.

The SVMI teachers developed another alternative. They began using the results of the quiz itself to push students to deepen their understanding of the mathematics. They might, for example, model correct answers and ask students to explain why is it correct. Or they might show two different methods that ended up with the right answer and ask students why they both work. Or they might give student work with a flaw or misconception and ask students to explain it. Tasks like these require students to critique the work of others and articulate and communicate their understanding of mathematics. The result is that students who didn't get it the first time are driven to examine their fundamental misunderstandings, and at the same time, students who got good scores but may nonetheless not have full command of the underlying concepts are pushed to deepen their understanding.

Re-engagement shows how the discoveries of lesson study can go far beyond a single lesson. When teachers demonstrated this method through their public lessons, other teachers embraced it and applied it to all kinds of lessons.

A Common Core for Teacher Education

Educators of future teachers in the U.S. face a problem: They don't know what material their students will end up teaching. Third grade math in Georgia, for example, may cover entirely different material from third grade math in California. As a result, in the words of David Cohen of the University of Michigan, teacher educators have been forced to train their students to teach nothing in particular.

Democracies similar to the U.S., on the other hand, all have common curricula, with exams that cover that curriculum. Teacher educators can therefore cover that material with their prospective teachers in detail.

"Although it is unrealistic to expect that a brand-new teacher will be expert at every aspect of the common core, teacher educators can ensure that their student teachers don't have gaps that are downright dangerous for students."

With the advent of the common core, there's now an opportunity for the U.S. to do the same. Furthermore, teacher education itself could now have a common curriculum, creating a coherent, nationwide system for training teachers that guarantees that any teacher entering a classroom for the first time will have the essential skills the students will need. This is a particularly good moment for such an effort, because there's an unprecedented national agreement now that teachers matter. Research has shown that the students of an excellent teacher learn far more than students of a weak teacher, even when the students are in the same school and have the same demographics. And consistent, high-quality teacher education is the way to produce consistent, high-quality teachers.

At the CIME conference, Deborah Ball of the University of Michigan presented a powerful argument for a common curriculum for teacher education, along with a road map for the challenges that will need to be addressed.

Other professions, she pointed out, have specific, detailed standards for trainees entering the profession, which teaching lacks. For example, for plumbers to be certified, they must know how to install copper and copper alloy piping; build a plumbing trap; and vent a sanitary drainage system. They also must undergo a five-year apprenticeship with 1700 to 2000 hours of on-the-job training before becoming a fully-fledged master plumber. Because of these agreed-upon standards, you can be confident when you call a plumber that he or she will know what to do.

Similarly, a doctor has to meet very specific standards. For a chest exam, for example, doctors must know how to observe respiratory efforts and note presence or absence of respiratory distress; confirm midline tracheal position with gentle palpation anteriorly; and percuss the chest on left and right. All of these standards have been spelled out in detail. In addition, medical students get hands-on training from attending physicians – sometimes literally, by placing a student's hands properly on a dummy patient.

There's no analogue in teaching. The closest to it is a standard like this: "Teachers use a variety of instructional strategies to engage students in challenging academic content." That standard is extremely vague in comparison with those for plumbing or medicine: It doesn't say what those instructional strategies are or what the content is. An analogously specific standard would be something like, "The teacher elicits solutions to a geometry task and conducts a discussion comparing the solutions," or, "The teacher poses a focused task to engage students in considering how to apply the definition of a fraction." But no standards at that level of specificity exist. In addition, student teachers are never taught many of the physical skills they'll need in the classroom, like how to use physical distance and voice in approaching a child to ask a question.

Developing a common curriculum for teacher education would require developing these standards. Teacher educators, then, would need to come to an agreement about what teachers must know how to do the first day they enter the classroom and what they can figure out on their own with experience. They would also need to identify the situations that will inevitably arise in a classroom and to focus most of the effort on preparing teachers for them rather than for the much rarer, uncertain and highly complex situations. For example, math teachers shouldn't be surprised when a kid subtracts upward. No teacher should have to go out and discover that in the classroom, Ball argued, because it's really predictable that that's what kids will do.

Ball's group has begun this work by trying to identify what they call "high-leverage practices," the set of skills that powerfully promote student learning and hence are most important for beginning teachers to have. Although it is unrealistic to expect that a brand-new teacher will be expert at every aspect of the common core, teacher educators can ensure that their student teachers don't have gaps that are downright dangerous for students. So Ball's team has gathered teachers and people who study teaching to create a comprehensive map of the work of teaching to identify which of tasks are essential for beginners to be skilled at before they are given a classroom of their own and which they can pick up as they practice.

In addition to agreeing on this type of content, teacher educators would need to agree on what student teachers need to learn about the practice of teaching. While student teachers have always had practice in teaching, this has generally been done in an ad-hoc way, with only the number of hours practicing being specified rather than the specific type of practice needed.

Take, for example, the job of leading a mathematical discussion with second-graders. Rather than simply throwing novice teachers into that task, a sequence of activities would need to be identified that would teach someone who has never done this before what they need to do. This might include videos, discussions, practice doing it themselves, analyzing and discussing a video of their own work, etc. After going through this sequence some number of times, a student teacher should then be able to perform the task themselves competently.

→ *see page 14*

High-leverage practices for all teachers

Here are a few examples of the things that Ball's group has identified as candidates for high-leverage practices all teachers need to be able to do:

- Explaining specific content ideas and processes
- Choosing and using representations, examples and models of core content
- Setting up and managing small-group work
- Recognizing and identifying common patterns of student thinking
- Selecting and using specific methods to assess students' learning on an ongoing basis
- Conducting a meeting with a parent or caregiver and being able to explain a child's difficulties in a comprehensible and useful way

Critical content for elementary math teachers

And here are a few examples Ball's group has identified as candidates for critical content for elementary math teachers to understand deeply:

- Place value
- Computational procedures with whole numbers and decimals
- Fractions
- Modeling mathematical ideas
- Reasoning
- Constructing viable mathematical arguments and critiquing the arguments of others

→ *from page 13*

The particular sequence of tasks would depend on what the student teacher is learning to do; for example, Ball said, learning to recognize common patterns of student thinking would probably require different tasks than learning to lead a discussion.

Finally, a common curriculum for teacher education would require an assessment system. Just as we don't allow a pilot to fly real airplanes or a doctor to treat real patients until passing a test proving their competency, we shouldn't allow teachers to teach real children until they've done the same, Ball argued.

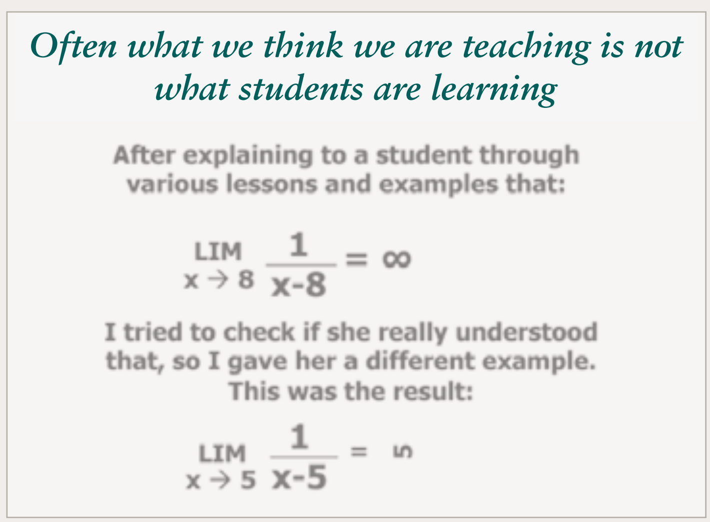
An assessment system would need to cover everything logically necessary, be guided by the wisdom of practice, and be consistent with research. Research isn't necessary for some things that are logically clear, like the importance of being able to conduct an effective, respectful conversation with a parent. But for other things, research is essential, like identifying the common patterns of student thinking teachers ought to be prepared for and the most helpful responses to them. Furthermore, research would be needed to link the assessments to student learning gains: Students of teachers who do well on the teacher assessments should do well on their own assessments, and the assessments should be refined until that can be proven. These assessments would be linked to the specific common core state standards. They would also offer diagnostic info to help teachers to continue to improve their work. And there would be detailed exemplars of the goal. For example, it would describe what a classroom discussion led by a very good beginning teacher looks like.

Problem: When student teachers were shown this work by a student, they often figured that the child understood how to do it because the answer was correct. They didn't notice the 1 that had been carried inappropriately. As a result, when given the problem 16 + 27+ 48, they often wrongly predicted that the child would carry a 2, when in fact the child again carried a 1. The child didn't understand that you didn't simply always carry a 1.

$$\begin{array}{r} {\scriptstyle 1} \\ 13 \\ 21 \\ 45 \\ \hline 79 \end{array}$$

Deborah Ball of the University of Michigan presented her vision of a common core curriculum for teacher education.

Randy Philipp of San Diego State University spoke about the need for teachers to learn to understand how their students understand mathematics. Often, he explained, teachers assume that children are understanding math the way they do, when it can be very different. The slide below from his talk gave one example.



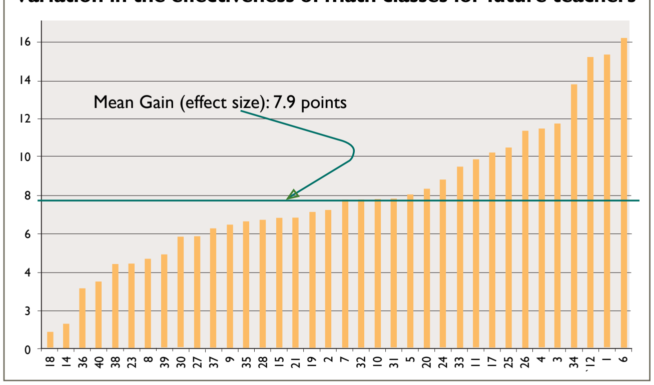
Research Findings

Raven McCrory on the effectiveness of math classes for future teachers

How effectively do math courses teach future teachers math they need for the classroom?

According to research performed by Raven McCrory of Michigan State University and her team, they're doing it surprisingly well.

McCrory's team had student teachers take a test of mathematical knowledge for teaching before and after taking mathematics classes at 57 institutions in Michigan, New York City and South Carolina. Students gained between a half-point and 15 points on a 100-point test with a standard deviation of 10 points – an enormous effect size, but also an enormous range of variation. So the team tried to understand why some classes were so much more effective than others.



On average, students gained nearly 8 points on a test of mathematical knowledge for teaching after taking a math class. But the variation in gains from one class to another was enormous.

The team found bigger gains in classes that used a textbook rather than informal materials; in which students did math in class rather than listening to a lecture; and in which students came in better prepared, with higher pretest scores. Students with weak preparations particularly needed an active, hands-on instructional style, and students with strong preparations especially benefited from having a textbook, apparently since they knew how to make better use of one. Instructor experience, rank, highest degree, or attitude didn't matter, nor did class size.

Raven McCrory of Michigan State University found that mathematics courses do a surprisingly good job of preparing future teachers for the classroom.

Maria Teresa Tatto on the Teacher Education and Development Survey

How do the countries that do math education best prepare their math teachers? Maria Teresa Tatto of Michigan State University presented results from the Teacher Education and Development Survey, a study that compared the preparation of math teachers in 17 countries to understand how teacher education policies and practices affect student outcomes.

The study suggests that the United States could improve the mathematics scores of its students by imitating the practices of higher-scoring countries. In high-scoring countries such as Taiwan and Singapore, students get rigorous math instruction in high school, creating a pool of well-qualified student teachers. University teacher-preparation programs are highly selective and demanding. And young people are drawn to the profession because teaching offers excellent pay, benefits, and job security.

Furthermore, the study strongly supports the benefits of university-based teacher-preparation programs, rather than alternative programs that put talented liberal arts graduates in the classroom quickly with minimal preparation. All the countries with the best teachers have rigorous teacher education programs. Teaching, the study supports, is a teachable skill rather than a knack than only some people have, so the U.S. can strengthen its teaching workforce by strengthening teacher education.

Maria Teresa Tatto of Michigan State University compared math education in 17 countries to find the critical elements of teacher preparation programs.

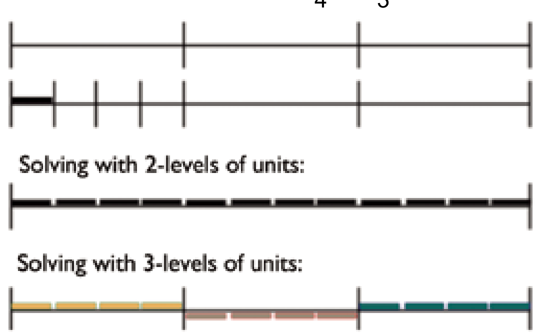
Andrew Izsák on how teachers use drawn models

Pictures are key to making sense of many topics in mathematics, and they are central to the Common Core State Standards. For example, drawings of the number line, of rectangles showing area, or of shaded regions can each aid understanding of fractions enormously. Relatively little research has been done, however, on how thoroughly teachers understand drawn models and the extent to which they can use drawn models effectively with their students.

Andrew Izsák of the University of Georgia started working on this by studying an individual teacher's use of drawn models of fractions in depth. He has found previously that a key skill teachers need is the ability to operate with three levels of units simultaneously. Consider the problem of determining what is $\frac{1}{4}$ of $\frac{1}{3}$ without the formal procedure for multiplying fractions. You could start by dividing a unit length into three equal-sized parts to construct $\frac{1}{3}$. Then you could divide the $\frac{1}{3}$ into four equal-sized parts to construct the length you want. The question is, what do you get?

One way to answer this question is to concatenate the new length you constructed and to see that 12 copies exhaust the whole. This solution only requires reasoning with two levels of units because at each stage you only attend to two units at a time. A second way to solve the problem is to see that there are three groups of four pieces in the whole. This requires attention to three levels of units simultaneously.

Levels of units: what is $\frac{1}{4}$ of $\frac{1}{3}$?



Furthermore, understanding concepts like improper fractions requires three levels of units: For example, understanding the improper fraction $\frac{5}{4}$ in terms of the Common Core definition (five copies of $\frac{1}{4}$ of the whole) requires simultaneous attention to the $\frac{5}{4}$, the whole, and the $\frac{1}{4}$.

Izsák found that not all teachers operate with three levels of units, and those that did not were severely constrained in their ability to effectively use drawn models to help their students understand fraction arithmetic.

To build on the study described above, Izsák and collaborators at the University of Georgia studied 14 middle school teachers who were participating in an in-service training on fractions that particularly focused on drawn models and questions about the meaning of dividing fractions. As is true for many teachers, these teachers only learned formal methods for computing with fractions when they were students and did not develop strong conceptual meanings for division with fractions. The problem $1\frac{3}{4} \div \frac{1}{2}$, for example, can be understood as representing the question "How many halves are there in $1\frac{3}{4}$?," and interpreting it that way allows the question to be represented visually. Answering a question like "How many halves are there in $1\frac{3}{4}$?" requires the ability to work with three levels of units at once: halves, quarters, and wholes.

Izsák led the design of a test similar to the University of Michigan test of Mathematical Knowledge for Teaching that included questions on drawn models. The research team then used psychometric modeling to split a sample of 201 teachers into groups with similar patterns of correct and incorrect answers. The teachers fell into two groups – those who understood how to work with three levels of units (who got higher scores) and those who did not.

Mathematics teacher educators, Izsák says, have primarily focused on helping teachers understand the meaning of division of fractions. But his research suggests that if they don't also make sure that teachers are comfortable operating with three levels of units simultaneously, they won't be able to use drawn models effectively with their students.

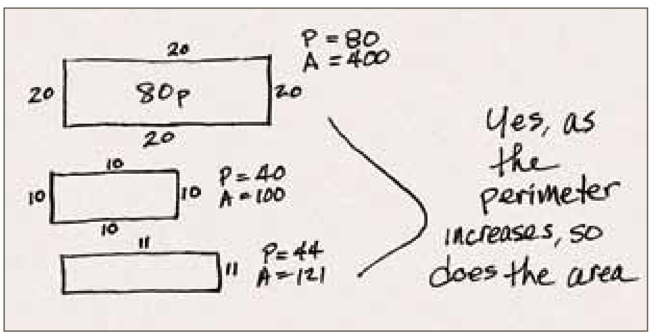
Andrew Izsák of the University of Georgia found the critical things teachers needed to understand in using drawn models of fractions.

The challenge in preparing preservice teachers

Denise Spangler of the University of Georgia presented the results of a study of preservice teachers to determine how well they were acquiring the mathematical skills they needed for teaching. The preservice teachers, they found, were mostly able to solve the problems themselves – though not always, even though the material had just been covered in a skillfully taught class. But even those who could solve the problems correctly themselves often didn't have the depth of knowledge necessary to respond skillfully to student work.

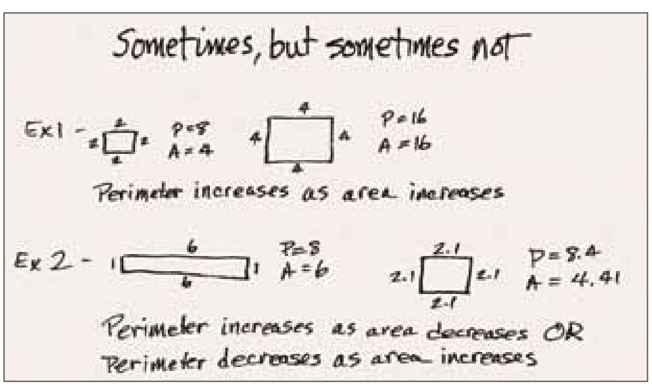
For example, consider this problem: Rectangle I has a larger perimeter than Rectangle II. Can you conclude that Rectangle I also has a larger area than Rectangle II? Why or why not?

First student response:



The preservice teachers typically recognized the answer as wrong, but most wanted to scrap the student's work and start over. They didn't have strategies to build on the student's reasoning. In this case, few of them noticed that the drawings weren't proportional to the lengths of the sides.

Third student response:



A correct answer! Hallelujah! Most preservice teachers stopped there and didn't analyze the work further. But notice that in the student's second example, the perimeter is actually 14, not 8, so the example doesn't support the student's reasoning. Also, it's very impressive that the student uses a square with non-integer sides and uses a mixture of squares and rectangles, which few of the preservice teachers noticed.

In general, preservice teachers with a weaker command of the mathematics tended to have difficulty seeing children's mathematical thinking, especially when it's different from their own; to assume they know what children are thinking instead of asking; to push children to do it their way; to ask bite-sized questions, with leading questions; to start over rather than building from existing ideas; to not scrutinize correct answers; and to not make an effort to connect solution strategies.

Those with higher content knowledge tended to ask more open questions; to try to get students to figure things out for themselves; to push students to analyze their solutions and go on from there rather than starting over; to pay attention to process as much as final answer; to link solution strategies; and to extend correct solutions to push for generalizations.

So how do math educators help their preservice teachers acquire this higher level of content knowledge? One key was to focus on teaching preservice teachers to do tasks, rather than focusing on preparing lessons. In particular, they need to be given assignments analyzing student work so that they can learn to see mathematics through their students' eyes and practice different ways of responding.

Denise Spangler of the University of Georgia described the depth of mathematical knowledge that is critical for teaching.

Collaborations between mathematicians and mathematics educators

Every semester, Jim Lewis used to read student evaluations for all mathematics courses taught at the University of Nebraska-Lincoln as part of his job as department chair. And every semester, he saw the same thing: Instructors in the mathematics courses for elementary school teachers received terrible evaluations. "This course is irrelevant for my work as a teacher," the prospective teachers would say. "Why do I have to learn this stuff?"

At the same time, he heard from his instructors how weak the prospective teachers' mathematical preparation was, even about basic issues like place value that are clearly essential for elementary school teachers to understand.

So Lewis decided to teach the mathematics classes for elementary school teachers himself, to figure out what was going on and try to fix the problem.

To make sure he didn't fall into the same difficulties, he gave himself extra support: He collaborated with a math educator. He knew that in order to reach the prospective teachers and get them engaged in the work of his class, he'd have to connect the mathematics to teachers' everyday work in the classroom. Not having taught elementary school himself, he needed help to do that. He also knew that the prospective teachers didn't have a lot of respect for what mathematicians had to contribute to their education, so he needed someone whose opinion they valued to back him up and convince them that the mathematical work was important.

So in 1999, he approached Ruth Heaton, then a junior professor in UNL's Department of Teaching, Learning, and Teacher Education. Heaton had been noticing the same problem, but she didn't have the time to teach the mathematics herself. Furthermore, she recognized that a mathematician might be able to enrich her own teaching of mathematical pedagogy. Mathematicians are, after all, precisely the people who know how to think deeply about mathematics, including the fundamental mathematics taught in elementary school.

The pair described their collaboration at the 2011 CIME conference. Collaborations between mathematicians and math educators, they say, are one of the most powerful methods to improve math education.

The pair began teaching together by scheduling their classes (a math content class and a math pedagogy class) to meet back-to-back in a two-year pilot program. This allowed them to participate in one another's classes without asking the university to pay them both for teaching one class.

They now have an "immersion semester" required of all students preparing to be elementary school teachers in which they are focused on the teaching and learning of mathematics. Heaton and Lewis designed a block of four courses, for a total of ten credit hours, all with an emphasis on mathematics teaching and learning: a mathematics content class, a mathematics pedagogy class, a field experience that involved working in an elementary classroom two days each week, and a class with master teachers on creating learner-centered classrooms. The pair worked to integrate the classes, creating a common syllabus and, when possible, common assignments.

In an evaluation of the immersion semester, students who had finished Lewis's and Heaton's classes were viewed by cooperating teachers as better prepared to teach all subjects in the elementary classroom, not just in mathematics. And unlike in previous versions of the course, student evaluations were excellent.

Mathematician Jim Lewis and math educator Ruth Heaton (not shown) teamed up to improve the math education classes at the University of Nebraska-Lincoln.

Mathematics test for teachers in Massachusetts

The weak mathematical preparation and skills of many teachers have led the Massachusetts Board of Elementary and Secondary Education to pass a math test in order to become credentialed as a teacher, beginning in 2009.

In the first year, only 27 percent of prospective teachers passed. Despite their dedication to their work, they didn't have the appropriate preparation for the test. In response, some challenged the legitimacy of the test and requirements, and others ridiculed the teachers who hadn't passed. But the Board of Elementary and Secondary Education stayed the course and refused to lower the passing score for the test. They did, however, conditionally certify those who came close but didn't quite pass, requiring them to retake and pass the test within five years. This raised the pass rate to nearly 40 percent.

Richard Bisk of Worcester State University reported that his university has developed three courses to meet the new guidelines and help prospective teachers pass the test, one on number and operations, one on geometry, measurement, probability and statistics, and one on algebra for teachers, emphasizing the connections to arithmetic.

By March 2011, across the state, 58% of prospective teachers taking the test for the first time passed it.

Richard Bisk of Worcester State University described the impact of a mathematics test for teachers in Massachusetts.

The Mathematical Sciences Research Institute (MSRI), located in Berkeley, California, fosters mathematical research by bringing together the foremost mathematical scientists from around the world in an environment that promotes creative and effective collaboration. MSRI's research extends through pure mathematics into computer science, statistics, and applications to other disciplines, including engineering, physics, biology, chemistry, medicine, and finance. Primarily supported by the U.S. National Science Foundation, the Institute is an independent nonprofit corporation that enjoys academic affiliation with nearly 100 leading universities as well as support from individuals, corporations, foundations, and other government and private organizations.

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Main Office 510-642-0143 • Fax 510-642-8609
Mailing Address: 17 Gauss Way • Berkeley, CA 94720-5070
www.msri.org