MSRI WORKSHOP

MATH & RACIAL JUSTICE

the role of mathematics in today's movement for racial justice

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A compendium of the MSRI Workshop on Mathematics and Racial Justice, June 9–11 and 16–18, 2021

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INTRODUCTION

> "*[R]acism may wear a new dress, buy a new pair of boots, but neither it nor its succubus twin fascism is new or can make anything new. It can only reproduce the environment that supports its own health: fear, denial, and an atmosphere in which its victims have lost the will to fight.*"
>
> —Toni Morrison

> "*But all our phrasing — race relations, racial justice, racial profiling, white privilege, even white supremacy — serves to obscure the fact that racism is a visceral experience, that it dislodges brains, blocks airways, rips muscle, extracts organs, cracks bones, breaks teeth. You must never look away from this. You must always remember that the sociology, the history, the economics, the graphs, the charts, the regressions, all land with great violence on the body.*"
>
> —Ta-Nehisi Coates

> "*Never forget that justice is what love looks like in public.*"
>
> —Cornel West

USING MATHEMATICS AS A TOOL TO CRITICALLY ANALYZE SYSTEMIC RACISM has a long history in the United States. W. E. B. Du Bois in 1903 predicted quite prophetically that "the problem of the 20th century is the problem of the color line." Du Bois was also among the first to invoke mathematics and statistics to analyze issues of racial injustice through his *Data Portraits Visualizing Black America*. The first Black person known to have earned a graduate degree in the U.S. was Kelly Miller, who went on to use what he had learned as a graduate student at Johns Hopkins to challenge the flawed statistics of eugenics in Fredrick Hoffman's 1896 book *Race Traits and Tendencies of the American Negro* and, as a faculty member at Howard University, taught mathematics as a tool for understanding social issues. The first known Black woman to enter graduate school in mathematics was Anna Julia Cooper, who later dedicated her life to the struggle for racial justice. Almost a century later former mathematics teacher and civil rights leader Bob Moses declared that "mathematics literacy is the literacy of the 21st century," and that the failure to provide equitable mathematics education for all has helped maintain the color line that still threatens our democracy.

The 2021 MSRI Workshop on Mathematics and Racial Justice was born out of an effort from members of the mathematics community to engage with the national conversation about racial justice that came to the forefront in Summer 2020 after the video recording of the brutal murder of George Floyd by a Minnesota police officer captured the world's attention. The protests and demonstrations in the months that followed brought the issue of police violence, racial injustice, and anti-Blackness into classrooms, boardrooms, and dinner tables around the country and around the world.

In response, many institutions began to ask whether there was more that they could do. MSRI has a history of support for diversity and inclusion in the mathematical sciences dating back to the work of former MSRI Director William Thurston and Deputy Director Lenore Blum that led to MSRI hosting the first Conference for African-American Researchers in the Mathematical Sciences (CAARMS) in 1995, so it was not a surprise when MSRI Deputy Director Hélène Barcelo reached out to members of the organizing committee with an offer to provide a platform for a conversation about race and the math community. We were not initially certain how we should act; we were certain, however, in our conviction that mathematicians can bring unique skills and abilities to the struggle for racial justice that are urgently needed in the world. We decided, with the support and assistance of the MSRI leadership, on organizing a mathematics research workshop that would provide an opportunity for the broader mathematical sciences community to learn about the ways in which scholars are using mathematics as a tool for understanding and exploring issues of racial injustice.

Our country's history of racism endures despite gains made during the civil rights era and this legacy continues into our present. Through the workshop, we intended to engage mathematicians and other scholars within the mathematical sciences who might work to dismantle this legacy.

Creating a complete litany of racial injustices is impossible. Educational, financial, medical, judicial, and political institutions are not without culpability in committing injustices towards Black people. George Floyd's murder on May 25, 2020, was only the latest in a string of documented murders of Black people by the police. The unjustified and highly publicized murders of Trayvon Martin (2012), Michael Brown (2014), Tamir Rice (2014), Eric Garner (2014), Freddie Gray (2015), Sandra Bland (2015), Jamar Clark (2016), Philando Castille (2016), Ronell Foster (2018), Elijah McClain (2019), and Breonna Taylor (2020) are just a small sample of the lives taken in the decade leading up to our decision to organize this workshop.

We intended to engage mathematicians and other scholars within the mathematical sciences who might work to dismantle this legacy

The murders of unarmed Black people at the hands of the state are disturbing echos of the history of the extreme violence of state sanctioned Jim Crow and the lynching of Black people in the U.S. In fact, while we were drafting the introduction to this document 10 people were murdered in an anti-Black terrorist attack in a grocery store in a predominantly Black section of Buffalo, NY, and the algorithms that govern social media are suspected to have played a role in inciting this violent act. These public murders of Black people and the parallel with the U.S. history of lynching is especially disturbing at a time when many believe the U.S. to be "post-race." The U.S. is certainly not "post-race," indeed many of the most compelling problems facing the U.S. today pivot on race.

As mathematicians, who see every day the potential for analyzing and guiding solutions for some of the world's most pressing problems, we recognized it was time for the mathematics community to rally around scholars using mathematics to analyze, study, and guide solutions for the problems of racial justice and to educate and inspire mathematics educators and researchers to take an active role in creating and implementing those solutions.

How can mathematics be used to identify and dismantle the ways in which biases manifest in social constructs, specifically those constructs concerning racial justice?

This workshop was designed to promote and stimulate mathematical research that sheds light on issues critical to racial justice and focused on the following guiding question: *How can mathematics be used to identify and dismantle the ways in which biases manifest in social constructs, specifically those constructs concerning racial justice?* For the purposes of this workshop, we defined racial justice as *the result of intentional, active, and sustained anti-racist practices that identify and dismantle racist structures and policies that operate to oppress*. We selected four key areas on which to focus: Bias in Algorithms and Technology; Fair Division, Allocation, and Representation; Public Health Disparities; and Racial Inequities in Mathematics Education.

The content of the scholarly work that was presented and the research expertise and personal perspective that all the speakers generously shared were profoundly rich. The conversations that took place in each session were dynamic and compelling, and all provided participants with a sense of agency and ability to become involved in mathematical research projects which are rooted in racial justice issues. This compendium is a record of the answers shared by mathematicians over the two-week long workshop.

Although the workshop was organized by and centered on the concerns of Black Americans, many of its lessons are applicable to other marginalized groups both in the U.S. and abroad. We hope that this compendium serves to summarize the activity of the workshop in such a way that it can serve as a foundation for further work using mathematics to advance justice.

— *Workshop on Mathematics & Racial Justice Organizing Committee*
Caleb Ashley, Ron Buckmire, Duane Cooper,
Monica Jackson, Omayra Ortega, and Robin Wilson

OVERVIEW

THE KEYNOTES BY ROBERT BERRY AND REDIET ABEBE set the stage for the discussions that followed: Berry presented a historically-informed view of the way mathematics education, as it is often implemented, dehumanizes people of color; and Abebe demonstrated the power of data and computer science to study social problems and guide their solutions. After the keynotes, the workshop was divided into four primary theme areas, which have also guided the organization of this compendium: Bias in Algorithms and Technology; Public Health Disparities; Racial Inequities in Mathematics Education; and Fair Division, Allocation, and Representation, briefly summarized below.

For more details, including videos, notes, and handouts from the sessions, visit the workshop website: www.msri.org/workshops/1012

Bias in Algorithms and Technology

Algorithms undergird and dominate many aspects of modern society. Although central, the presence of algorithms in our daily lives is far from innocuous. Part of this danger is inherent, bound up in the fact that the operating parameters and the explicit roles of algorithms are suppressed or hidden. Bias in design and implementation further compound the potential threat of algorithms to be harmful social forces.

Determining if an algorithm acts as a mechanism by which structural inequalities are perpetuated in society is fundamentally important on moral, ethical, and legal grounds. The existence of types of "automated social bias" has been established in a variety of algorithms, from those governing housing and lending practices, to those employed in policing and the criminal justice system. Speakers on this topic explored many facets in the creation and implementation of algorithms that affect social justice.

Public Health Disparities

Everyone deserves the right to good health, yet unnecessary and unjust inequities persist in our healthcare system. These inequities are the result of policies and practices that create an unequal distribution of money, power, and resources among communities based on race, class, gender, place, and other factors. Healthy People 2020, a program of the US government's Office of Disease Prevention and Health Promotion, states that "Racism has been linked to low birth weight, high blood pressure, and poor health status." A racial justice approach to health equity requires that we address how issues of racism, disproportionate distribution of wealth opportunities, and privilege within society produce negative outcomes for the Black community. Speakers on this topic discussed the racial disparities that exist within healthcare, how mathematics has been used to perpetuate these problems, and how mathematics can be used to identify and mitigate these disparities.

Racial Inequities in Mathematics Education

Mathematics education and racial justice intersect in many ways. Mathematics has long been a gatekeeper and catalyst for academic success in higher education in the United States. In the classroom, these experiences play out at the individual level from teacher-to-student and student-to-student. At the systemic level, there are multiple overlapping and interconnected inequities based on race that affect students of color in general and Black students in particular. Some examples are a focus on standardized testing over rich problem-solving opportunities and equitable teaching practices, funding formulas for public education that are directly connected to long-lasting racially segregated housing patterns, a deficit view of Black children in mathematics education research, overlooking Black students for advanced coursework, and the dehumanized, impersonal view that mathematical identities do not intersect with racial identities. In this section, we will explore the barriers towards dismantling racial inequities in mathematics education that still persist despite numerous, longstanding movements to eradicate or at least ameliorate them. Further, we intend to use this space to envision what racial justice in mathematics education can look like for Black students.

Fair Division and Allocation

The question of how finite resources can be shared is an important and interesting problem with mathematical underpinnings, historical significance, and increasing relevance to modern life. The principle of fairness has particular resonance to questions of racial justice in response to historically unfair systems and treatment. Both theoretical and applied mathematics can (and should) be involved in any discussion, analysis, or decision-making process concerning fair division, allocation, and representation. Theoretical considerations include presentation of definitions, axioms, and theorems, and applications include algorithms for fair division of entities among multiple parties and fair allocation of goods that are not divisible. Fair representation has applications that include the apportionment to states of seats in the U.S. House of Representatives and the contemporary disputes about how the states and other regions are divided into districts for election of representatives.

PART 1

KEYNOTES

Mathematics has the potential to make powerful changes in society, but that power is not always used responsibly.

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SEEKING RACIAL EQUITY AND SOCIAL JUSTICE IN MATHEMATICS TEACHING AND LEARNING

Based on a keynote lecture by Robert Q. Berry III, University of Virginia, as well as his chapter, "Disrupting Policies and Reforms in Mathematics Education to Address the Needs of Marginalized Learners," in Toward Equity and Social Justice in Mathematics Education, edited by Tonya Gau Bartell

THROUGH A HYBRID POLICY ANALYSIS using a critical race theory lens informed largely by the work of Derrick Bell, Robert Berry makes the case that policies and reforms in mathematics education were not designed to address the needs of marginalized learners; rather, these policies and reforms are often designed and enacted to protect the economic, technological, and social interests of those in power.

The Social Conditions of Historically Excluded Learners

The term "historically excluded learners" does not ascribe a sweeping set of attributes to Black, Latinx, Indigenous, and poor students; rather, it acknowledges the shared histories and experiences of these groups. Students from Black, Latinx, and Indigenous backgrounds prioritize community, family, spirituality, and connectedness to the world. Too often, these shared histories and experiences are overlooked and not valued in school settings.

There is a history of education policies intentionally aimed at keeping members of these groups from gaining access to education. It was illegal in many states to educate enslaved Black people. Additionally, education was used as a tool to "civilize," Christianize, or control Black and Native people so that they would not contribute to social upheaval. It was illegal to teach Native American children in their native languages. Schooling for Native American children adopted the "kill the Indian to save the man" policy.

After the end of Reconstruction in 1877, Black children were often intimidated and terrorized when they attempted to access education; today's education reform policies often overlook the generational impact of this terror. Education research, policy, and reform "has been violent to marginalized people, such as [I]ndigenous groups, who are represented by perspectives that are neither kind to their cultural worldview nor accurate regarding their priorities," as Zeus Leonardo writes. Schools are social institutions set up by those in power and are organized to support and value the types of cultural and social capital held by those in power. Policies and reforms in education often portray marginalized learners as in need of "fixing" and their cultures, families, and communities as deprived and deficient.

Today's STEM Education

More than half of the children of the U.S. are part of a historically excluded group, and they are projected to be in the majority in the entire U.S. population by 2043. Yet they make up only 13% of the STEM workforce and 16% of all STEM undergraduate degrees awarded. Historically excluded students who are equally prepared for Advanced Placement coursework are less likely than other students to actually take those courses. The number of bachelor's and master's degrees earned by members of historically excluded groups have increased since 1993, but the number of doctorates earned in these fields has flattened at about 7% since 2002.

David Bressoud contends that because U.S. schools are funded locally, there is a tremendous variation in calculus instruction available in high schools, with the most privileged students having the greatest access. This has a profound effect on historically excluded learners because they are more likely, for example, not to have access to calculus instruction in high school and therefore more likely not to have access to calculus requirements by universities. Consequently, the cycle of lack of access to STEM degrees and careers persists.

Theoretical Framework: Interest Convergence Principle

Interest convergence is an analytical viewpoint for examining how policies and reforms are dictated by those in power to advance their political, social, and economic interests. Derrick Bell, a former attorney with the NAACP during the Civil Rights Era, employed his interest-convergence principle to explain how the United States Supreme Court issued the landmark ruling in *Brown v. Board of Education of Topeka* in 1954, which revoked the "separate but equal" doctrine that had legally sanctioned segregation in public education and daily life.

Bell argued that the *Brown* decision was not the result of a moral reckoning in America, but rather that the court was more interested in providing "immediate credibility to America's struggle with Communist countries to win the hearts and minds of emerging third world peoples" than in doing what was morally right. Under the interest-convergence principle, the Brown decision is best understood as progress requiring the coincidence of a pressing issue, more than as a commitment to social or racial justice. Bell writes, "Even when interest-convergence results in an effective racial remedy, that remedy will be abrogated at the point that policymakers fear the remedial policy is threatening the superior societal status of Whites, particularly those in the middle and upper classes."

Interest convergence provides a framework to discuss power dynamics as framed by systemic interests and a loss/gain binary. In STEM education, this binary is often framed as a negotiation for providing broader access to historically excluded learners in exchange for access to a structure leading to upward mobility. An example of the binary in STEM is the discourse of increased participation of historically excluded learners in STEM careers leading to high-paying jobs with little focus on whether the climate in the STEM workforce is ready for the increased diversity. This binary position is an individualistic venture and does not focus on the structures or policies marginalizing communities broadly.

Gutstein argues that mathematics education policies and reforms are motivated primarily by the desire to maintain global economic superiority. The "Educate to Innovate" campaign is one example of this attitude, as the following quote demonstrates.

> "*Whether it's improving our health or harnessing clean energy, protecting our security or succeeding in the global economy, our future depends on reaffirming America's role as the world's engine of scientific discovery and technological innovation. And that leadership tomorrow depends on how we educate our students today, especially in math, science, technology, and engineering.*"

This framing in terms of economic superiority suggests that the problems of technology and science innovation are affected by all people in the same or similar ways, but Gutstein points out that the proposed solution is based on the assumption that the benefits will trickle down to all of those whose interests are served. When we consider that U.S. productivity has increased since the 1970s, yet income- and wealth-polarization have grown, we must question who benefits from the solutions to such technological and scientific problems. Consequently, while the policies and reforms claim that success will benefit all citizens, only those with the most power might receive those benefits. Policy reforms are more about the interests of those with power and less about historically excluded people's needs and interests.

Martin found that mathematics education policy and reform documents promote a market enterprise working for the financial benefit of a select few. He stated that the "status of African American, Latino, Native American, and poor students has not been a primary determinant driving mathematics education reform. When discussions do focus on increasing participation among these students, it is usually in reference to workforce and national economic concerns."

The Elementary and Secondary Education Act (ESEA) of 1964 (reauthorized in 1994, 2001, and 2015) addresses educational inequities through legislation. One can question whether the learners are the primary beneficiaries of this legislation. The 2015 Every Student Succeeds Act has a purpose statement, "To provide all children [a] significant opportunity to receive a fair, equitable, and high-quality education, and to close educational achievement gaps." But the legislation does not address how those gaps have formed.

Unpacking Policies and Reforms: Racial Commodification

A common theme among policy and reform documents is a call for increased participation of historically excluded learners in STEM fields. These calls usually reference increased and new demands on the U.S. economy, the drive to stay ahead technologically of international competitors, and a need to secure the U.S. from international security threats. Rarely are there references focused on the needs of marginalized people and communities.

Positioning marginalized people's increased participation in mathematics to meet interests that may not include their own commodifies them by affixing a market value to their labor and intellectual development. Basile and Lopez describe this positioning as the method by which racial hierarchies are replicated and label it racial commodification. From a critical race theory perspective, we must consider whose interests are protected and how policies and reforms maintain the protection of those already in power. In *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, we see examples of commodification:

> "*Moreover, there are large, persistent disparities in mathematics achievement related to race and income — disparities that are not only devastating for individuals and families but also project poorly for the nation's future, given the youthfulness and high growth rates of the largest minority population.*"

In this example, and throughout the document, there is little discussion of the conditions, contexts, and experiences of marginalized students. It appears that this document is suggesting that the youthfulness of the largest minority population seems to be a segment that needs to protect the interests of those with power, thus the commodification of this population of learners.

Meaningful Policy Change Requires Decentralizing Whiteness

This review of policies and reforms suggests that economic, technological, and security interests have been — and continue to be — the drivers of many policies and reforms. These policies and reforms situated mathematics education in a nationalistic position of being color-blind, and in a position where race, racism, conditions, and contexts do not matter. When race, conditions, and contexts are not examined, schools and communities are positioned as neutral sites rather than cultural and political sites. Despite the evidence that racism and marginalization exist in schools and communities, many still adhere to the belief that color-blind policies and pedagogical practices will best serve all students.

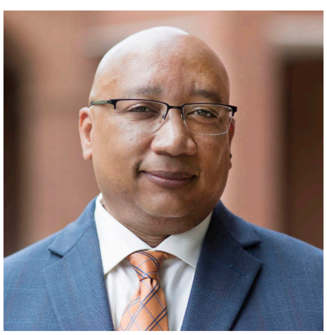
To have any meaningful policy gains, we must decentralize whiteness when discussing policies and reforms. There is significant evidence suggesting that whiteness is at the center of many educational policies and reforms. By decentralizing whiteness, we disrupt its power and privilege. Decentralizing whiteness opens the space and broadens opportunities to consider the roles that histories, contexts, and experiences play in the development of reforms and enactment of policies.

Activities such as the Black Lives Matter (BLM) movement will demand that the perspectives and worldviews of historically excluded peoples will be given consideration. BLM is a grassroots effort that is unapologetic in its rhetoric and challenges structural racism, anti-blackness, and institutionalized violence in school reform, policy, and research. There is compelling evidence that schools and schooling were created to maintain the power and privilege of whiteness. Problems and issues in education must be framed as part of the history and legacy of racism and as an issue of civil rights and social justice, viewed through a critical lens.

Given the growing body of research focused on context, identities, experience, and conditions of historically excluded learners, mathematics education policies and reforms can draw from this body of work in establishing new policies and reforms. This body of research considers issues of race, racism, contexts, identities, and conditions as variables that impact the mathematical experiences of marginalized learners. It challenges the dominant discourses and pushes the field of mathematics education to consider sociological, anthropological, and critical theories and encourages researchers to consider outcomes other than achievement as the primary measure of success. One finding from the research is that educators must create opportunities for students to experience mathematics learning using the resources they bring to classrooms; teachers must know and understand learners' identities, histories, experiences, and cultural contexts and consider how to use these to connect students meaningfully with mathematics.

There is a need for policies and reforms that focus on leveraging communities' and community-members' knowledge and experience in mathematics education. Mathematics teaching and learning not only occurs in classrooms but also occurs in other spaces as well. By leveraging these resources, we situate mathematics teaching and learning as a way to structure experiences that are contextual and provide opportunities for exchange of mathematical ideas. The use of context in mathematics education can help learners to recognize and build upon the cultural and social resources they bring to the mathematics classroom.

About the presenter



Dr. Robert Q. Berry III is the Samuel Braley Gray Professor of Mathematics Education and the Associate Dean of Diversity, Equity, and Inclusion in the School of Education and Human Development at the University of Virginia and the immediate Past President of the National Council of Teachers of Mathematics (NCTM). Berry co-edited the 2020 book High School Mathematics Lessons to Explore, Understand, and Respond to Social Injustice, which focuses on teaching mathematics for social justice. His articles have appeared in the Journal for Research in Mathematics Education, Journal of Teacher Education, Educational Studies in Mathematics, and the American Educational Research Journal. Berry is a two-time recipient of NCTM's Linking Research and Practice Publication Award and received the University of Virginia's All-University Teaching Award in 2011. He received his Bachelor of Science degree from Old Dominion University, his master's degree from Christopher Newport University, and holds a Ph.D. from the University of North Carolina at Chapel Hill.

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ROLES FOR COMPUTING IN SOCIAL CHANGE

Based on a keynote lecture by Rediet Abebe, University of California, Berkeley

MATHEMATICIANS AND COMPUTER SCIENTISTS are increasingly aware of the ways in which their tools can play important roles in social change and social justice but can also be used to serve the status quo and even exacerbate existing inequalities. When asking themselves how to use their work in service of a particular cause, these researchers should be aware both of some common pitfalls of such work and of the many ways in which their work can be used for good.

Addressing Pitfalls

One pitfall, *solutionism*, is to assume that computing can and will solve any and all problems, discounting the role of other types of interventions. Another, perhaps more dangerous, is *tinkering*, assuming that certain problematic features of society are fixed and tinkering around the edges to optimize outcomes given these immutable problems rather than trying to tackle them head-on. Another pitfall to avoid is *diversion*, distracting from the roots of problems and serious efforts to address them.

These pitfalls may cause some to question whether computing can play a meaningful role in creating a more just world. In a paper published in 2020, Abebe and colleagues asked the question, *How can computing support (and not supplant) other routes to broad social change?* Their answer was to identify four recurring roles computing can play in social justice work: diagnostic, formalizer, rebuttal, and synecdoche. As a diagnostic, computing can help researchers understand social problems. As a formalizer, it helps define social problems explicitly. As a rebuttal, it can illustrate the limitations of technical interventions. As synecdoche, it can bring renewed attention to long-standing problems. This chapter elaborates on the diagnostic and formalizer roles.

Computing as Diagnostic

Using computing as diagnostic can help researchers measure social problems and diagnose how they manifest in technical systems. When computing is used as a diagnostic, there is no attempt to solve social problems; rather, the approach seeks to provide evidence that problems exist and perhaps to identify aspects of technical structures that create these problems. For example, Latanya Sweeney, a professor of the practice of government and technology at Harvard, wrote a paper about racial bias in advertisement delivery. She showed that online searches for names commonly associated with African Americans tended to include ads related to arrests, whether or not someone by that name had ever been arrested. Abebe's research on search results for queries related to HIV and AIDS is another example of computing used as a diagnostic tool. (See sidebar.)

One danger of using computing as a diagnostic is that diagnosing a problem is not the same thing as treating it. If researchers focus too much on identifying a problem, they may never move toward work that will alleviate it. Worse, attempts to fix the problem can backfire. Simply knowing a problem exists does not always lead to (or even motivate people to seek) good solutions. In her book Race After Technology, Ruha Benjamin, a sociologist at Princeton University, wrote, "Data, in short, do not always speak for themselves and don't always change hearts and minds or policy." In the U.S., people know that mass incarceration of Black and Latinx men and chronic homelessness are huge, pervasive problems, but efforts to address them have been ineffective.

Computing as Formalizer

Using computing as a formalizer exploits the fact that computing requires explicit specifications of inputs and objectives to sharpen our understanding of social problems. Social problems are often presented and understood in a vague way: Social workers want to make decisions in the best interest of a child; employers want to hire the most qualified applicants; housing assistance programs aim to help as many people as possible as much as possible. But what is the best interest of a child? Which applicants are most qualified? How do we quantify the amount of help a social program provides? In order to use computing to understand and address social problems, researchers must explicitly define and state utility functions that represent their values.

One example of computing as a formalizer comes from Abebe's work in studying homelessness and housing instability, particularly among Black women. She and her colleagues noticed that many people with housing insecurity face shocks, such as late paychecks or illness or caretaking duties that limit their ability to work. These income shocks, both large and small, can lead to eviction and homelessness for people without a safety net. They developed a model of economic welfare that includes income, wealth, and these income shocks and use it to show how optimal allocations of subsidies might be different under this model than models that use only income and wealth. Their work shows how quantifying problems to use in computational models can yield policy insights.

Lessons in Computing for Social Change

Computer scientists and mathematicians who want to use their work for social good have a responsibility not just to acknowledge but to shape how their research is understood and used in society. Statements such as "I'm just an engineer" or "I'm just a scientist" evade responsibility for the impact of one's work. Researchers need to pay attention to problems, such as bias in online searches, when they happen, but ideally they will anticipate such problems before they happen. If people who programmed search engine ad preferences had considered how subpopulations might be affected, they may have been able to prevent people with Black-sounding names from being shown disproportionately prison-related ads, for example.

Researchers also need to build partnerships with domain experts and affected communities based on mutual trust and respect. People understand their own problems better than anyone else does, but sometimes researchers parachute in believing they have computational solutions for particular problems without getting a strong background in the problem from social scientists, humanities researchers, social workers, or those affected by the problem.

About the presenter

Dr. Rediet Abebe is an Assistant Professor of Computer Science at the University of California, Berkeley and a Junior Fellow at the Harvard Society of Fellows. Abebe holds a Ph.D. in computer science from Cornell University and graduate degrees in mathematics from Harvard University and the University of Cambridge. Her research is in artificial intelligence and algorithms, with a focus on equity and distributive justice concerns. Abebe co-founded and co-organizes Mechanism Design for Social Good (MD4SG) and is serving as Program Co-chair for the inaugural ACM Conference on Equity and Access in Algorithms, Mechanisms, and Optimization (EAAMO '21). Her dissertation received the 2020 ACM SIGKDD Dissertation Award and an honorable mention for the ACM SIGEcom Dissertation Award for offering the foundations of this emerging research area. Abebe's work has informed policy and practice at the National Institute of Health (NIH), the Ethiopian Ministry of Education, and the United Nations Food Systems Summit. Abebe also co-founded Black in AI, a non-profit organization tackling equity issues in AI. Her work is influenced by her upbringing in her hometown of Addis Ababa, Ethiopia.

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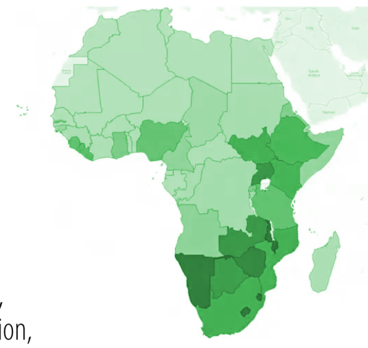
Case Study: HIV-related Keyword Searches in Africa

Six primary categories of HIV-related searches;
Heat map shows percentage of total search traffic containing the words "HIV" or "AIDS"

Drugs
drug, treatment, patients, abuse, therapy, drugs, resistance, antiretroviral, substance

Symptoms
pain, sign, lymph, swollen, nodes, sore, symptom, symptoms, throat, infection

Breastfeeding
positive, baby, mother, breastfeeding, breast, mothers, child, born, feeding, babies



Stigma
stigma, issues, discrimination, related, ethical, legal, prevention, safety, pdf, workplace

Healthy Lifestyle
food, positive, people, person, diet, healthy, living, eat, good patients, medication, nutrition, foods, lifestyle

Natural Cures
cure, oil, black, healing, heal healed, seed, herbs, natural, cures

Access to accurate information about transmission, diagnosis, and treatment of any disease is important for public health. In 2019, Abebe and her colleagues published a paper about the availability of accurate information about HIV and AIDS in Africa, where the disease has an extremely high burden. They used keyword searches related to HIV and AIDS to try to understand what people were trying to learn and the quality of the information they had access to. They used natural language processing to cluster the searches into several broad themes: symptoms, drugs, healthy lifestyles, stigma, natural cures, and breastfeeding.

The bottom-up approach afforded by user-generated data from searches is invaluable in shaping other types of research into public understanding of HIV. For example, "honey bee venom cures aids" and "olive leaf extract cures hiv" were two searches related to natural cures. Without having done this study, a surveyor asking people about HIV information may not even know to inquire about honey bee venom or olive leaf extract.

Abebe and her colleagues wanted to know whether people were receiving scientifically accurate information in these searches and whether there were any differences between the quality of information in the different categories of questions.

They found that searches for natural cures tended to result in lower-quality websites than searches for drug treatments of HIV. One reason is that health agencies do not usually have information about natural remedies on their sites because natural remedies do not cure HIV or AIDS. Abebe and her colleagues believe their work shows that there is a place for public health organizations to meet people where they are by making high-quality, accurate, accessible information available that addresses the questions people have about natural remedies.

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PART 2

BIAS IN ALGORITHMS AND TECHNOLOGY

By examining the bias that exists in algorithms and technology, we can understand the ways in which these tools can be used for oppression or liberation..

DESIGNING FOR EQUITY

Based on a plenary talk by Sharad Goel, Harvard University

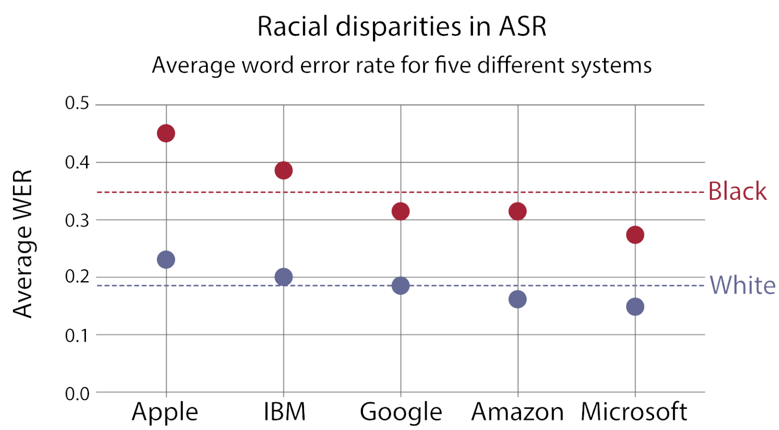
QUESTIONS OF WHAT FAIRNESS IS AND HOW TO MEASURE IT arise frequently at the Stanford Computational Policy Lab (SCPL), an academic center with the explicit goal of using interdisciplinary research and technical innovation to drive social impact beyond the university. The team includes researchers, engineers, and journalists with expertise in a variety of socially significant domains such as criminal justice, education, and healthcare; at the time of the workshop, Sharad Goel was Director. The team strives not only to understand how systems work but to intervene in their design and implementation to create fairer and more equitable algorithms. Recent SCPL projects serve as examples of both the challenges of defining and achieving fairness in machine learning algorithms and a consequentialist paradigm for designing equitable decision-making algorithms.

Racial Disparities in Automated Speech Recognition

Automated speech recognition (ASR) is widespread. Millions of people walk around with Siri, Alexa, Google Assistant, or other speech recognition tools in their pockets. In addition to their convenience, these programs help with tasks such as dictation, translation, and subtitling, so they are important accessibility tools for some people with disabilities. ASR has blossomed in the past decade, thanks in part to advances in deep learning and an explosion of training data. For a paper published in 2020, SCPL audited ASR systems from Amazon, Apple, Google, IBM, and Microsoft to determine whether there were racial disparities in the programs' performance. They compared human- and machine-generated transcripts for approximately 20 hours of audio of Black and white speakers living across the United States and found striking differences in the word error rates for white and Black speakers. The systems varied in overall quality, but all had error rates approximately twice as high for Black speakers as for white speakers, as illustrated in the figure on the next page:

Audio from Black speakers in the study was easy for most English speakers to understand, but ASR systems transcribed it less accurately than audio from white speakers. Black speakers' speech tended to have elements of African American Vernacular English (AAVE), a variety of English that is common among Black speakers in the U.S., who make up about 12 percent of the population. Although AAVE has been marginalized historically, it is no less correct than varieties of English associated with white speakers. The researchers wanted to determine whether features of AAVE were associated with the errors, and if so, which features were most associated. They coded examples for grammatical features such as zero copula (constructions such as "they gone" and "she tall") and future be (as in "he be here tomorrow") and acoustic features such as final consonant cluster reduction ("ban'" instead of "band" or "goin'" instead of "going") and haplology ("Missippi" instead of "Mississippi"). They found that audio clips with more AAVE features were indeed transcribed less accurately than audio clips with fewer of those features, supporting the idea that AAVE is transcribed less accurately than some other varieties of English. Further, they found that acoustic aspects, not grammatical aspects, of AAVE speech seemed to drive the errors in transcription.

ASR Error rates were twice as large for Black speakers



ASR's bias against speakers of AAVE did not occur in a void. It was no accident that the algorithms had not been trained well to understand a variety of English that is predominantly spoken by a group that is marginalized and discriminated against in the U.S. To fix this problem, the speech recognition community needs to invest resources in assessing and reporting inclusivity and equity in their tools and in ensuring that ASR systems and the institutions that build them are broadly inclusive.

Because the ASR errors were largely driven by the fairly minor acoustic differences between AAVE and other varieties of English, using more recordings of AAVE speech to train speech recognition algorithms would likely improve ASR performance on Black speakers. To improve equity, ASR developers should collect more data on AAVE speech as well as other varieties of English and use that data to train algorithms on a broader range of English speakers.

As ASR becomes increasingly common in everyday life, the ASR community would do well to learn from legislative progress made in other domains, such as computer vision.. For example, some states and countries have placed restrictions on the use of automated facial recognition by government agencies and private companies. Technologies such as ASR have similarly complicated fairness concerns, and ASR researchers and designers should participate in processes that ensure that their use is legal and ethical.

A Deontological Approach to Fairness: ASR and Pretrial Detention

This discussion of ASR has operated under the intuitive assumption that fairness would mean that speech recognition algorithms would have similar error rates for white and Black speakers. But researchers in algorithmic fairness sometimes struggle to formalize and measure fairness. Sometimes metrics for fairness that seem intuitive can lead to unintended and inequitable outcomes.

For example, pretrial risk assessment algorithms have been considered as a possible alternative to human decisions about whether people should be detained before trial based on their risk of reoffending. One way to define fairness is called classification parity. An algorithm is considered fair if error rates are approximately equal for people in different groups, for example Black and white defendants. In this case, one way to formalize error is with the false positive rate, the number of people who were considered high-risk but did not reoffend divided by the total number of people who did not reoffend. This notion of fairness may be called deontological because it prioritizes rules rather than consequences.

One pitfall of using false positives as a metric for fairness is that if two populations have different risk profiles, they will naturally have different false positive rates. For example, if population A has an overall lower risk of reoffending than population B, population B will often have a higher false positive rate than population A. One way to lower this false positive rate would be to arrest more low-risk people in population B, but this solution makes population B worse off, rather than better-off. In the real world, due to differing socioeconomic backgrounds, life experiences, policing practices, and other factors, Black and white populations in the U.S. do have different risk profiles, so the error rates are hard to compare.

In the speech recognition setting, there is reason to believe that Black speakers are not harder to understand than white speakers and therefore that ASR programs should have similar error rates for these two populations. In contrast, pretrial risk assessments do not have the same guarantee. Factors other than faulty risk-assessment algorithms can lead to different error rates for Black and white defendants.

Consequentialist Approach to Fairness: Free Transportation to Court Appearances

Consider the example of the risk of defendants to miss court dates, sometimes called "flight risk." This term is a misnomer, because in many cases defendants who miss court dates have not left the area to evade the law but instead faced barriers to appearing in court, such as a lack of transportation or childcare. Many people are incarcerated simply because they have failed to appear in court.

In contrast to the deontological measures of fairness for ASR and pretrial detention that were based on rates of false positives, a consequentialist approach considers the outcomes — in this case aiming to maximize the number of defendants who appear in court. SCPL has been using this approach in work with the Santa Clara County Public Defender office to distribute free rides to court. The question is who should receive those rides.

One consequentialist method of allocating those rides would be to optimize for the increase in number of appearances based on getting rides. Some people would take a free ride if it was offered but could easily drive themselves or find another ride if not. Hence, from a consequentialist point of view, such a ride would not do as much good as it would if it went to someone who would have missed the appearance without it. On the other hand, some people might not use the ride at all, wasting it entirely.

Using statistical methods, researchers can estimate the likelihood that a ride voucher would cause someone to make their court appearance who would otherwise have missed it and give the rides to the people most likely to be helped. But that method may give rides to differing proportions of people in one group than in another. A different allocation method, more in line with the deontological approach, would be to ensure that the same proportion of different demographics (perhaps white and Black defendants) are allocated rides. Neither method, nor a method between the two, is inherently better or worse. The optimal choice depends on the preferences of the entity distributing rides.

Multi-Armed Approaches. SCPL takes an algorithmic approach to allocating rides that depends on both the increase in number of appearances and parity between different demographic groups, using so-called "multi-armed bandit" models. The idea of their algorithm is to first create a utility function that takes into account the preferences of various stakeholders. Then randomly distribute rides to a small initial population and use the treated population to train a model that predicts outcomes for other distributions of rides. From there, they generate estimates for the potential outcomes under all the possible actions and solve for the policy that maximizes utility. As SCPL implements the policy, they continue to update the model using the new data. They believe this approach is more aligned with stakeholder preferences than algorithms that satisfy mathematical fairness constraints based on parity, which will usually result in suboptimal outcomes.

Deciding on a utility function for allocating free rides, or in other applications, is still a difficult prospect, requiring decision-makers to balance often-competing preferences. But a model that considers outcomes and updates itself as real data comes in can do more to effect social good than models that rely only on mathematical fairness constraints alone.

About the presenter

Dr. Sharad Goel is a professor of public policy at Harvard Kennedy School and an affiliate member of the Harvard Department of Computer Science. He was the founding director of the Stanford Computational Policy Lab, a group that develops technology to tackle pressing issues in criminal justice, education, voting rights, and beyond. In his research, Goel looks at public policy through the lens of computer science, bringing a new, computational perspective to a diverse range of contemporary social issues, including policing practices, electoral integrity, online privacy, and media bias. Before joining the Harvard faculty, Goel completed a Ph.D. in applied mathematics at Cornell University, worked as a senior researcher at Microsoft, and worked as an assistant professor at Stanford University.

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SOURCES AND CONSEQUENCES OF ALGORITHMIC BIAS

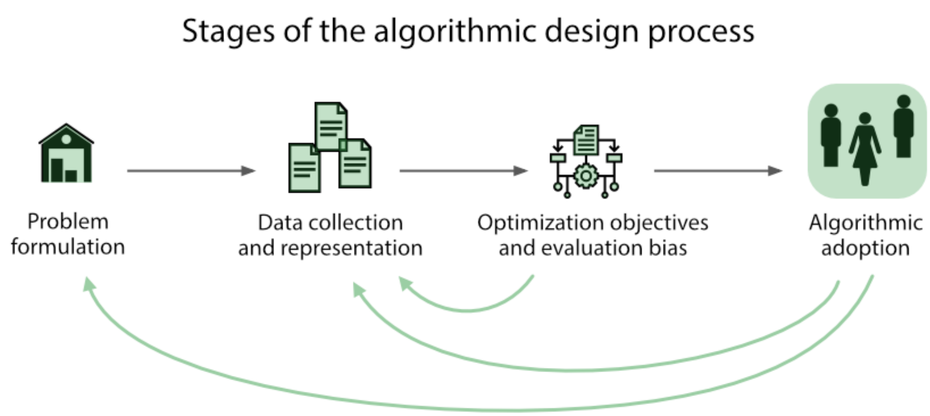
Based on a plenary talk by Maria De-Arteaga, University of Texas at Austin

MACHINE LEARNING AND ALGORITHMIC DECISION SUPPORT are increasingly being used in high-stakes settings such as healthcare, human resources, criminal justice, and public services. Many people have raised concerns about the fact that these algorithms can perpetuate and compound discrimination. This chapter examines potential sources of bias and presents a case study where these biases can lead to over- and under-policing.

Taxonomy of Sources of Bias

To understand the sources of algorithmic bias, consider the stages of building and deploying machine learning systems: problem formulation; data collection and representation; optimization objectives and evaluation; and adoption of the algorithm. Bias may creep into the algorithm in any or all of these stages, and biases introduced at one stage can interact with and feed back into bias at other stages. Furthermore, it may creep into not only the algorithm itself but also the process by which someone uses the algorithm to make a decision.

Sources of bias exist at every stage

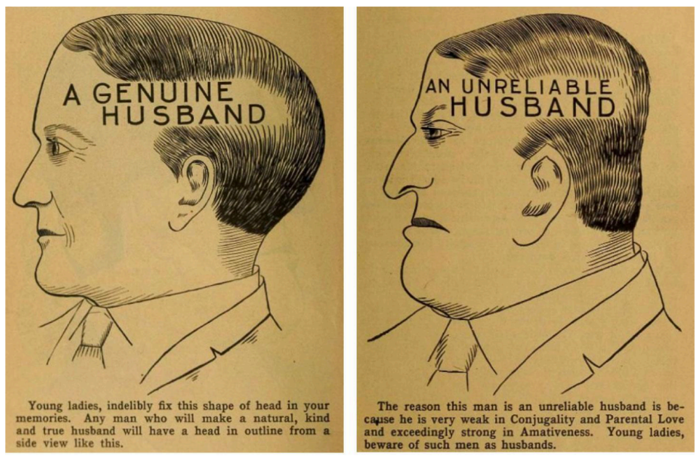


Problem Formulation

In the problem formulation stage of developing a machine learning system, the algorithm designer must decide what problem to try to address and what tools and data will be used to address it. Bias can be ingrained in the underlying assumptions at this stage.

Discriminatory assumptions. Consider the machine learning algorithms used in facial recognition software. Some applications of facial recognition, such as finding missing children, would be considered positive by almost everyone, but many applications of facial recognition are problematic and biased. For example, systems that use facial recognition to determine a person's gender are inherently transphobic, and predicting someone's personality, sexuality, or criminality using facial recognition amounts to little more than high-stakes, automated phrenology (Chinoy 2019).

Some facial recognition systems are little more than high-stakes, automated phrenology



Pages from *Vaught's Practical Character Reader*, a phrenology book published in 1902.
Source: Archive.org.

The facial recognition software Faceception, for example, claims to be able to classify people as having a variety of traits such as high IQs or a propensity for terrorism or pedophilia. When AI systems are used in tasks such as these that are inherently discriminatory, the question of ethics is different from the question of fairness. A more accurate algorithm, or one that performs as well across racial, ethnic, or gender lines, does not necessarily do less harm than a less accurate one. Many facial recognition algorithms perform worse on Black faces than white, but closing that gap will not prevent those algorithms from being used in harmful ways.

Proxies and mismeasurement. Aside from tasks based on discriminatory assumptions, proxies and mismeasurement can also bring bias into the algorithm at the problem formulation stage. When designers are creating a machine learning system that is meant to assist experts in their decisions, there is often a construct that the designer, or society, has decided should be the grounds of a decision. This construct may be complex and multifaceted, and it may be difficult for people to agree on a definition. The quantifiable and observable proxy that is used to represent this construct in the design process can introduce bias.

For example, an algorithm that assists in hiring decisions may observe whether people have gotten promotions rather than their actual potential. An algorithm that measures the risk of recidivism for a defendant may observe whether they have been re-arrested, not whether they have re-offended. A 2019 paper in *Science* (Obermeyer et al) studied a widely-used healthcare algorithm that used healthcare spending as a proxy for healthcare needs. Given historical disparities in access to care and how many resources were being devoted to patients, the result was that equally sick Black patients were receiving less investment than white patients. As a result, the algorithm was concluding that Black patients were healthier than equally sick white patients.

A failure to understand and correct the gap between proxy and construct can compound previous disparities and lead to misleading comparisons between machine and human decision-making. When we make head-to-head comparisons and judge that a machine learning algorithm is making better decisions, sometimes the machine is actually performing better, but sometimes the human is making decisions on different grounds and optimizing using different proxies than the machine is. This can even lead to self-fulfilling prophecies. If we evaluate the performance of the algorithm with respect to the proxy it was built to optimize, then we may conclude incorrectly that the algorithm did a good job on the construct itself. For example, if the biased healthcare algorithm informs our allocation of healthcare resources, the spending disparity will increase.

How will the algorithm be used? Designers should consider how an algorithm will be used to make decisions at the problem formulation stage. What is the overarching goal of the system? What are the mechanisms of entry into the population subjected to the algorithm? What is the space of possible decisions that the algorithm is informing? A 2018 paper by Shira Mitchell and colleagues delves into these aspects of prediction-based decisions.

Problem formulations matter because algorithms that solve seemingly the same task can be embedded within entirely different problem formulations, which can directly impact fairness considerations. For example, two papers, one by Elizabeth Herr and Larry Burt in 2005 and one by Ruyi Ge, Juan Feng, Bin Gu, and Pengzhu Zhang in 2017, examined the risks of student loan default. On the surface, the tasks were quite similar, predicting which students were most likely to default. But the decisions they were used to inform were different. Herr and Burt wrote, "interventions that focus on students' persistence and academic success were seen as the primary actions needed to help prevent student loan default." On the other hand, Ge et al wrote, "deterring borrowers with social media stigma and shaming on online social media could be a low-cost enforcement option for Chinese P2P [peer-to-peer] lending platforms." An intervention that will offer additional academic support programs to certain students is fundamentally different from one that will shame them on social media.

Data Collection and Representation

After the problem formulation stage comes data collection and representation. In some cases, algorithm designers engage in the collection of new data, but often in machine learning, they will use data that are already available in organizational and information systems.

Sampling bias. The first type of bias that may arise is sampling bias: the data that is being used to train the algorithm is not representative of the population it will be used on when it is deployed. This problem can be exacerbated by the fact that expense and convenience, rather than the quality of data, often drive the selection of data sources. For example, a 2015 paper on biases in geotagged tweets (Malik et al.) showed that geotag users are not distributed randomly in the U.S. but disproportionately live in areas that are more urban, more affluent, younger, and have a higher Asian, Black, or Hispanic/Latinx populations. Geotagged tweets are sometimes used to drive disaster response and management, so this bias could lead to misallocation of resources in high-stakes situations.

Sampling bias may be mediated by access to technology and resources, membership in a previously underserved community, and lack of trust in authorities. Thus, when certain groups are not represented in sample data, this bias often compounds previous injustices.

How well are people represented? Another problem with data collection and representation is how well people are represented. The predictive power of features collected may differ across subpopulations. For instance, in the context of health care, the symptoms that have been studied, taught, and recorded in data may only hold diagnostic power for some. Many dermatologists are trained to identify symptoms in pale skin, so Black and Brown individuals may not be correctly diagnosed or treated. The symptoms of some conditions, such as cardiac arrest, differ in men and women, and historical studies that used mostly male subjects may not lead to good outcomes in women. Cultural context can also affect the predictive power of certain attributes. For example, American credit scores penalize people who have few or no credit cards, so immigrants to the U.S. may be viewed as less creditworthy simply because accumulating lines of credit was not considered financially responsible in their home countries.

Adapting to a proxy. The ability to adapt to a certain choice of features may also differ across groups. When an algorithm designer introduces a proxy, they also introduce an incentive to improve with respect to that proxy. Not everyone has the ability to adapt. In the case of standardized tests, for example, wealthier students invest in tutoring and pay to retake tests, while poorer students do not always have those options. Regardless of the predictive power of standardized tests when they were first deployed, they now reflect less on students' potential than on their ability to afford extra help. The features that have different predictive power do not have that property by chance. They depend on who decided what data to collect and what experiences and perspectives are included or excluded.

Human labels are not truth. Another problem at the data collection stage is the fact that human labels can end up being interpreted as truth. For example, algorithms that are used by human resources departments may predict who is likely to get hired or fired or have a long career with the organization. Thus, they are predicting the assessments of managers, who are human, and therefore risk perpetrating past biases. This problem is perhaps obvious when it comes to hiring and promotions, where the subjectivity of human decisions is widely recognized, but may be less obvious in domains where people tend to think that there is an objective truth. A 2019 paper by Adewole Adamson points out this problem in cancer diagnosis. There is no gold standard the algorithm can learn from, so individual human expertise is being encoded as truth. Algorithm designers must consider how those encoding labels might lead to disparate performance across groups.

Optimization Objective and Evaluation Bias

The third stage, optimization objective and evaluation bias, is where we start considering the algorithm itself and how bias can creep into the design of the algorithm. The first important question a designer must ask is, "What objective will the algorithm optimize?" Furthermore, how will its performance be evaluated? These questions are related to problem formulation, but they are also part of the algorithm design process.

Optimizing overall performance has often been the obvious default choice in machine learning, but there is room to question whether it should be. Even if data is representative of a target population, if the choice of performance metric is centered around overall performance, then it will center majority populations.

A 2019 paper by De-Arteaga and coauthors considered bias in a recruiting algorithm. They collected a data set of 400,000 online biographies and attempted to predict each person's occupation based on the biography, as an automated recruiting algorithm might do as the basis for showing people job ads. They found that when an algorithm was optimized for overall performance, accuracy rates tended to be correlated with previous gender imbalances in an occupation. For instance, approximately 15% of the surgeons in their data set were women, but only 11% of those the algorithm correctly labeled as surgeons were women. An algorithm that performed that way would show job ads for surgeons to fewer women, reinforcing the historical gender imbalance.

Algorithmic Adoption

Fairness concerns do not stop at an algorithm's prediction but continue to the decisions made as a result. The way in which algorithmic recommendations are integrated into decision-making must be considered. One example is a study by Alex Albright of the adoption of risk assessment tools in bail decisions in Kentucky. Albright writes, "I show that this [raw racial disparities in initial bond] increase was not simply a consequence of different risk scores by race. Rather, the recommended default was also more likely to be overridden (in favor of harsher bond conditions) for black defendants than similar white defendants."

Understanding the Gaps

It can be tempting for algorithm designers to frame bias as a data problem, absolving themselves from taking responsibility for the harms caused by biased algorithms. A more useful alternative is to understand that the bias is in the gap between the question that motivates someone to use an algorithm and the question the data and algorithm actually answers. When designers understand these gaps and center the decisions made based on algorithms, they will be able to reimagine what questions they should be answering, rethink the integration of algorithms into decision-making, scrutinize proxy objectives, evaluate decisions rather than predictions, and anticipate long-term, contextual risks.

About the presenter

Dr. Maria De-Arteaga is an Assistant Professor at the Information, Risk, and Operations Management Department at the University of Texas at Austin, where she is also a core faculty member in the Machine Learning Laboratory. She received a joint Ph.D. in Machine Learning and Public Policy from Carnegie Mellon University. Her research on algorithmic fairness and human-centered machine learning studies the risks and opportunities of using machine learning to support experts' decisions in high-stakes settings. Her work has been awarded the Best Thematic Paper Award at NAACL 2019, the Innovation Award on Data Science at Data for Policy 2016, and has been featured by UN Women and Global Pulse in their report Gender Equality and Big Data: Making Gender Data Visible. She is a recipient of a 2020 Google Award for Inclusion Research, a 2018 Microsoft Research Dissertation Grant, and was named an EECS 2019 Rising Star.

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Case Study: Bias in Victim Crime Reporting and its Effect on Predictive Policing

Predictive policing is increasingly being deployed in cities and countries around the world, and has come under scrutiny due to a lack of transparency and concerns about biased outcomes. One focus of these critiques has been the potential for dangerous feedback loops when using arrest data as a basis of crime prediction.

Predicting policing, not crime

The issue is that the proxy — arrests — may not represent where crimes are being committed due to biases in arrests. In a paper about this phenomenon, Lum and Isaac write,"predictive policing is aptly named: it is predicting future policing, not future crime." They demonstrated that using data on drug arrests in Oakland, CA, as inputs to the model used by PredPol, one of the large companies providing predictive policing systems, would result in high concentrations of policing in racial and ethnic minority neighborhoods. In another paper, Ensign et al used a generalized Pólya urn model to theoretically analyze how feedback loops in arrest-based predictive policing systems arise.

In response, proponents of predictive policing have agreed that there is a concern there but say that they do not use this type of data in their models. PredPol, for example, says that their algorithms are unbiased by nature due to the fact that the data collected and analyzed is primarily victim data. When they do use arrest data, they exclude certain types of drug-related offenses and traffic citation data, which are usually police-initiated, because it is known to reflect officer bias.

In a study conducted in Bogotá, Colombia, Akpinar, De-Arteaga, and Chouldechova demonstrate how differential victim crime reporting can lead to geographical outcome disparities in hotspot predictions, even when the predictive policing algorithms do not use arrest data. These disparities result in both over- and under-policing.

Their analysis is based on a simulation patterned after district-level crime statistics collected by the Cámara de Comercio de Bogotá (Bogotá Chamber of Commerce). The survey contains information from approximately 10,000 participants from 19 urban districts and all socio-economic statuses. They were asked to report whether they had been victims of a crime and, if so, whether they had reported the crime. De-Arteaga and colleagues used this data to simulate reported and unreported crime data. These results are then fed into a predictive policing algorithm to compare how the algorithm would perform if it were based on the true crime distribution instead of just those that were reported and how these disparities compared across districts.

The researchers found that there were large disparities in crime reporting between districts. In one district, Chapinero, 9% of people were victims of a crime, and 28% reported. In Usaquén, 18% of people were victims of a crime, and 13% of them reported it. When trained only on reported crime data, some districts required more than double the crime rate of other districts to be selected as hotspots.

Potential solution: Rescaling

One potential solution to the problem would be to rescale according to victim reporting rates. Researchers ran simulations based on this approach and found that it only alleviates the problem partially. Although the rescaled model does get closer to the model of assuming that you have all crime data, the rescaling increases predictions in a district by a fixed factor, irrespective of the cell-specific crime. Each district is quite large, so it is still possible that the wrong cells within the district are being selected for increased policing. Hence the disparities are addressed at an aggregate district level, but they have been translated to a smaller scale. Some of the districts involved, such as Usaquén, are heterogeneous, so in order to recover the data, one would need a cell by cell rate of victim crime reporting, which is unattainable in practice.

The problem is not confined to the researchers' choice of model, a self-exciting process model, which is being considered for deployment in Bogota and already used by PredPol. They also looked at other types of models that are used in predictive policing and found very similar results. Therefore the problem will be present in any of the predictive policing models currently in use.

Crime reporting rates must be considered

In summary, differences in victim crime reporting rates can lead to geographical bias in common hotspot prediction algorithms, even when no data from arrests or police-initiated contact is used. These algorithms can therefore lead to misallocation of police patrols in the form of both over-policing of some neighborhoods, and under-policing of others. Victim crime reporting rates are known to be driven by socio-economic factors, types of crime and other demographics. More work is needed for an in-depth discussion of the interplay between predictive disparities and these factors in the Bogotá context.

Finally, we want to bring attention to the ongoing police brutality in both Bogotá and Colombia as a whole, where police are violently repressing civilians' protests and disproportionately targeting Black and indigenous communities.

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PART 3

PUBLIC HEALTH DISPARITIES

Most studies of public health disparities erroneously identify race as a causal variable, when racism is actually the cause of negative health outcomes in certain populations..

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THE PANDEMIC WITHIN THE PANDEMIC

Based on a plenary talk by Darius McDaniel, Leidos

> "*Of all the forms of inequality, injustice in health care is the most shocking and inhumane.*"
>
> —Martin Luther King, Jr.

THE REALITY OF RACE-BASED INEQUALITY IN HEALTHCARE was amplified and highlighted by the COVID-19 pandemic. Black people are often at higher risk than white people of contracting certain diseases, and overall they tend to have worse outcomes with the same diseases. Statements such as these can make racial health disparities seem abstract and depersonalized, but every statistic represents the pain, grief, and anxiety of thousands or millions of people. Darius McDaniel's presentation tied the abstract and statistical to the particular and personal.

A Journey into and through Biostatistics

For McDaniel, his own journey as a biostatistician is key to his understanding of racial health inequities in the U.S. In his first year as an undergraduate at Alabama A&M University, his father had to have triple bypass surgery. At the time, McDaniel did not know that due to higher rates of several conditions such as diabetes, hypertension, and cardiovascular disease, African Americans have heart attacks or require bypass surgeries more often than white Americans. The following year, McDaniel participated in a Research Experiences for Undergraduates (REU) program, the Summer Institute in Biostatistics, that changed his perspective on how to utilize mathematics within medicine and public health. Through the program, he learned that statisticians can participate in, and even lead, health-related research studies. In a subsequent summer, he participated in another REU on statistics and published his first paper. Those experiences allowed him to start attending conferences, meeting graduate students, and learn about potential opportunities in statistics.

McDaniel earned a master's degree in public health informatics from Emory University, which allowed him to go to South Africa working as a statistician and data analyst, where he could see firsthand the lingering effects of apartheid. There, he worked on multidrug-resistant tuberculosis and drug-resistant HIV, both of which affect Black South Africans at a much higher rate than white.

McDaniel's experience participating in a collaboration between the CDC and the Infectious Disease Center in Johannesburg encouraged him to bring a global lens to his work. After he graduated, he worked for the CDC for three years in pediatric infectious diseases. He left the CDC to work at Emory University studying Alzheimer's disease until the COVID-19 pandemic hit. At that point, he made his way back to the CDC to work on the COVID-19 response.

A History of Dehumanization and Marginalization

In her book *Medical Apartheid*, Harriet Washington lays out some of the history of medical racism and poor treatment of Black patients by medical practitioners. One of the concepts she points out is the immunity belief: white doctors believed that Black people were immune to certain infectious diseases, a belief built on the related idea that Black people have an innate immunity to physical and emotional pain.

Examples of the immunity belief and other dehumanizing ideas have appeared many times throughout history. For instance, Marion Sims, the "father of modern gynecology," experimented on Black women and children with no anesthesia. He believed Black people did not feel pain like white people and that there were developmental differences between Black and white humans that caused Black people to be less intelligent than white. A statue of Sims stood in Central Park until 2018.

In the infamous Tuskegee study, the U.S. Public Health Services and CDC withheld syphilis treatment from hundreds of Black men for 40 years to watch the progression of their disease. And further, in 1951, a Black woman named Henrietta Lacks went to the hospital for pain in her abdomen. After removing some of her cervix for a biopsy, doctors used her cells for research without her or her family's knowledge or consent.

In McDaniel's work on Alzheimer's disease, he and his coauthors discovered that there are racial differences in certain biomarkers associated with mild cognitive impairment; African Americans have lower levels of these biomarkers than white patients at the same levels of impairment. Diagnostic criteria were not being interpreted in the light of these racial disparities, perpetuating the cycle.

Today, Black people are two- to three-times more likely than white people to die in childbirth, often because their pain and symptoms are not taken seriously. The same is true for COVID-19 patients. For example, Indiana physician Susan Moore, who is Black, documented her experience with COVID-19 before ultimately dying from the disease. Her doctors did not believe she was having the symptoms or the amount of pain she was having. "You have to show proof that something is wrong with you for you to get the medicine," she said in a video. "I maintain, if I was white, I wouldn't have to go through that."

Medicine's racist history has repercussions today not only in poor health outcomes but also in hesitancy over medical treatments, including vaccination. A survey conducted in January 2021 by Katherine Kricorian and Karin Turner found that Black people were significantly more reluctant than white people to receive a COVID-19 vaccine but that encouragement from a medical professional of their own race would make them more likely to receive it. As of October 2021, a Kaiser Family Foundation report indicates that some progress has been made, but that Black people are slightly less likely than white people to be vaccinated in the 43 states for which the information was available.

Mathematical and Data Misuse

In her book *Weapons of Math Destruction*, Cathy O'Neil looks into the use and misuse of algorithms and machine learning in many aspects of society, including policing, sentencing, ad delivery, and school admissions. Data scientists and algorithm designers can and should disarm these models to reduce the negative impact they have on communities of color.

The answer is not to do away with algorithms in decision-making altogether. In medicine, for example, algorithms trained to classify images can detect disease at levels that the human eye cannot. But not all healthcare algorithms are currently used equitably, an issue that is personal for McDaniel. His grandmother needed a kidney transplant, and there is evidence that Black patients do not receive kidneys as often as equally sick white patients. She did receive a transplant, but McDaniel recalls dehumanizing conversations with physicians about his grandmother's condition and treatment. But those who design and deploy the algorithms must educate themselves on potential equity issues, from data collection to analysis to their use in algorithms. (See the chapter "Sources and Consequences of Algorithmic Bias" for more discussion about fairness in the design and deployment of algorithms.)

About the presenter

Darius McDaniel is a biostatistician who has worked in the research areas of neurology, cardiology and infectious diseases with focuses on Alzheimer's, tuberculosis, HIV, RSV and COVID-19. At the time of this workshop, he served as a statistician and epidemiologist with Leidos, a part of the COVID-19 response in U.S. Dialysis Centers at the Center for Disease Control and Prevention. He has a broad background in mathematics, statistics and informatics, with specific training and proficiency in both biostatistics and informatics. He has held statistical research positions at Emory University's School of Medicine and School of Public Health, the Center for Disease Control and Prevention and the University of Pennsylvania School of Nursing. He also serves as a biostatistics/informatics instructor in the School of Public Health at Emory. He received his M.S.P.H. in Public Health Informatics from Emory and his B.S. in Mathematics and Applied Statistics from Alabama A&M University. In Fall 2021, he began his Ph.D. in Biostatistics at Drexel University's School of Public Health.

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RACE AND CAUSALITY IN HEALTH DISPARITIES RESEARCH: TIME FOR A NECESSARY PARADIGM SHIFT

Based on a plenary talk by Emma Benn, Icahn School of Medicine at Mount Sinai

EVERY YEAR, RESEARCHERS PUBLISH HUNDREDS OF STUDIES that demonstrate race/ethnicity-related health disparities. In a typical such study, a researcher may set out to determine the existence or strength of, say, the relationship between race and hypertension. She constructs a statistical model considering factors that are associated with both race and hypertension that might hinder her effort to get an unbiased effect of race, such as diet, educational attainment, income, and access to healthcare. Then she conducts the study, adjusts for those factors using her statistical model, and determines whether there is a statistically significant difference in the risk of hypertension in Black and white populations, or estimates the effect of race on hypertension.

Circularity in Health Studies

In 1995, statistician Jay Kaufman and epidemiologist Richard Cooper published a paper pointing out the problem with the traditional approach to studying racial health disparities. They wrote, "when we set out to measure phenotypic traits in blacks and whites and find them 'different,' we conclude that blacks are 'different' because they are black." They note that studies of racial health disparities tend to assume, either implicitly or explicitly, that these disparities are due to genetic differences between the races. But race is a social construct only tenuously rooted in genetics. They write, "To believe that skin color has a unique association to outcomes ranging from IQ to blood pressure to prostate cancer by sheer chance is a questionable, if not preposterous, proposition."

Kaufman and Cooper coined the concept of circularity, "where efforts to understand social factors have the effect of emphasizing racial differences." That is, efforts to study and eventually reduce health disparities could potentially further stigmatize the groups that researchers intend to help in the first place. This becomes a circular slump: researchers hypothesize there are racial differences. They conduct studies that account for potential confounders and determine racial differences exist. Then they conclude that the racial difference is based on something inherent about the biology, genetics, or phenotype of race, even though race is socially constructed.

Can We Attribute a Causal Effect to Race?

In 2013, evolutionary biologist Alan Templeton wrote a paper addressing the question of whether different races really exist and concluded: "Humans are an amazingly diverse species, but this diversity is not due to a finite number of subtypes or races. Rather, the vast majority of human genetic diversity reflects local adaptations and, most of all, our individual uniqueness." His work, along with that of Kaufman and Cooper, supports the idea that researchers need to stop trying to assign a biological rationale for observed racial differences.

The fact that study after study finds race/ethnicity-related health differences and yet race has little biological meaning creates difficulty in attributing a causal effect to race. Paul Holland, a statistician working for the Educational Testing Service, raised this point in a 2003 paper. He argued that the measured effects of race do not have a causal interpretation. He believed that causes of outcomes should be experiences that individuals undergo, not attributes that they possess. Causal variables themselves must reflect the possibility of manipulation. Skin bleaching and plastic surgery aside, race is not mutable. Race therefore does not fit into an inferential framework, although it may play an important role in causal studies for descriptive reasons. Holland wrote: "In my opinion, RACE can play an important descriptive role in identifying important societal differences such as those in wealth, education, and health care. The attribution of cause to RACE as the producer of these differences is, to me, the most casual of causal talk and does not lead to useful action."

Racism vs Race as a Cause of Health Disparities

Race may play a role as an effect modifier — that is, an intervention or exposure may have a different effect on an outcome across racial/ethnic groups. Findings of that nature can help researchers delve deeper into the effects of discrimination and bias. But many studies stop at race and never delve into naming racism, rather than race, as a cause of health disparities. Unless the amount of melanin in skin can cause an outcome, caution should be used when ascribing a causal role to race rather than to racism or its downstream effects. Furthermore, the overarching goal of medical research is not to describe differences, it is to reduce disease and improve health. So a focus on race as a causal factor is less helpful than a focus on causal factors that can be changed.

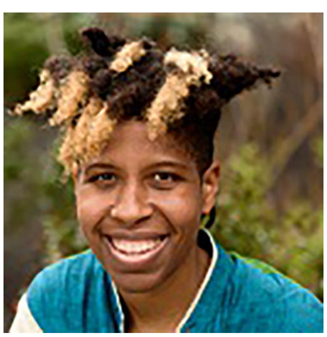
A Shift to More Nuanced Variables is Necessary

Changing the paradigm in studies of racial/ethnic health disparities from using race as an input to looking at more nuanced variables related to race is no trivial matter. Most data sets do not actually contain the information needed to reflect the constructs related to race and racism. Current studies that attempt to adjust for confounding factors related to race may, for example, correct for years of education. But due to de facto segregation in schools and the underfunding of majority-Black schools, years of education may not reflect educational attainment accurately. The relationship between education and health may also be confounded by factors such as air quality, lead exposure, crime, and access to healthy food. It is almost impossible for budget-constrained scientists to carry out studies that measure all of the possible factors that may contribute to the relationship between race and health.

If researchers are to move from describing racial differences to identifying mutable targets for intervention, then race cannot be the endpoint. In 2020, the convergence of the COVID-19 pandemic and widespread acknowledgement of the crisis of systemic racism in the U.S. motivated researchers to move beyond individual-level associations between race and health to studies that look at broader systems and structures. When researchers suspect that biological and genetic differences between races may indeed contribute to different health outcomes, genetic ancestry data would be a better variable to include than the more blunt tool of race, as a 2021 article by Akinyemi Oni-Orisan and coauthors argues. They write:

> “*We do not believe that ignoring race will reduce health disparities; such an approach is a form of naive "color blindness" that is more likely to perpetuate and potentially exacerbate disparities. Although ignoring race could improve equality (by leading to identical medical treatment for everyone), we believe that equity is necessary to address disparities. We urge our colleagues in medicine and science to refrain from haphazardly removing race from clinical algorithms and treatment guidelines in response to recent initiatives attempting to combat anti-Black racism. The ultimate goal, we believe, would be to replace race with genetic ancestry in an evidence-based manner. But we have not yet reached a point where genetic-ancestry data are readily available in routine care or where clinicians know what to do with these data. Until we do, ignoring race and thereby reverting to the United States' outdated system of health care, in which clinical research findings are generated in populations of European descent and extrapolated to the treatment of non-European populations, is neither equitable nor safe for those other populations.*”

About the presenter



Dr. Emma Benn is an Associate Professor in the Center for Biostatistics and Department of Population Health Science and Policy at the Icahn School of Medicine at Mount Sinai (ISMMS). She is also the Founding Director of the Center for Scientific Diversity and Associate Dean of Faculty Well-being and Development at ISMMS. She serves on the Faculty Diversity Council, Quality Leadership Council, HBCU Working Group, and Anti-racism Task Force at ISMMS.

Dr. Benn has collaborated on a variety of interdisciplinary research projects over the course of her career and is particularly interested in health disparities research. She developed a graduate-level course at ISMMS, Race and Causal Inference, designed to increase the methodologic rigor by which students and trainees investigate health disparities with a goal of finding effective causal targets for intervention. Dr. Benn, through her leadership of the Center for Scientific Diversity, is committed to increasing diversity, inclusion, and equitable advancement in (bio)statistics and the *biomedical research workforce, more broadly, as well as reducing racial/ethnic disparities in faculty promotion in academic medicine. Dr. Benn is the co-founder of the NHLBI-funded Biostatistics Epidemiology Summer Training (BEST) Diversity Program and a former co-Chair of the ENAR Fostering Diversity in Biostatistics Workshop. She currently serves on the American Statistical Association's Task Force on Antiracism and LGBTQ Advocacy Committee. She also serves as a mentor for the JSM Diversity Workshop and Mentoring Program and the Math Alliance.*

Dr. Benn was co-PI of the NIGMS-funded Applied Statistics in Biological Systems (ASIBS) Short Course aimed at increasing the statistical competency and research capacity of early-stage researchers nationwide. She currently is co-PI of the NHGRI-funded Clinical Research Education in Genome Science (CREiGS) Short Course aimed at exposing doctoral students, postdocs, and clinical and research faculty to computational tools in genome science in addition to effective strategies for engaging underserved communities in genomics research. Dr. Benn's contributions to diversity and inclusion in statistics and STEM have been celebrated by various organizations including Mathematically Gifted and Black, Graduate Women in Science, and the American Statistical Association.

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PART 4

RACIAL INEQUITIES IN MATHEMATICS EDUCATION

Mathematics education has been used to encourage conformity and a narrow vision of productivity; instead, we seek ways to create liberatory mathematics classrooms for all students..

TEACHING TO TRANSGRESS: MATHEMATICS AS THE PRACTICE OF FREEDOM

Based on a plenary talk by Brittany Mosby, Tennessee Higher Education Commission

> “*We learned early that our devotion to learning, to a life of the mind, was a counter-hegemonic act, a fundamental way to resist every strategy of white, racist colonization. Though they did not define or articulate these practices in theoretical terms, my teachers [Black women] were enacting a revolutionary pedagogy of resistance that was profoundly anticolonial.*”
>
> — bell hooks, *Teaching to Transgress*

> “*Education either functions as an instrument which is used to facilitate the integration of the younger generation into the logic of the present system and bring about conformity or it becomes the practice of freedom, the means by which men and women deal critically and creatively with reality and discover how to participate in the transformation of their world.*”
>
> — Paulo Freire, *Pedagogy of the Oppressed*

LIBERATORY EDUCATION IS EDUCATION THAT BECOMES “the practice of freedom,” to use Paolo Freire’s term. It can be transgressive and disruptive of the status quo in the classroom because its goal is to create beings who are able to change their society, not to create employable workers. Liberatory education is community- and dialogue-centered rather than centered on individualism. It is an anti-oppressive, humanizing pedagogy whose goal is self-actualization, not only of the students but also of their instructors. It engages with the world outside the classroom. To practice freedom within education requires tearing down the walls between the classroom and the outside world.

Assessing the Motivations for Increasing STEM Participation

Some people use nationalistic aims, whether military or economic, as motivation for increasing the participation of Black and Brown students in STEM fields. (See the chapter

"Seeking Racial Equity and Social Justice in Mathematics Teaching and Learning" for further discussion of mathematics education and its intersection with American nationalism.) Education as the practice of freedom, however, sits in direct opposition to those aims. Instead of focusing on creating people who can produce more and more economic value for the country, liberatory education focuses on creating holistic, lifelong learners who are able to question and challenge society and who have the tools necessary to do so.

Freire holds that there are two main types of pedagogy. The first, which he refers to as the banking method, views the educator as pouring knowledge into the empty vessels of the students. Freire sees this method as dehumanizing and oppressive. Students are seen as bringing nothing into the classroom; their only purpose is to receive and regurgitate information imparted to them by an educator. The other type of pedagogy is the problem-posing method, which Freire sees as humanizing and liberating. In this method, educators engage learners in dialogue with the expectation that learners bring to the classroom innate knowledge, intrinsic instincts and cultural knowledge about the subject. In this case, educators are open to learning from their students in addition to imparting their knowledge in a process that is more like a dialogue than a one-way street.

Teachers of STEM subjects, including math, often use the banking method, which should especially concern those who work with traditionally marginalized students. Those who wish to promote racial justice, social justice, and economic justice in the world should reframe mathematics and mathematics education as a practice of freedom by centering liberation and liberatory practices within their mathematics pedagogy.

Effect of a Liberatory Attitude towards STEM Education

In 2016, rapper B.o.B. tweeted about his skepticism that the Earth is round. His tweets, and people's replies, ballooned into diss tracks by B.o.B. and Neil deGrasse Tyson and dozens of hand-wringing articles about how science education is failing and society was becoming anti-scientific. Journalist Lizzie Wade, however, wrote an article for *The Atlantic* that took a different attitude:

> "*Take a look especially at the tweet that started it all: "The cities in the background are approx. 16 miles apart … where is the curve? please explain this." There's something touchingly genuine about this to me, some deep seated desire to work through confusion and toward truth. This isn't a man who never learned science, or who has some fundamentalist objection to examining empirical evidence about the world. This is a man who has looked at the world around him and decided that mainstream science isn't doing a good job at explaining what he sees. So he's collecting evidence, seeking out literature by well-versed "experts," and working out a better theory on his own.*"

Educators should see someone like B.o.B. not as an anti-science conspiracy theorist but as a curious person who has looked at the world around him and has decided that mainstream science is not doing a good job of explaining what he sees. He is seeking out literature by experts and trying to devise experiments to test his theory. Instead of immediately dismissing wrong answers, educators seeking to make mathematics the practice of freedom in their classrooms will view education as a process of discovery and support students wherever they are in that process.

A Framework for Liberatory Mathematics Pedagogy

Mathematics courses are frequently used as gatekeepers to STEM majors and careers. But opportunities to thrive in mathematics are disparately and disproportionately distributed, and current mathematics pedagogy is often anti-liberating. Therefore, the mathematics classroom is a logical and necessary place to begin working towards liberatory pedagogy.

Many educators think that teaching mathematics requires the banking method of pedagogy in a way the humanities do not, but mathematics was viewed as one of the humanities by ancient Greeks and Romans. It is a humanistic endeavor by nature, guided by the curiosity and interests of the humans who have developed the field over millennia. Today's mathematics curricula have a long list of learning objectives that must be met for students to be adequately prepared for the next course in their sequence. With this pressure to cover material, instructors may find it challenging to create space for liberation and the practice of freedom in the mathematics classroom. They may find it helpful to reflect on several questions related to their classroom environments:

- *Would you describe your approach as student-centered? If so, at what level do students participate in the creation of the course itself or its content?*
- *Are your end-of-course evaluations the only chance for students to provide feedback on the course?* Instructors should try to build in ways for students to share feedback about what is and is not working before the end of the course.
- *Do you have good answers for the questions, "Why do we need to know this?" and "When will we use this?"* Mathematics is frequently taught in isolation from other subjects, so students are often told that the math they are learning will be useful in some vague way in future math or science endeavors. Educators should have better answers than that for why students are taking their courses.
- *How do you acknowledge student cultures and foster community in the classroom?* Too often, students feel like they are competing against each other rather than working together in a community. Instructors who want to create liberatory classrooms should know how they are working against this dynamic.

Envisioning a Liberated Mathematics Classroom

There are several key elements of a liberated mathematics classroom.

- First, it *affirms students' existing cultural knowledge and mathematical intuition*. Teachers in this classroom are not overly concerned with pointing out where students are wrong, but rather with working in tandem with them to deepen their knowledge of a subject and work towards more complete understanding.

- A liberated mathematics classroom also *highlights the utility of mathematics as a language to understand problems across multiple fields*. When students ask, "When will I use this?" or "Why do I need to know this?" a teacher should be able to point to the utility of mathematics as one reason. Teachers should be incorporating problems from multiple other fields into their classrooms to demonstrate the versatility and power of the techniques students are learning.
- It encourages *metacognition and agency* in the learning process. When a teacher is in community with students, the students are able to take responsibility for their learning, which requires them to think about how they are learning and whether they are learning effectively. They should also be equipped to fix problems they have with learning.
- It *does not rely solely on lecturing* to impart knowledge. This assertion may be the point of biggest pushback on the framework of liberated education because mathematics educators feel so much pressure to get through a large number of learning objectives. Lectures may be necessary sometimes, but even in lectures, students should be engaged, active participants, not empty vessels.
- It *balances rote, single-skill practice with complex, contextualized, multi-step problems*. Fluidity with some rote skills is a helpful, and often necessary, step for students who want to apply mathematics to more complex problems, but even when focusing on rote skills, teachers can incorporate student discovery into the process. For example, instead of merely memorizing the multiplication table, students can find patterns and symmetry in the multiplication and figure out why. Then teachers can incorporate contextualized problems that use multiplication.
- Finally, a liberated mathematics classroom is *decolonized*. It decenters whiteness, maleness, and European-ness. It introduces students to non-Western foundations of mathematics, such as those in the Arab world and Africa, and includes the history of women's contributions to mathematics.

Altogether, a classroom where mathematics is the practice of freedom is a classroom that *centers mathematics as a process of discovery*. The mathematics classroom is a place of collaboration, among students, between students and instructors, and between the students and the content as co-creators of that content. Rather than being overly concerned with smacking down wrong ideas, it empowers students along their journeys. This process is radical and often uncomfortable, sometimes even to students, who may be used to being passive recipients of information rather than active co-creators.

Connecting the Classroom to the World

> "*Students, as they are increasingly posed with problems relating to themselves in the world and with the world, will feel increasingly challenged and obliged to respond to that challenge. Because they apprehend the challenges as interrelated to other problems within a total context, not as a*

theoretical question, the resulting comprehension tends to be increasingly critical and thus constantly less alienated.”

— Paulo Freire, *Pedagogy of the Oppressed*

Many people are familiar with memes about word problems that are full of pseudo-context that do not mean anything to students. “Susie has 52 watermelons and Johnny has 67. How many do they have together?” Why do they have so many watermelons in the first place? Educators can do better by using student-centered, possibly student-developed, problems in the classroom and incorporating culturally relevant concepts. Black students, and students from other communities of color and traditionally marginalized groups, have a cultural tradition of focusing on the community.

Mosby and her colleagues recently studied the participation of HBCU students in the field of public interest technology. Over half of students at HBCUs are majoring in social sciences: psychology, business, social work, economics, and related fields. The number jumps to two-thirds when nonperformance humanities such as religion and philosophy are included. One in four HBCU students are majoring in STEM fields. In a survey of HBCU students in Tennessee, the most commonly cited reason students gave for why they chose their major was that they wanted to help people and/or their community.

Teachers can honor their students’ desire to serve their communities by leaning heavily into interdisciplinary education and show them that mathematics can increase their ability to have a positive impact on society. Mathematics as the practice of freedom requires tearing down disciplinary silos and allowing students the freedom to lean into their cultural motivation for learning.

The current aim of many policy reforms, particularly with respect to HBCUs, is to promote STEM research and increase the number of STEM majors. Increasing the diversity of the technology sector is a worthwhile endeavor because a more diverse workforce should be better equipped to address issues of bias in algorithms and other matters of racial justice in technology. On the other hand, focusing only on STEM majors does a disservice to Black and Brown students who choose other majors. Mathematics is useful in nearly every other discipline, so mathematics educators should honor their students who have chosen other majors by empowering them to apply their mathematics knowledge to problems in their chosen fields. For example, statistics students who did social justice capstone projects at one school looked at topics including the effects of youth smoking, police-involved deaths, infant mortality and healthcare expenditures, discrimination in hiring, and the distribution of terrorism around the world.

Valuing Students with Contexts and Connections

Mathematics educators owe it to themselves and their students to step outside of the traditional barriers of the mathematics classroom and to think of it as a more interdisciplinary, more liberating, and more free place where learning can take place and students can make connections between themselves, the content and the world. Liberatory education is not just a set of tools, and it is not a how-to manual for teaching in the classroom. It is understanding and valuing students' cultures, cultural knowledge, and communities, and then finding meaningful ways to connect them back to what happens in the classroom. By doing that, teachers can equip them with the skills that they need to free themselves and to engage in the fundamental changing of their society.

About the presenter

Dr. Brittany L. Mosby is the Director of Historically Black Colleges and Universities (HBCU) Success programs and initiatives at the Tennessee Higher Education Commission, the coordinating board for the state's postsecondary education. As Director, Mosby develops policy, programming, and partnerships that foster student success, facilitate institutions' ability to fulfill their missions, and further the state's higher education attainment goals. Prior to joining the Tennessee Higher Education Commission, Dr. Mosby was a tenured associate professor of mathematics at Pellissippi State Community College. A third-generation Spelman College graduate, Dr. Mosby is a direct beneficiary of the intergenerational social mobility made possible by HBCUs and is passionate about preserving the legacy and promoting the enduring sustainability of the unparalleled HBCU experience. Mosby also received a Master of Science in mathematics from Carnegie Mellon University, and a Doctor of Education in higher education policy and leadership as a Peabody Honors Scholar at Vanderbilt University.

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RETHINKING EQUITY AND INCLUSION AS RACIAL JUSTICE MODELS IN MATHEMATICS (EDUCATION)

Based on a plenary talk by Danny Martin, University of Illinois at Chicago

DIVERSITY, EQUITY, AND INCLUSION (DEI) EFFORTS have often fallen short of affirming the humanity of some students, particularly Black students; instead, they often commodify students for marketing purposes. Such efforts have received a significant amount of attention in the past few years, especially in university and college contexts, and this chapter interrogates DEI efforts against the persistent realities of white supremacy and anti-Blackness and reframes the idea of justice in mathematics education for Black learners.

The following questions are central to the issues at hand, and Danny Martin invited his audience to reflect on them as the presentation began:

- What do race and racism have to do with mathematics education?
- How can I begin to think differently about mathematics education so that I can increase my understanding of racism in mathematics education?
- Beyond equity and inclusion, what are some justice projects that can respond to the material realities, needs, and desires of Black people inside and outside of mathematics education?

Understanding Black Learners

Martin's research focuses on issues of race and identity, particularly the experiences of Black learners, including their self-constructions of themselves as learners, doers, and creators of mathematics in school and non-school contexts. In Martin's research (2006), self-construction must be co-construction because everyone must contend with what others say about them. Negotiating others' assertions about the self is part of how one sees oneself. Martin's work is guided by two questions. First, what does it mean to be a learner and doer of mathematics in the context of being Black? Second, what does it mean to be Black in the context of learning and doing mathematics? These are two superficially similar but in fact different empirical, existential questions.

Two key concepts come into play when addressing these questions:

- *Mathematics identity* encompasses a person's self-understandings as well as how they are constructed by others in the context of doing mathematics. These identities can be expressed in narrative form as a negotiation between one's own assertions and the external assertions of others. They are always under construction.
- *Mathematics socialization* includes experiences that individuals and groups have within a variety of contexts such as school, family, peer groups, and workplaces that facilitate, legitimize, or inhibit meaningful participation in mathematics.

As Martin and other scholars have attended to these ideas over the past 20 years, conceptual and methodological space has opened up for non-deficit, phenomenological perspectives of Black (and other) learners, focusing on socialization, identity, resilience, agency, and success, including what success means to an individual mathematics learner. Despite this shift, many Black learners continue to experience dehumanizing and violent forms of mathematics education. These dehumanizing and violent experiences are rooted in white supremacy and anti-Blackness. Violence is not necessarily physical but includes epistemological violence (knowledge and beliefs about Black learners), symbolic violence, and systemic violence.

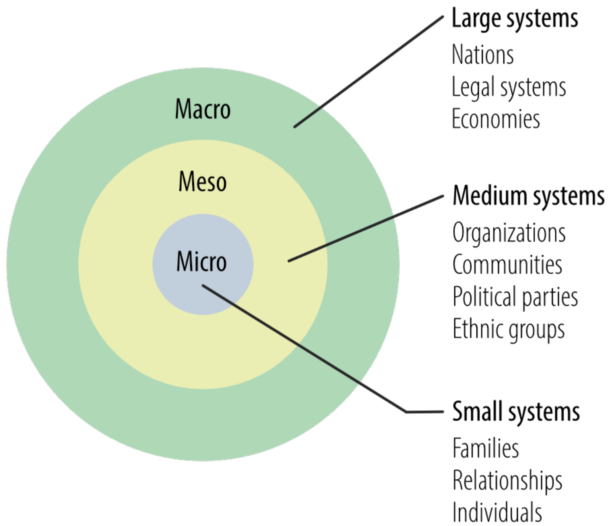
Critically (re)Framing Equity, Inclusion, and Mathematics Education

Some proponents of DEI initiatives may recognize that white supremacy, anti-Blackness, racial capitalism (benefiting economically from the racial identity of others), xenophobia, and other forces exist and that people of color have to contend with them. These proponents try to create DEI initiatives that reform education or produce incremental change, often drawing on white benevolence. The hope is that DEI and incrementalism will overtake the societal forces of white supremacy and anti-Blackness. In reality, it is not a fair fight. Those who study the history of racial progress or lack thereof in the U.S. understand that these are monumental forces that resist all efforts to reform them (Anderson, 2016).

White supremacy and anti-Blackness are adaptive, self-correcting systems. They function as multilevel projects of exclusion that adapt and self-correct across levels. Inclusion and equity at one level does not preclude exclusion and inequity at other levels. Equity and inclusion initiatives and practices are often accommodated, co-opted, or simply absorbed by white supremacy and anti-Blackness, resulting in the maintenance of those systems, retrenchment, or incremental advancement masquerading as racial justice. These DEI practices can impede, rather than support, Black learners' advancement, liberation, and flourishing in mathematics education and beyond because they promote less oppression while failing to change or dismantle the fundamental character of these systems.

Mathematicians and educators who are teaching in classrooms will generally be working at the micro-level. But no matter what progress they make there, if they are only focused on the micro-level and ignore the meso- and macro-levels, they will impede their own efforts to work towards the liberatory aspects of mathematics education. Mathematicians and educators must think about and understand the projects of exclusion that exist at every level.

Multilevel projects of exclusion



The different levels on which anti-Blackness and white supremacy work to resist and/or accommodate DEI initiatives. Inclusion at one level does not preclude exclusion at another.

Given the above characterizations of white supremacy and anti-Blackness more broadly, what do race and racism have to do with mathematics education? There is some general agreement among mathematics educators that mathematical engagement, learning, and participation are both cognitive and cultural activities. But they are also racialized; that is, they are experiences in which socially and personally constructed meanings for race emerge as salient interactional experiences related to mathematics (Martin, 2006, 2013, 2019). Several lines of research — how students experience racial-mathematical socialization outside of the classroom context, how they experience that socialization in the classroom, and how teachers are agents of racial-mathematical socialization in the classroom — investigate and confirm this fact.

Racial-Mathematical Socialization

Niral Shah at the University of Washington has shown that when students are working together in small groups, they can spontaneously engage in extra-mathematical discourse focused on race. For example, a Black student could be sitting around the table with other Black students or in a racially heterogeneous group. When the Black student does something mathematical that is considered correct, another student may comment, "Oh, you must have some Asian in you." Ebony McGee of Vanderbilt University has shown that if a Black student declares a major in a heavily quantitative field, someone might say, "You'd better get an Asian roommate." There are two circulating discourses emerging in those moments: the assumed lack of mathematical ability of Black learners, and the model minority myth for Asian learners.

Racial microaggressions — verbal or behavioral indignities, whether intentional or unintentional, that communicate hostile, derogatory, or negative messages about race — have received more attention from educational researchers in recent years. They come in different forms and represent slights to one's sense of self. Martin shared a snippet from a conversation he had with an African American woman in her 30s who was returning to school at a community college after having received a bachelor's degree. She said, "I was in high school, it was in ninth grade, and I had a teacher, I'll never forget his name… I'll never forget him. I had a C in algebra the first semester, I had a D the second and he told me, 'This as far as you can go in math.' I never took another math class from that point on until I came to Cal State Hayward." Martin replied, "And you bought into it?" She said, "Why wouldn't I? I am a 15-year-old Black student and my teacher, whatever race he is, White male, and you know he's telling me, 'This is it, you've gone as far as you can go.' He didn't have any patience. It wasn't like 'Maybe you need to look into another class, maybe another teacher, maybe another method of instruction,' You know, [math] wasn't stressed. It just wasn't." Such interactions are numerous, typical, and harmful. They also illustrate the way race influences mathematics education.

Structural Manifestations of Race

Beyond the experiential realm, race manifests in structural ways. Mathematics education, research, policy, and practice contexts can be characterized as instantiations of "white institutional space." Such spaces are characterized by the exclusion of non-whites from positions of power in various institutions, which results in the accumulation of white economic and political power; the development of a white frame (rationality) that organizes the logic of these institutions and normalizes white racial superiority; the historical construction of a curricular model based on the thinking of white elites; and the assertion of knowledge and knowledge production as neutral and unconnected to power relations (Moore 2008).

Martin and other scholars have incorporated that definition into their work on mathematical education. Martin has also suggested and shown that mathematics education research, practice, and policy as social practices are deeply involved in the production and reproduction of a racial hierarchy of mathematics ability that places white and Asian learners at the top and Black, Latinx, and Indigenous learners at the bottom. Historically, mathematics education reforms have been aligned with and put in service to a number of political projects promoting white supremacy, anti-Blackness, racial capitalism, nationalism, xenophobia, militarism, neoliberalism, etc. Merely bringing people of color into white institutional spaces does not change the way they function to uphold white supremacy. The spaces will pride themselves on inclusion, but the fundamental character of the spaces does not change without deep structural change.

Math is not Apolitical

These experiential and structural aspects of mathematics education show that those who wish to position mathematics and mathematics education as apolitical or outside the scope of race and racism are misinformed. Mathematics education is a social practice. Within this social practice, hierarchies of race, gender, and class among its participants are reproduced (and sometimes disrupted). Within K-12 classrooms, notions of smartness and competence are not inherent traits but are often socially negotiated. As such, smartness is not only an identity but an opportunity and identity that is made available to some students, in some conditions, and denied to other students in other kinds of conditions. Mathematics educators should ask themselves how those conditions come to be, who is orchestrating them, and how the conditions reflect and reify white supremacy, anti-Blackness, and the dehumanization of Black people.

Education is a political project: U.S. mathematics education reform movements in their political contexts

<table>
<tr><th>Reform Contexts</th><th>Reform Discourses
(Appeals to Inclusion)</th><th>Racial and Political Contexts</th></tr>
<tr><td>New Math (circa 1957)</td><td>Best and the Brightest
National Security</td><td>Jim Crow racism, legalized segregation, New right racial project, heightened nationalism, Cold War politics, Civil rights movement</td></tr>
<tr><td>Curriculum and Evaluation Standards (NCTM, 1989)</td><td>Mathematics for All</td><td rowspan="3">Post-civil rights era, neo-conservatism, Reagan–Clinton-era neo-liberalism, New Jim Crow, mass incarceration, neo-liberal racism (color evasiveness)</td></tr>
<tr><td>Everybody Counts (NRC, 1989)</td><td>Mathematics for All
Economic Necessity
Global Competitiveness
Human Resource Development</td></tr>
<tr><td>Principles and Standards for School Mathematics (NCTM, 2000)</td><td>Mathematics for All</td></tr>
<tr><td>Final Report of National Mathematics Advisory Panel (USDOE, 2008)</td><td>Mathematics for All
Economic Competitiveness
National Security/Antiterrorism</td><td>Bush-era neo-conservatism, neo-liberal racism, Post-9/11 nationalism , New Jim Crow, Far right racial project, No Child Left Behind Act</td></tr>
<tr><td rowspan="2">Common Core State Standards — Mathematics (2009)
U.S. Education Reform and National Security (2012)
Principles to Action (2014), California, Oregon, Seattle</td><td rowspan="2">Mathematics for All
College and Career Readiness
National Security/Antiterrorism</td><td>Obama-era post racialism, neo-liberal racism, New Jim Crow, Black Lives Matter, Far right racial project</td></tr>
<tr><td>Corrective racial politics of Trump election and presidency, MAGA, white rage, COVID, white supremacist insurrection, Biden presidency</td></tr>
</table>

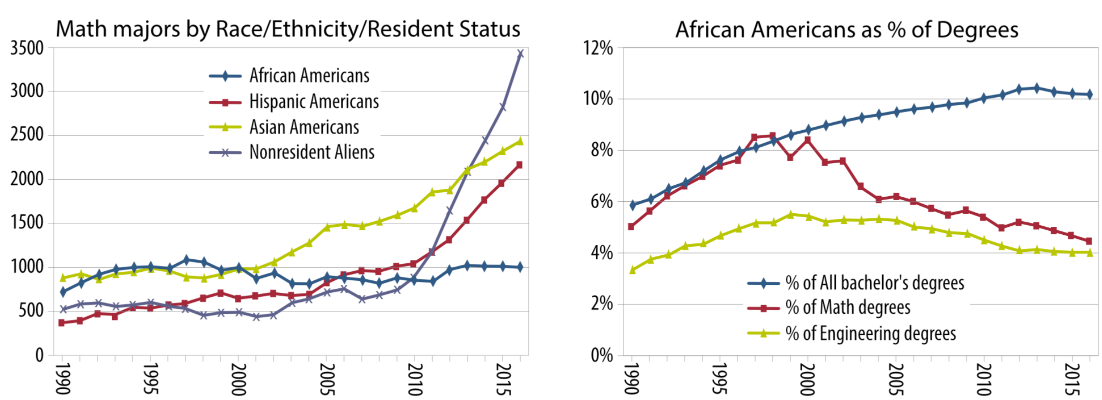
Consider a 2014 study that looked at the early childhood longitudinal study (ECLS-K) dataset for the kindergarten class of 1998–99, which followed the then-kindergarteners through eighth grade. Researchers were interested in the placement profiles of students around the 7th–8th grade transition, which is a pivotal point in the curricular space of school mathematics. There are typically two important factors in a student's mathematics placement: teacher recommendations and prior performance. Researchers were interested in studying the impact of teacher evaluation of student performance versus students' demonstrated performance on algebra placement. They found that teacher evaluations of student performance played a greater role for Black students than for white students and that they had an adverse effect. Black students had reduced odds of being placed in algebra, even after controlling for math performance.

Studies like this show that not everyone will be recognized equally for their hard work and achievement in mathematics. The authors conclude, "Black students confront an untenable impediment and that their Blackness, (or, as we suggest here, the teachers' implicit responses to these students' Blackness) serves…as an invisible…obstacle to gaining access to higher level mathematics courses, irrespective of their demonstrated performance." Mathematics education is not only a social practice; it is also a political project.

The table on the previous page catalogs several U.S. mathematics education reform movements from the late 20th and early 21st centuries and places them in the political context of the time. The table shows, for example, that mathematics education has been seen as vital to national security and economic competitiveness and has always occurred in the racial and political context of the time. The "New Math" reform era, which focused on "the best and the brightest," assumed those best and brightest were white men.

The "Mathematics for All" and inclusion rhetoric took off in the late 1980s, but it has had little effect on the rates of Black majors at universities, as shown in the charts below, where the blue line shows that the number of Black math majors was essentially flat from 1990–2016. In fact, because of population growth and growth in college attendance, it may also be true that the proportion of math degrees issued to African Americans may have also decreased, as shown in the figure on the right.

What has been the Effect of "Mathematics for All" for Black Inclusion in Mathematics?



Source: Bressoud (2018)

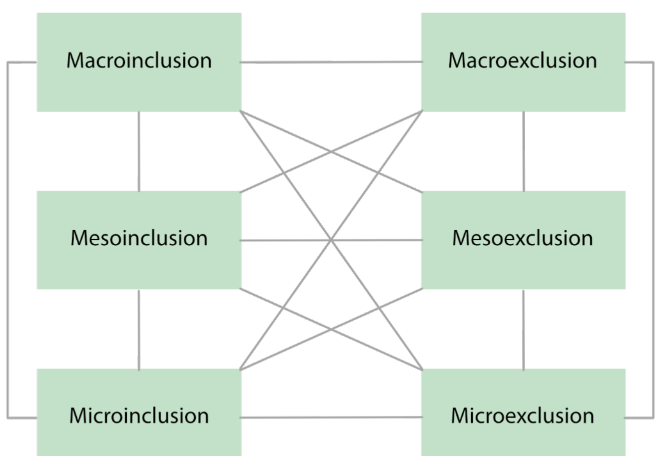
Models of Inclusion/Exclusion

Studies such as the one cited above regarding algebra placement, along with trend data such as that in the two charts, help bolster the argument that equity and inclusion efforts are mitigated by other forces. What makes this mitigation a challenge to equity and inclusion? Some mathematics education researchers (Faustino et al, 2018) have been working on developing frameworks describing macro and micro-inclusion and exclusion in mathematics education. Faustino and colleagues characterize microexclusions as "violence, not only physically but emotionally as well. Certain discourses can be violent, for instance when they provide a stigmatization of certain groups of students. Political structures can be violent by preventing certain groups of children and young people from access to adequate education. Economic structures can be violent by keeping groups of people in poverty. Violent microexclusions can be acted out in the mathematics classroom."

Macroexclusion obviously leads to microexclusion. Many people would like to believe that macroinclusion — rhetoric around diversity and inclusion on a societal level — leads to microinclusion — a sense of belonging and value in individual classrooms and workplaces. But recent analysis by Faustino et al. shows that macroinclusion can actually lead to microexclusion.

Martin has extended the model above to consider micro, meso, and macro levels of exclusion, since they are particularly relevant to discussions of Black learners. He draws on the work of Carol Anderson, for example, and her concept of white rage, where she documents that Black advancement is often the precursor to white rage and racial resentment. Some individual whites have supported efforts at true inclusion, but every landmark of advancement has been met with some kind of retrenchment effort. Martin's expanded model suggests that opening the proverbial gates at one level in mathematics education does not preclude closing the gates at another level (for example, increasing access to algebra in theory but denying high-scoring Black students in practice). This is the insidiousness of white supremacy and anti-Blackness.

Multilevel projects of inclusion and exclusion



Martin (2021). Adapted from Faustino, Moura, Gomes da Silva, Muzinatti, & Skovsmose (2017).

Martin's expanded model applies to other projects of inclusion/exclusion. For example, immigration is inclusive in spirit, but some immigrant groups are more valued than others due to their racial and ethnic identities. Not everyone gets to come to the U.S., and different countries have different quotas. Thus immigration as it has been implemented in the U.S. is both inclusive and exclusive. Desegregation was supposed to open doors for Black children, but it limited opportunities for Black teachers, which has had ripple effects to the present day. Hence desegregation as it was implemented in the U.S. is both inclusive and exclusive.

Implications of a Critical (re)Framing of Equity, Inclusion, and Mathematics Education

Given the history of exclusion in white institutional spaces, how do mathematicians and math educators begin to imagine Black futures in mathematics in Black spaces, where the hopes and desires of Black people are centered? In their 2018 book *Toward What Justice?*, Tuck and Yang proposed a model of refusal that is relevant in discussions of white supremacy and anti-Blackness in mathematics education, as well in discussions of DEI models that promote incremental justice:

> “*We refuse:*
>
> - *Justice projects which require us to prove our humanity or worth*
> - *Justice projects which require us to frontload a lot of learning or consciousness-raising*
> - *Justice projects which require us to appeal to the people who abuse us*
> - *Justice projects which require us to gather an audience of white settlers who are presumed to have agency*
> - *Justice projects that presume compromise as the main avenue for achieving solidarity*”

Justice-minded mathematicians and educators should refuse justice projects that do not alleviate Black suffering (Dumas, 2014). Equity and inclusion are not going to alleviate black suffering on their own. Instead, they often aim to create small, incremental changes that do not disrupt the status quo.

Liberatory mathematics education. As an alternative to inclusion into white spaces and incremental change, Martin and his colleagues (Martin, Price, & Moore, 2019) propose a Black liberatory mathematics education: the framing and practice of mathematics education that allows Black people to flourish in their humanity and brilliance, unfettered by whiteness, white supremacy, and anti-Blackness.

Black liberatory mathematics education starts from the axiom that Black children are brilliant. It prioritizes liberation over rhetoric of integration and freedom. This form of mathematics education is skeptical of liberal notions of inclusion and equity, appeals to democracy and citizenship, neoliberal multiculturalism, and refuses all forms of violence against Black learners and others. The “freedom” to participate, integrate, and be included into anti-Black spaces characterized by various forms of violence is not freedom. This framing does not suggest Black people should not learn mathematics. Rather, it foregrounds liberatory goals and collective advancement. In 1970, S. E. Anderson wrote, “[learning mathematics] is necessary not because American capitalism’s advanced forms of technology require this background, but because Black Liberation Struggle against the American racist-capitalist system requires knowledge of [mathematics and] 20th century technology."

In terms of practice, a Black liberatory mathematics education is designed and directed first and foremost by liberation-seeking Black people including parents, caregivers, community members, Black teachers, and Black students. In his book *Black Power: The Politics of Liberation in America*, Kwame Ture wrote, "Only black people can convey the revolutionary idea—and it is a revolutionary idea—that black people are able to do things themselves. Only they can help create in the community an aroused and continuing black consciousness that will provide the basis for political strength. In the past, white allies have often furthered white supremacy without the whites involved realizing it, or even wanting to do so. Black people must come together and do things for themselves."

Black people and white people see the world differently because society ensures that their experiences will be different. In 2002 and 2003, as part of an oral history project undertaken by the University of California, Berkeley, Nadine Wilmot, who is white, interviewed David Blackwell, one of the first African Americans to receive a Ph.D. in mathematics. This is an excerpt from their conversation:

> *Wilmot:* *In the beginning of this interview, I remember I was asking you these questions. I was like, "What was it like for you as an African American in graduate school?" And after the interview you said, "Nadine, this question you're asking." You said, "Nadine, I'm always black. Everything I do is black."… You had this very interesting tape way of saying it, which I thought was very important. Partially because it just kind of illuminated the difference in the way you and I talked about race, but also because it was a…I just thought it was just a very important statement.*
>
> *Blackwell:* *"Well, it's just so obvious. It colors everything you say, do and think."*

About the presenter

Dr. Danny Bernard Martin is Professor in the Department of Curriculum and Instruction and the Department of Mathematics, Statistics, and Computer Science at the University of Illinois at Chicago. Dr. Martin's research has focused primarily on understanding the salience of race and identity in Black learners' mathematical experiences. Prior to coming to UIC in 2004, Dr. Martin was a faculty member in mathematics for 14 years at a Bay Area community college.

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Defining Terms: A Shared Understanding

A shared understanding of certain terms is necessary for productive conversation about issues around race, racism, and education. This section clarifies important terms, often citing definitions from a variety of sources in the literature.

Racial literacy

"Racial literacy provides a vocabulary and a set of tools to help individuals interpret how race and racism interact with education, and it promotes engagement of racial themes into curricular and pedagogical projects and approaches." (Flynn et al, 2020). Racial literacy requires people to move beyond viewing racism as an individual trait (often in the knee-jerk response of "I'm not racist") to understanding it as a societal force. Caffrey et al (2018) write that it involves "(1) understanding the intersections of power and race, (2) being able to locate and analyze racial systems, (3) possessing the grammar and vocabularies associated with racial discourse, such as *White supremacy*, *anti-Blackness*, *racialization*, *racial identity*, and *intersectionality*, while (4) differentiating among terms such as *ethnicity*, *nationality*, *discrimination*, *prejudice*, and *stereotyping*."

Race

"Race is a sociohistorical concept. Racial categories and the meaning of race are given concrete expression by the specific social relations and historical context in which they are embedded…The meaning of race is defined and contested throughout society, in both collective action and personal practice. In the process, racial categories themselves are formed, transformed, destroyed, and re-formed. At the micro level, race is a matter of individuality and the formation of identity. At the macro level, race is a matter of the formation of social, political, ideological, and economic structures" (Omi and Winant, 1986). Race is not a biological fact but a socially constructed concept with material consequences. Categories of race are contested across time and geography. Race in contemporary times is different from race in the 1950s or 1850s. The category white, for example, has been adjudicated and litigated and re-litigated in courts (Lopez, 1996). Who gets to be white in the U.S. has been a longstanding, ongoing question because when society decides who is white, those who are not considered white are stratified in particular ways.

Racism

"Racism is a structure because racial dominance exists in and is reproduced by the system through the formulation and applications of rules, laws, and regulations and through access to and the allocation of resources. Finally, racism is a process because structures and ideologies do not exist outside the everyday practices through which they are created and confirmed. These practices both adapt to and themselves contribute to changing social, economic, and political conditions in society" (Essed, 2002). Racism is dual-pronged. It is not just individual inclinations, prejudices, discriminations, beliefs, or ideologies. It also has to do with systems and structures that work together in concert to create common understandings of where people are situated in society. Racism produces race. Edouardo Bonilla-Silva (1997) writes, "Actors in racial positions do not occupy those positions because they are of X or Y race, but because X or Y has been socially defined as race."

White supremacy

White supremacy is "a political, economic, and cultural system in which whites overwhelmingly control power and material resources, conscious and unconscious ideas of white superiority and entitlement are widespread, and relations of white dominance and non-white subordination are daily reenacted across a broad array of institutional and social settings." (Ansley, 1997).

Anti-Blackness

Anti-Blackness is an interlocking paradigm of institutions, attitudes, practices, and behaviors that work to dehumanize and oppress Black people in order to benefit non-Black people, and specifically, to benefit and maintain white supremacy. "When social systems are racialized by white supremacy, whiteness becomes the default of humanity and Blackness is stripped of its humanity, becoming a commodity, becoming disposable," as the Black Liberation Collective website describes it. Scholars who study anti-Blackness are concerned with the antagonism between Blackness and humanity. Black people are regarded as being outside the scope of humanity, which permits atrocities such as the murder of George Floyd and killing of Breonna Taylor by police officers. Black people are not regarded as human, so the pain they experience is not acknowledged as human pain.

Intersectionality

"Fundamentally, race, class, and gender are intersecting categories of experience that affect all aspects of human life; thus, they simultaneously structure the experiences of all people in this society. As any moment, race, class, or gender may feel more salient or meaningful in a given person's life, but they are overlapping and cumulative in their effects…yet race, class, and gender intersect with other categories of experience, such as sexuality, ethnicity, age, ability, religion, and nationality" (Anderson and Collins, 2012).

Racial justice

Different people and groups have defined this term differently. The National Education Association describes it as, "The systematic fair treatment of people of all races, resulting in equitable opportunities and outcomes for all. … Racial justice — or racial equity — goes beyond 'anti-racism.' It is not just the absence of discrimination and inequities, but also the presence of deliberate systems and supports to achieve and sustain racial equity through proactive and preventative measures" (see www.nea.org/resource-library/nea-and-racial-justice-education). The MSRI Workshop on Mathematics and Racial Justice defines it as, "Racial justice is the result of intentional, active, and sustained anti-racist practices that identify and dismantle racist structures and policies that operate to oppress, disenfranchise, harm, and devalue Black people. This workshop will bring together mathematicians, statisticians, computer scientists, and STEM educators as well as members of the general public interested in using the tools of these disciplines to critically examine and eradicate racial disparities in society."

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PART 5

FAIR DIVISION, ALLOCATION, AND REPRESENTATION

Mathematics can play a pivotal role in addressing resource allocation and political representation.

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ELECTIONS AND REPRESENTATION

Based on a plenary talk by Michael Jones, Mathematical Reviews

IN THE VERY SIMPLEST CASE, when voters are choosing between precisely two options, it is not hard to determine how an election should work. The option preferred by a majority of voters should win. But the real world is messier than that. Many elections have more than two candidates, and different voters' preferences may lead to seemingly contradictory outcomes. In the case of the U.S., the Electoral College further complicates voting for President.

Mathematically Defining an Election

Mathematically, an election can be defined as the following: There are n candidates, C_i, for i ranging from 1 to n. Each voter has preferences of the candidates that lead to a linear ordering on candidates. We define a collection of voters' preferences to be a profile P. An election procedure or social welfare function f maps the set of all profiles P to a partial order over the candidates and chooses the top candidate (or candidates for an election that will result in multiple winners) or ranks the candidates in order if a full ordering is desired.

The classical approach in social choice theory is to think about how an election procedure satisfies different types of axioms or properties. The following axioms are one possible set for a reasonable election procedure.

1. *Resoluteness*: Each set of preferences results in a unique winner.
2. *Anonymity*: Voters are treated equally.
3. *Neutrality*: Candidates are treated equally. (If every voter's preferences are reversed, then the group preference is reversed.)
4. *Positive responsiveness*: Gaining support does not hurt a candidate. If a voter changes their vote from a losing candidate to the winning candidate and all other votes remain fixed, then the winning candidate still wins.

In 1952, Kenneth May proved that for an odd number of voters and exactly two alternatives, an election procedure satisfies these four axioms if and only if it is simple majority rule. But as soon as an election has three or more alternatives, that is no longer the case. This conundrum was the focus of much research in the 20th century.

Another important set of potential election axioms are the following:

1. *Pareto efficiency*: If all voters prefer *a* to *b*, then the outcome of the social welfare function must rank *a* above *b*.
2. *Independence of irrelevant alternatives*: The ranking of a and b by the social welfare function only depends on each voter's relative ordering of these two outcomes and not on how they rank another possible outcome.
3. *Non-dictatorship*: No one voter determines the social welfare ordering.

In 1951, Kenneth Arrow proved a theorem that is probably the most famous result in social choice theory: when voters have three or more alternatives, there is no social welfare function that satisfies Pareto efficiency, independence of irrelevant alternatives, and non-dictatorship. This theorem is often called Arrow's impossibility theorem. Similar theorems exist for many other choices of possible election system axioms.

One goal of mechanism design of election systems is for voters to reveal their true preferences rather than voting strategically. For elections of three or more candidates, however, Allan Gibbard and Mark Allen Satterthwaite independently proved that the only election procedure that is non-manipulable is a dictatorship. In this context, manipulation refers to what is sometimes called "strategic voting": voters will misrepresent their true preference in order to get a preferred outcome. For example, in a plurality voting system, a voter may vote for candidate B instead of their favorite, candidate A, because they think candidate B is more likely to defeat candidate C, their least favorite candidate.

With a "perfect" voting system therefore off the table, how can social choice theorists determine election procedures that achieve various societal goals?

Election Procedures for Three or More Candidates

Plurality voting. Plurality voting is the election procedure that is most familiar to voters in the U.S. Each voter selects one candidate, and the candidate with the most votes win. This procedure has a major drawback, which is that a candidate can be the least preferred by a majority of the electorate and still win. For example, consider an election between candidates C_1, C_2, and C_3 in which one third of the electorate plus a small number ranks the candidates in the order C_1, C_2, and C_3; one third of the electorate ranks them C_3, C_2, and C_1; and one third minus a small number ranks them C_2, C_3, and C_1. Candidate C_1 will win despite being the least favorite of almost two thirds of the voters. Plurality voting tends to encourage a two-party system, in which many people find their views not well-represented by either party.

Approval voting. Approval voting is another election procedure. In this method, each voter approves of some subset of candidates and the candidate with the most approval votes wins. This procedure is sometimes used by organizations (including the Mathematical Association of America and American Mathematical Society, two of the largest professional mathematics organizations in the U.S.) to choose committee members. It was also used in municipal elections in Fargo, ND, in June 2020. One problem with approval voting is that knowing a voter's preferences is not sufficient to determine how they will vote. Their cutoff for "approval" of a candidate is not clear from knowing their rankings. Approval voting also fails to measure the intensity of preferences for a particular candidate.

Borda count. The Borda count is used less often in political elections and more often in sports, where it may be used to rank players or teams. Named for French mathematician and engineer Jean-Charles de Borda, in this method, each voter will rank all n candidates. A voter's favorite candidate receives $n-1$ points, their second favorite receives $n-2$, and so on, until their least favorite candidate receives 0 points. The candidate with the largest sum of points wins. As is the case with several other election procedures, the Borda count requires every voter to rank all of the candidates, which can be an imposition when there are many candidates. But on the positive side, it uses all information about voter preferences to determine the winner, not just voters' top few candidates. The Borda count is susceptible to strategic voting. A voter may choose to rank their favorite candidate's most promising rival last, no matter their true feelings about the candidate, to improve their chance of electing their favorite. (Informed of the possibility of strategic voting, Borda said, "My scheme is only intended for honest men.") But research by Don Saari showed that the Borda count minimizes the likelihood of manipulation when there are three candidates.

Ranked choice voting. Ranked choice voting may be the alternative election procedure with the most momentum in the U.S. at present. It has been adopted in several locations, including New York City and Oakland. In ranked choice voting, also known as "single transferable vote," voters rank candidates in order of preference. If one candidate wins a majority of first place votes, they win the election. Otherwise, the candidate with the fewest first place votes is eliminated and their votes are transferred to voters' second place choices. The process is repeated until one candidate has a majority of votes.

Proponents of ranked choice voting point to its potential for reducing the vitriol of political campaigns as one of its strengths. Candidates can tout their points of agreement with other candidates to campaign for second place votes. Ranked choice voting has its drawbacks as well. In some elections, some ballots may be exhausted before a winner is determined because some voters do not rank all the candidates. Those voters, then, are not having their full preferences considered. It also fails to satisfy the axiom of monotonicity, the condition that improving the ranking of a candidate on some ballots while leaving the relative positions of other candidates in place cannot hurt the first candidate.

For example, suppose 10 voters prefer $A > B > C$; 10 voters prefer $B > C > A$; and 9 voters prefer $C > A > B$. Under ranked choice voting, no candidate receives a majority of the first-place votes, so candidate C is eliminated. The nine voters who prefer $C > A > B$ have their votes transferred to candidate A, who wins the pairwise election 19 to 10 over candidate B. If two voters who prefer $B > C > A$ switch their rankings to $A > B > C$ and the election was re-run, then the outcome should not hurt candidate A if ranked choice voting satisfies monotonicity. However, in this case, B now has only 8 first-place votes and is eliminated. The 8 votes are transferred to candidate C who wins the election over A: 17 to 12.

Case Studies of Ranked Choice Voting

Ranked choice voting started in the 1850s in Europe, where it was used in multi-winner elections. In the 1870s, William Ware adapted it to single-winner situations, but it was not used in the U.S. until 1915, when Ashtabula, OH, used it for city council elections. After some success in the early 1900s, the 1940s and '50s saw reforms back to simple plurality voting. By 1962, the only place still using it was Cambridge, MA, where it was used for their city council and school board elections. In recent decades, the method has seen a resurgence. It is currently used for some elections in several cities and states in the U.S., including Oakland, CA. Below is a table illustrating the 2010 Oakland mayoral election, the first election in which the city used ranked choice voting.

2010 Oakland Mayoral Election

Candidate	Round 1	Round 2	Round 3	Round 4	Round 5	Round 6	Round 7	Round 8	Round 9	Round 10
Jean Quan	29,266	29,299	29,391	29,514	29,645	30,500	30,884	31,655	35,033	**53,897**
Don Perata	**40,342**	**40,374**	**40,455**	**40,606**	**40,728**	**40,814**	**41,364**	**42,188**	**45,465**	51,872
Rebecca Kaplan	25,813	25,831	25,890	26,026	26,117	26,496	26,831	27,475	32,719	
Joe Tuman	14,347	14,357	14,471	14,552	14,780	14,949	15,202	15,462		
Marcie Hodge	2,994	2,999	3,033	3,155	3,200	3,250	3,625			
Terence Candell	2,315	2,316	2,386	2,497	2,613	2,680				
Don MacLeay	1,630	1,636	1,677	1,719	1,852					
Greg Harland	966	968	1,059	1,087						
Larry Lionel "LL" Young Jr.	933	939	976							
Arnold Fields	733	738								
Write-in	268									
Continuing votes	119,607	119,457	119,338	119,156	118,935	118,689	117,906	116,780	113,217	105,769
Exhausted ballots	0	149	262	435	376	893	1,655	2,766	6,284	13,667

Don Perata, the plurality winner, was leading the count through nine rounds of "instant runoffs," but in round 10, Jean Quan pulled ahead and won. One of the reasons for her victory was that Quan actively encouraged supporters of other candidates to rank her in second place. She catered to voters who did not see eye-to-eye with her on every issue but shared some values and policy positions.

In 2017, the Justice Department filed a lawsuit against Eastpointe, MI, claiming that their election procedure was discriminatory. The suit was based on the fact that there were very few, if any, Black citizens elected to city council, which violated section two of the Voting Rights Act, which prohibits voting practices or procedures that discriminate on the basis of race, color, or language. Eastpointe had winner-take-all elections that ended up diluting the voting power of the African American population. In 2019, as a result of the lawsuit, they started using ranked choice voting, and it increased the diversity of the candidates elected. Furthermore, they were required to educate voters about ranked choice voting, which helped smooth the adoption of the process.

Representation and Apportionment

In the U.S., one of the primary problems of representation is determining districts for the House of Representatives. Apportionment — determining how many representatives each state has — is one part of this problem. Drawing district boundaries is another. (For further discussion of drawing district boundaries, see the chapter, "A Once in a Lifetime Opportunity to Address Gerrymandering," which follows this chapter.)

Let h be the total number of representatives in the House of Representatives. Suppose state i has population p_i and that together the sum of state populations is P. Then each state should receive $q_i = (p_i/P)\,h$ representatives, but this number, called the quota, is exceedingly unlikely to be an integer. The apportionment problem is to determine how to round the quotas to integers so that their sum is h.

Alexander Hamilton proposed a solution to the problem: round each quotient down and give extra representatives to the states with the largest fractional remainders. As an example, consider the three-state, 1000-person country below.

Hamilton's solution

State	Population	Quota	Floor	Apportionment
1	502	6.526	6	6
2	377	4.901	4	5
3	121	1.5730	1	2
T	1000	13	11	13

George Washington used the first-ever veto by a U.S. President to reject Hamilton's method because it gave Vermont, and not Virginia, an extra representative. Instead, Thomas Jefferson produced, and Washington adopted, a competing apportionment method that gave Virginia more representatives. Today, a method called the Hill-Huntington method is used after every census to determine apportionment.

Apportionment is also important during presidential primaries, and recent elections have given some interesting results. During the 2008 election, Republicans used plurality voting to award delegates from each state, while Democrats used Hamilton's method, with the consequence that John McCain secured the nomination by early March, but the Obama-Clinton race continued through June. In following elections, Republicans have required primaries before April 1 to use a proportional method but have not dictated the method.

To see the differences in results different methods can produce, consider the example shown in the table at the top of the next page. The columns labeled (1)–(6) show the number of delegates given to each candidate by an apportionment method used by either Republicans or Democrats in a presidential primary. Bold numbers in those columns indicate delegate amounts that are more than one away from the quota. In order, the methods are referred to as (1) nearest-integer extremes, (2) nearest-integer sequential, (3) Hamilton, (4) large, (5) iterated lower quota, and (6) sequential upper quota.

Apportionment methods compared

i	v_i	q_i	(1)	(2)	(3)	(4)	(5)	(6)
1	29,130	25.473	**27**	26	25	26	**28**	26
2	20,000	17.489	17	18	18	18	17	18
3	17,720	15.495	15	15	16	16	15	16
4	16,750	14.647	15	15	15	14	14	15
5	16,550	14.472	14	14	14	14	14	15
6	15,350	13.423	13	13	13	13	13	**11**
Total	115,500	101	101	101	101	101	101	101

The choice of which apportionment method to use depends on one's goals. Commonly-used apportionment methods often favor smaller or larger states, or candidates with more or less popularity. For example, in the chart above, nearest-integer extremes and iterated lower quota both gave extra delegates to the most popular candidate. Some primary elections require that candidates with support below a particular threshold are dropped from the calculations before allocating delegates. That practice can have the unintended effect of taking delegates from another candidate, a phenomenon called the elimination paradox.

The elimination paradox

i	v_i	q_i	Hamilton	q_i*	Hamilton*
1	6625	3.313	3	3.524	4
2	2775	1.388	2	1.476	1
3	599	0.300	0		

In this vote, disregarding votes for the least popular candidate causes a different candidate to lose delegates.

Different allocation methods, like different election procedures, have different strengths and weaknesses, and therefore may be more appropriate for one context than another. In the case of delegate allocation in primaries, for example, methods that favor less popular candidates may generate ideas and participation and therefore be beneficial early in a race, and those that favor more popular candidates will consolidate support, which would be helpful towards the end of a race. Mathematicians and mathematics educators have an opportunity and a responsibility to be involved in analyzing and solving problems with election procedures and allocation methods and to use real data in the classroom to encourage younger generations to think about the application of mathematical ideas in politics.

About the presenter

Dr. Michael A. Jones is the Managing Editor and an Associate Editor at the American Mathematical Society's Mathematical Reviews, where he has worked since 2008. Previously, he held faculty positions at Montclair State University (New Jersey), Loyola University Chicago, and the U.S. Military Academy at West Point. He earned his Ph.D. from the Department of Mathematics at Northwestern University in game theory under Donald Saari in 1994. His research interests are focused not only on theoretical aspects of game theory, social choice, and fair division, but the application of mathematics to the social sciences, including political science, economics, psychology, and law. Since 2009, he has taught a summer course for high school students ("The Mathematics of Decisions, Elections and Games") as part of the Michigan Math and Science Scholars program at the University of Michigan.

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A ONCE IN A DECADE OPPORTUNITY TO ADDRESS GERRYMANDERING

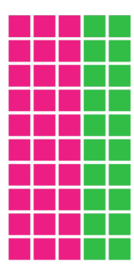
Based on the plenary talk by Stephanie Somersille, Somersille Math Education Services

THE U.S. CENSUS SERVES AS THE BASIS FOR REDISTRICTING the country's congressional districts, state and local representation, and allocation of resources. Since it is conducted only every ten years, the period immediately following the census provides an opportunity to address problems in current districting, especially gerrymandering. This chapter describes the problem of gerrymandering, introduces the new Geography and Election Outcome metric that can be used to evaluate the fairness of congressional districts, and considers alternative voting methods that may be less susceptible to the ills of gerrymandering.

The Problem of Gerrymandering

Every 10 years, the United States conducts a census. After the results are tabulated, some states gain or lose congressional seats and districts are redrawn based on population changes. Based on the 2020 census, for example, California and New York lost seats, and Texas, Florida, and North Carolina gained seats. (See the preceding "Elections and Representation" chapter for more discussion on allocation of seats.)

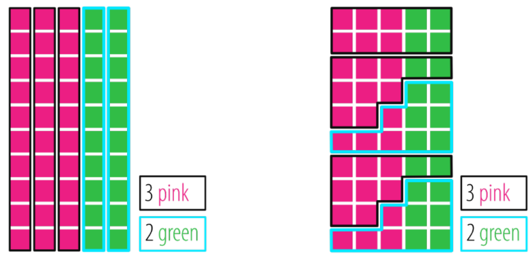
The example below illustrates the difficulty of deciding how to divide voters into districts, even in the simple case of 50 voters, 30 of whom prefer the pink party and 20 of whom prefer green. The voters will be divided into five districts of 10 people each.



How should you divide this region into five districts?

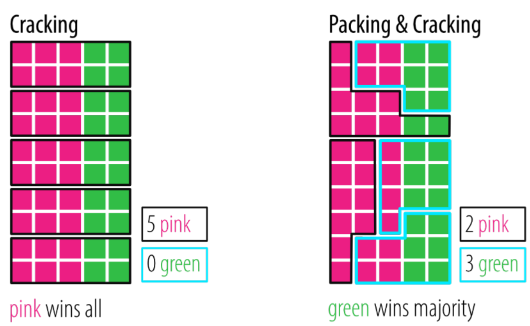
There are many ways to divide these voters into districts that grant proportional representation to the pink and green parties with three majority-pink districts and two majority-green districts.

Proportional representation



There are also several ways to divide voters into districts that do not give proportional representation to the parties. If someone from the pink party chooses districts, they may "crack" the green vote by creating five districts that pink wins, completely erasing green voters' preferences. On the other hand, the green party could win a majority of districts, despite green being the preference of a minority of voters, by "packing and cracking:" packing pink voters into a few heavily-pink districts and cracking the rest of the pink voters into districts that the green party wins with a slim majority.

Non-proportional representation



The term "gerrymander" comes from 1812 Boston, when Governor Elbridge Gerry, tasked with redistricting Massachusetts, drew a contorted district that political cartoonists likened to a salamander.

The original "Gerry-Mander"

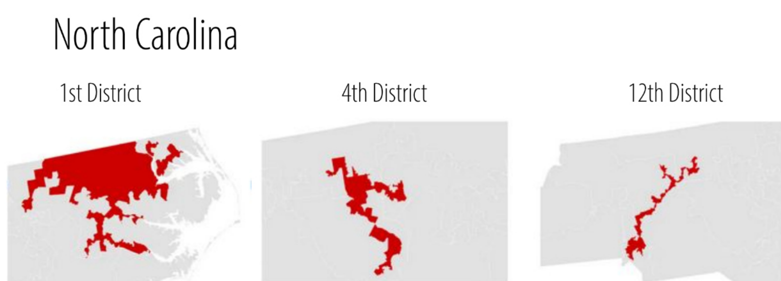
Is Gerrymandering Legal?

The legality of gerrymandering is a complex issue. Since the 20th century, federal courts have deemed extreme cases of gerrymandering to be unconstitutional but have struggled with how to define the types of gerrymandering, quantify gerrymandering, and determine whether one set of districts is more fair than another.

The Voting Rights Act of 1965, which was amended in 1970, 1975, 1982, 1992, and 2006, prohibits racial discrimination in voting. It was intended to prevent two types of discrimination: first, vote denial based on race or language minorities, including literacy tests and moral character tests; second, vote dilution, which can prevent racial minorities from electing their preferred candidates. The second type is where gerrymandering comes in.

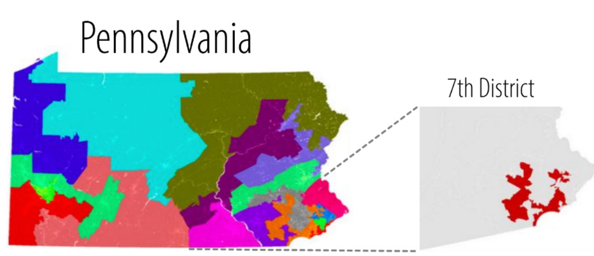
In 1986, the Supreme Court decided the *Thornburg v. Gingles* case, which led to Gingles conditions for voting rights lawsuit claims. The first condition is that the racial minority must be sufficiently numerous and compact to form a majority in a single district, sometimes called a majority-minority district. Next, minority groups must be politically cohesive and vote similarly. Finally, the majority also votes as a bloc in a way that can defeat the minority groups' candidates of choice. The remedy in Voting Rights Act lawsuits in which minorities are not able to elect their preferred candidates is to draw majority-minority districts so that they can potentially elect candidates of their choice.

The idea of majority-minority districts can be appealing, but things can go wrong when people start drawing districts. Several of North Carolina's districts, for example, violate common sense.



Where's the common sense?

Pennsylvania's 7th district, nicknamed "Goofy kicking Donald Duck," looks especially bad in the context of the entire state. It also illustrates the fact that cities are easier to gerrymander than rural areas because the concentrated population yields a great deal of potential connected districts with enough residents. Goofy kicking Donald Duck is in the Philadelphia area, a major population center in Pennsylvania.



Goofy kicking Donald Duck

Gerrymandering is problematic because you have the politicians choosing their voters, not the voters choosing their politicians. If the politicians can draw districts and determine who's going to vote for them, incumbents can all but guarantee their reelection and decide whose votes matter and whose don't. That is why opponents of gerrymandering describe it as moving away from the ideal of "one person, one vote."

Somersille recalls that when she first learned about gerrymandering, she was surprised to find that mathematicians were usually not involved in drawing congressional districts. As a theoretical mathematician, she was initially wary of getting involved with a problem whose rules were less clear-cut and assumptions were often hidden, unlike the problems she usually worked on in her research. In her volunteer work related to social justice, she eventually found the problem of gerrymandering so compelling that she was moved to bring her mathematical background to the table to try to address it. Mathematics alone will not solve gerrymandering. In addition to mathematicians, a broad coalition of people with many different areas of expertise are necessary: politicians, lawyers, lawmakers, social scientists, and of course those with the most at stake, community members themselves.

Quantifying Gerrymandering

One of the obstacles to addressing gerrymandering in court is that there is no widely agreed upon metric defining gerrymandering. The "you know it when you see it" definition of gerrymandering is unsatisfying. Being able to measure the problem is crucial in addressing it. The currently popular metrics fall into two camps: map data metrics and election data metrics.

Map data metrics. Map data metrics, such as the Polsby–Popper ratio, the Roeck ratio, the convexity coefficient, and the convex hull, are based on the geometric irregularity of the shapes of districts. The Polsby–Popper ratio, for example, is proportional to the ratio of the area of the district to the square of its perimeter. The convexity coefficient is based on finding the probability that the straight line between any two points in the district is itself entirely contained within the district.

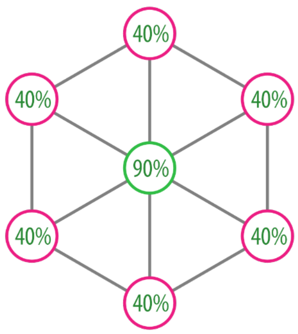
Map data metrics have a few weaknesses. For one, the physical geography of a district influences the metric. A river, mountain range, or coastline can make a district seem gerrymandered when it is not. Furthermore, with modern technology, it is easy to generate thousands of maps with the same, say, Polsby–Popper score and choose the one that best suits one's agenda.

Election data metrics. Election data metrics, on the other hand, are based on voting patterns in recent elections and tend to assume the existence of two dominant political parties. These metrics include the mean-median difference, efficiency gap, partisan bias metric, and declination function. They include data such as the number of wasted votes — votes for the losing candidate and votes for the winning candidate beyond the majority necessary to win — and comparisons between statewide and district-wide election margins.

These metrics can have shortcomings as well, such as flagging proportional outcomes as gerrymandered in regions that are dominated by one party.

Geography and Election Outcome Metric

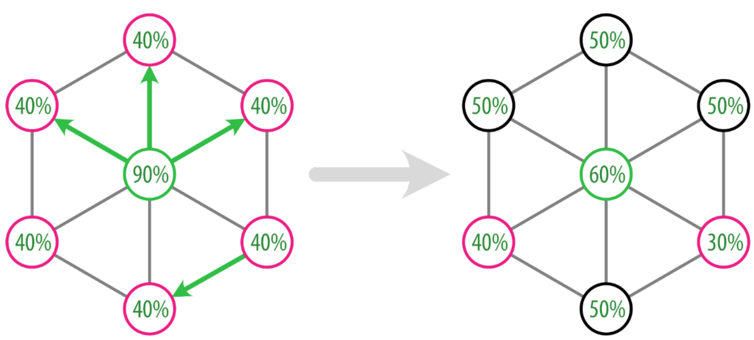
Somersille and colleagues have developed a new metric, the Geography and Election Outcome (GEO) metric, that uses both geography and election data to measure gerrymandering. The idea is to look at wasted votes and determine whether swapping some voters with a neighboring district would cause the relevant party to gain an additional seat.



Percentages shown are vote shares for the green party; Pink-circled districts were won by the pink party.

Can more voters' views be represented?

As an example, consider a fictional state with seven districts arranged as above. Each district is labeled with the percentage of the vote won by the green party. If the middle district, which is heavily green, transfers some of its vote share to three neighboring districts, those districts will be competitive, and the middle district will still be won by the green party. Furthermore, if one of the majority pink districts transfers some green votes to a neighboring district, another district will also be competitive.



Votes can be traded between the center district and the top three districts as shown. The center district will remain a safe win for the green party.

The black districts are now competitive.

Creating more competitive districts

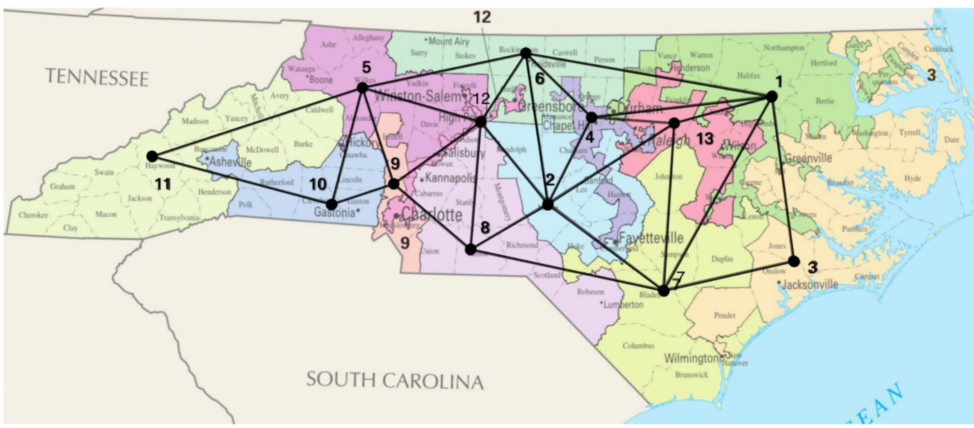
In this example, the green party would get a GEO score of 4 because by transferring votes from some districts, it can make four districts it was losing into competitive districts. On the other hand, the GEO score for the pink party is 0 because there is no way for the pink party to make districts competitive by transferring votes from one district to one other district. A GEO score of 0 for one party and a high GEO score for the other (relative to the number of districts) is usually a red flag for gerrymandering because it means the map is drawn as favorably as possible for one party. A large difference in GEO scores between two parties is also a sign that a map may be gerrymandered.

The GEO metric is promising for several reasons. It is understandable, which is helpful for use in court. It recognizes the importance of both geography and election data in determining whether a district is gerrymandered. Finally, it flags districts as potentially packed or cracked.

Applying the GEO Metric to States

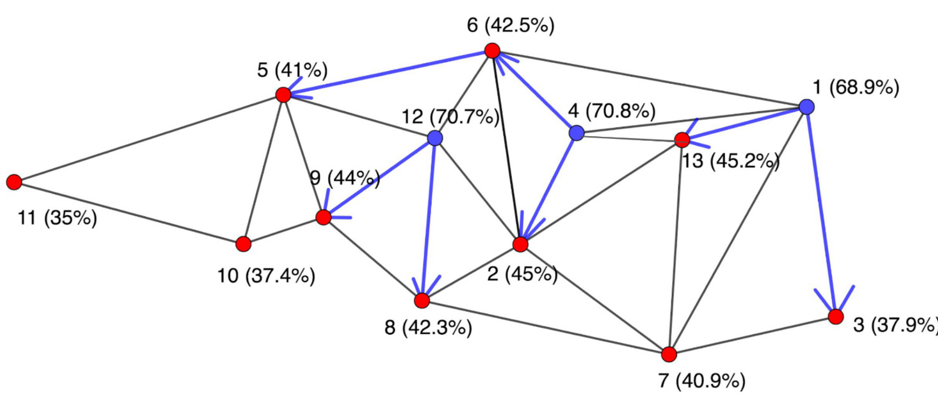
To find the GEO metric in a real state, take a map of districts and make each district a node of a graph. Add an edge between them if the districts border each other.

GEO metric node mapping



North Carolina. Then take election data and determine whether swapping votes between adjacent districts would increase the number of competitive districts without sacrificing any currently won districts. In the below example, where the percentages listed are the percentages of votes won by the Democrats in North Carolina during the 2016 presidential election, the Democratic party can make seven districts competitive by moving voters, so its GEO score is 7.

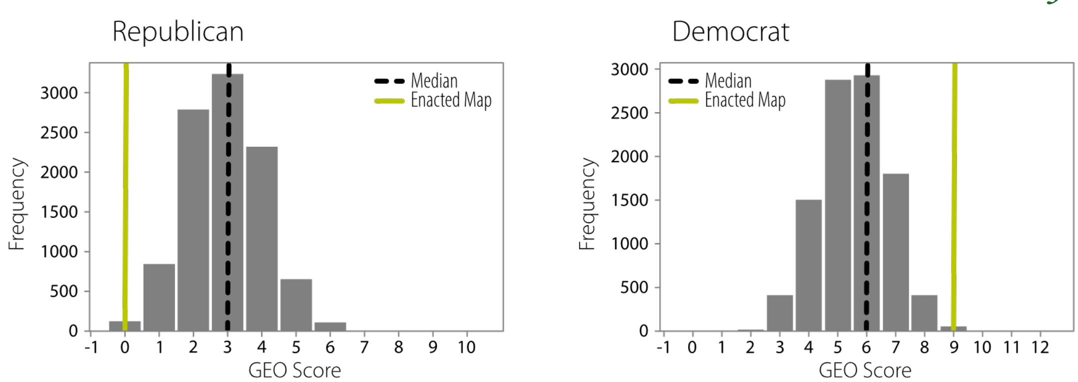
Movements that can make districts more competitive



One possible concern about the GEO metric is whether votes can really be moved across district lines by redrawing boundaries slightly, but mapmakers have the ability to generate ensembles of maps satisfying necessary criteria such as preserving counties where possible or maintaining majority-minority districts. One way of assessing a map for gerrymandering using ensemble generation is to investigate whether a map is an outlier among other maps with similar properties.

Pennsylvania. Somersille and her colleagues performed this test on Pennsylvania voting districts. They generated thousands of maps and found that the median map had a GEO score of 3 for Republicans and 6 for Democrats, but Pennsylvania's actual map is a 0 for Republicans and 9 for Democrats, suggesting that it is an outlier that is gerrymandered in favor of Republicans. (Pennsylvania had already been found to have gerrymandered districts and was required to redraw its map.)

Pennsylvania GEO

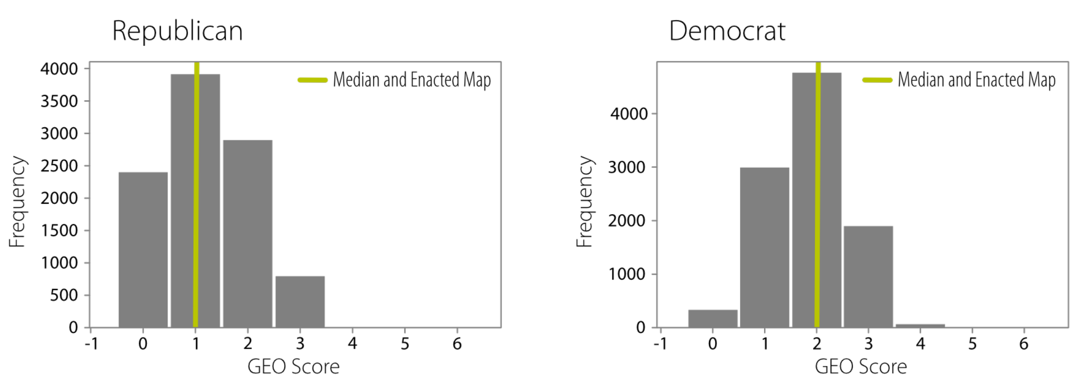


Not possible to improve their outcome

A lot of potential to improve outcome

Colorado. On the other hand, Colorado's map has a GEO score of 1 for Republicans and 2 for Democrats. Both of those numbers coincide with the median for generated maps, indicating that it is not gerrymandered.

Colorado GEO



Alternative Voting Methods

The framers of the Constitution did not envision a multiracial democracy in which all adults are allowed to vote. Initially, only white men could vote, and the plurality winner, winner-takes-all system made more sense in that less diverse setting. Voters may have expected to be in the majority on some issues and the minority on others; an entrenched minority group was not assumed to be part of the political landscape. Today, many minority groups, whether racial or religious or linguistic, are never able to elect their candidates of choice. The Electoral College was created in a very different environment from today's. The winners of the Electoral College and the popular vote have been different five times in history, two of which have been in the last six elections. The preceding discussion of redistricting took our current system as a given, but it is worth asking whether other voting systems could represent voters better.

Ranked choice voting. One alternative is ranked choice voting. It has been adopted in some U.S. cities and states, including the 2021 Democratic primary for New York City mayor, and in some other countries. It has advantages, including minimizing the problem of vote splitting. (See "Elections and Representation" in this volume for more discussion of ranked choice voting.)

Cumulative voting. Another voting method, often used in the corporate world, is cumulative voting. Say a company has a 10-seat board. Every voter gets 10 votes and can allocate them as desired, casting them all for one candidate or distributing them among several.

Potential and challenges. A voting rights lawsuit in Santa Clara, CA, provides an illustration about the potential for alternative voting systems and the challenge of having new methods adopted. Five Asian Americans sued the city in 2017 because the seven-person city council (six council members plus the mayor) had never included an Asian American. (At the time of the lawsuit, Santa Clara was approximately 30% Asian American and Pacific Islander.) Intentionally or not, the election process made minority representation unlikely. The six city council members were all at-large — they did not represent particular districts in the city — but each candidate ran for a particular place on the city council and had to win a plurality of votes for that seat. A court ruled that there was a voting rights violation and required Santa Clara to create six districts rather than have at-large council members.

A team consulting for the city, however, suggested ranked choice voting instead: voters would rank all candidates and the top six would be selected for the city council. This idea was not adopted because so many candidates could be included in the race that ranking them would become difficult for voters. Another option was a hybrid model in which the city was split into two districts and each one would elect three council members using ranked choice voting. A ballot measure to adopt this system failed in 2018; a similar measure to create three districts with two representatives each also failed in 2020. In the meantime, the court required Santa Clara to create six temporary districts, and in 2018, the first Asian American was elected to city council. The city will soon be making the district boundaries permanent, a move that creates the potential for gerrymandering at the present time and that may not continue to represent the city well as demographics shift within the city. Moreover, the lawsuit only required the creation of an Asian American majority district, but the city also has a substantial Latinx population. Ranked choice voting would be more likely to allow the Latinx population and future demographic groups to have representation on the city council.

The existing system makes adopting new voting systems expensive, time-consuming, and difficult, but there are many opportunities for mathematicians, as well as experts in other fields, to participate in the process of making current voting districts more fair. Several avenues of research allow mathematicians with differing interests to contribute to the work, including racial polarization in voting, ensemble map generation algorithms, the size of the space of all maps, voting as a mean field game, and ranked choice voting systems.

About the presenter

Dr. Stephanie Somersille is the founder of Somersille Math Education Services, a boutique math consulting firm that inspires and coaches students of all ages to achieve levels of expertise beyond their own expectations. Somersille helps students ready for more challenge as well as students who need support with math concepts and anxiety. She helps teachers better serve students with different levels of preparation and confront issues of racial and gender equity. Somersille is passionate about applying mathematical analysis and algorithms to issues of social justice, fair representation and public policy. She is currently consulting on projects involving redistricting and preventing extreme partisan gerrymandering. Somersille has held research/teaching positions at University of Texas, Austin; Instituto Superior Tecnico, Portugal; and Dartmouth University. Somersille holds a bachelor's degree in economics from Yale University and a Ph.D. in mathematics from the University of California, Berkeley, with thesis topics: partial differential equations, probability, and game theory. When she isn't working or volunteering with Brown Girl Surf and the Marine Mammal Center, she enjoys surfing, running, bike riding, ballet, and the symphony.

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FAIR DIVISION AND ALLOCATION

Based on a plenary talk by Michael Jones, Mathematical Reviews

HOW SHOULD ONE DIVIDE A RESOURCE among people who have different claims to that resource? This question of fair division has been asked in many different contexts throughout history and has invited answers from politicians, mathematicians, and even rabbis.

The Contested Garment Principle

Suppose someone goes bankrupt. They have three creditors, to whom they owe \$100, \$200, and \$300, and only \$300 with which to pay those creditors. How should the creditors be paid? Two basic ground rules are that the sum of amounts awarded must be equal to the total estate size and that no claimant may receive more than they claim. Three options for dividing the estate are \$100, \$100, and \$100, respectively; \$50, \$100, and \$150; and \$0, \$100, \$200. Different people may have different ideas of which allocation is the most fair.

The Talmud, the central text of Jewish law and theology, weighed in on the problem of dividing an asset between people with different claims on it. The following table shows the Talmudic prescription for how much each claimant should receive for various estate sizes. At first glance, these amounts may seem arbitrary: in the first scenario, each person receives the same amount of money even though they have different claims; in the second, they receive different amounts and different proportions of their claims; and in the third each receives half of their claim.

		Estate		
		100	**200**	**300**
Claims	**100**	100⁄3	50	50
	200	100⁄3	75	100
	300	100⁄3	75	150

Talmudic prescription for dividing among claimants

Perhaps surprisingly, the amounts awarded in each scenario are derived via a process, called the Talmud rule, which extends a better understood, two-claimant rule from the Talmud known as the "contested garment principle."

In 1985, Robert Aumann and Michael Maschler showed that the solution provided by the Talmud rule is equivalent to the nucleolus of the game. In cooperative game theory, the nucleolus of a game is the outcome that, in non-technical terms, minimizes the maximal dissatisfaction over all possible subsets of players, referred to as a coalition. The concept of the nucleolus of a game was only invented in 1969, so the fact that the Talmud's solution coincides with the nucleolus suggests that these two concepts capture some important properties of what humans find fair, even before the mathematization of game theory.

The Talmud rule is the only rule satisfying four desirable properties for bankruptcy problems:

1. *Equal treatment of equals*. Two claimants who have the same claim size will receive the same amount.

2. *Claims truncation invariance.* The idea of this property is that if one's claim is larger than the estate size, replacing the estate size with one's claim amount will not change the amount received.

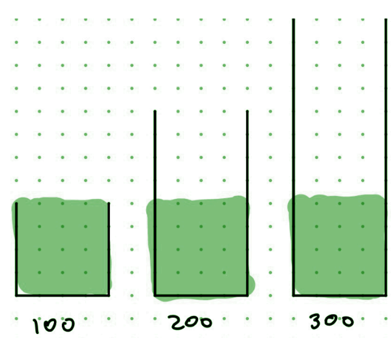
3. *Minimal rights first*. This amounts to the idea that each person can expect at least the difference between the estate and the other claimants' claims (or 0 if that difference is negative) and that any division rule is equivalent to a procedure that first assigns those amounts to all claimants and then divides the rest of the claim among them.

4. *Bilateral consistency*. This property states that when any subset of $n-2$ claimants leaves the group, the amount remaining will be allocated the same way to the remaining two-person subset of claimants based on their claim sizes as it was allocated in the larger group.

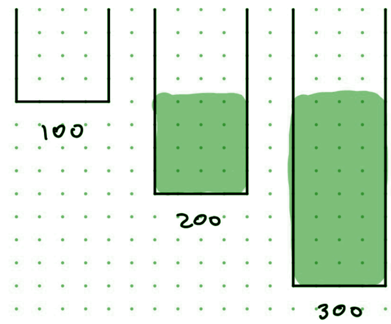
Visualizing Division Rules

Various division rules can be visualized as water being pumped into a tank, also called hydraulic rationing. To divide an estate worth \$300 between claimants with claims of \$100, \$200, and \$300, we imagine pumping 300 units of water into three tanks with volumes 100, 200, and 300. Water is pumped in to equal levels on all three tanks. Below is an illustration of the *constrained equal awards*. In this method, everyone gets the same amount until their claim is completely honored.

Constrained equal awards

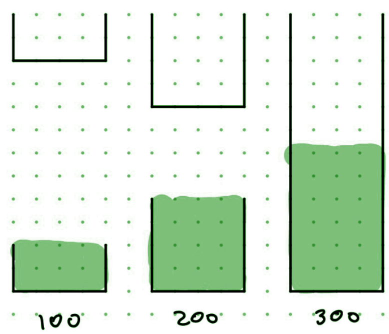


To visualize an allocation called *constrained equal losses*, imagine the three tanks hanging from the same point on the ceiling instead of sitting on the floor. Again, water is pumped in from the bottom to equal levels on all three tanks. In this method, everyone's claim is short by the same amount (as long as every claimant receives a positive amount).



Constrained equal losses

The Talmud rule solution is a combination of these two methods. For estate sizes up to half of the sum of the claims, the Talmud rule uses constrained equal awards for claims half the original size. For estate sizes greater than half the sum of the claims, the rule awards each claimant half their claim and uses constrained equal losses to allocate the remaining estate, again using claims half the original size. To visualize this rule, containers of half size are both sitting on the ground and suspended, and water is again added from the bottom.



The Talmud rule solution

Cake Cutting

Fair division in the context of a contiguous object such as land is referred to as cake cutting. If *n* claimants have equal rights to the land but possibly different preferences for parts of the land, then we seek an allocation that satisfies the following rules. First, it is proportional: each claimant gets at least $1/n$ of the land. Second, it is envy-free: no claimant prefers another claimant's allotment. And finally, it is Pareto efficient: there is no other allocation where one claimant does better and all others do at least as well.

In 2021, the Los Angeles County Board of Supervisors voted to transfer a beachfront property from Manhattan Beach to the descendants of Charles and Willa Bruce, African Americans who had the land taken from them by eminent domain in 1924. Cake cutting, for example, could help them divide the land fairly between those descendants.

Fair Division of Reparations

Awarding reparations is another task that fair division can inform. During World War II, the federal government forcefully removed and confined about 120,000 Japanese Americans, of whom 62% were US citizens. President Roosevelt authorized the internment using an executive order that allowed local military commanders to exclude any person of Japanese ancestry from the Pacific coast of the U.S. In 1944, the Supreme Court upheld the constitutionality of the actions as a "pressing public necessity" in the case of *Korematsu v. United States*. The Civil Liberties Act of 1988 recognized that internment had been based on "race prejudice, war hysteria, and a failure of political leadership" and granted reparations to surviving internees. Although this problem has some aspects of a fair division problem, the Civil Liberties Act applied a one size fits all solution: No matter how long you were interned at a camp or what you lost, you received the same amount of money. A fair division approach would treat different losses differently.

Several other current dilemmas could productively be treated as fair division problems. Reparations for slavery may be the most obvious example. Some cities, such as Detroit, MI, and Evanston, IL, are considering reparations for historic redlining. Georgetown University has been considering reparations to the families of enslaved workers who built the school. The University of South Dakota's land was taken from Indigenous people, and the university currently profits not only from tuition but also from mineral rights on the land. How should the university compensate the families of people whose land was taken?

Mechanism Design

Mechanism design is a term for constructing a game to achieve a particular outcome, such as a truthful revelation of preferences in an election. The Gibbard–Satterthwaite theorem in social choice theory states that there is no way to create a social choice function that cannot be manipulated. (See also the chapter on "Elections and Representation.") Despite this, however, mechanism design proves to be a powerful tool in economics. The 2007 Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel was awarded to Leonid Hurwicz, Eric Maskin, and Roger Myerson for their efforts in creating mechanism design theory.

As a junior faculty member at Montclair State University in New Jersey, Michael Jones was asked to assist the associate dean in devising a method to award travel funds. Like many state universities, faculty members at Montclair State were encouraged to attend conferences, but the university could not budget as much as faculty needed for travel. In mathematical terms, if R is the total sum requested by all faculty, and B is the total Dean's office budget for travel, when $B<R$, then awarding travel funds is a claims problem.

The initial method used by the Dean's office was to prorate everyone's travel grant by the budget shortfall based on the amount each requested. Using b_i for the award given to faculty member "i" and r_i for the amount they requested, $b_i = (B/R)\, r_i$. This allocation incentivized faculty members to request more than they actually needed. Jones developed a mechanism that was less susceptible to manipulation. Faculty members were still budgeted b_i, but their actual reimbursement depended on s_i, the amount they actually spent. If they spent less than they requested, their reimbursement was $(B/R)\,s_i$. If they spent more than they requested, they received only their full budgeted amount, $(B/R)\,r_i$. This mechanism incentivized people to request only what they anticipated using and to use less expensive travel options, so the total amount requested came closer to the university's budget.

Another illustration of mechanism design is a second price auction, where the item is purchased by the highest bidder for the price of the second highest bidder. Such an auction incentivizes people to bid their true valuation of the item being sold.

Matching Algorithms

A problem such as assigning students to schools is a matching problem. In 1962, David Gale and Lloyd Shapley published an algorithm to solve this problem subject to the constraint that there is no school/student pair that would prefer to be matched together rather than in their current pairings. (In their article, they referred to this as the "stable marriage problem," but that formulation is antiquated, sexist, heteronormative, and hardly applicable to the real-world process of finding a life partner!) Although Gale and Shapley did not publish their algorithm until 1962, it has been used by the National Resident Matching Program (typically called "the Match") to place medical residents in hospitals since the 1950s.

Their solution, known as a deferred-acceptance algorithm, is as follows: The students provide a linear ordering over all the schools, and the schools provide a linear ordering over all the students. Each school offers admission to the top student on their list. Each student who receives an offer then provisionally accepts the best offer they have received. Then any school whose offer was not accepted makes an admissions offer to the next student on the list. If a student had already provisionally accepted another offer but receives an offer from a school they prefer, they turn down the first offer and take the second. The process continues until admission offers are not turned down anymore. (The algorithm can be flipped so students are making offers to schools to start out with to create a different, but still stable, matching. When schools make offers, the matching favors the schools, and when students make offers, it favors the students.)

The deferred-acceptance algorithm can be used in conjunction with affirmative action measures. California's Proposition 209 (1996) states that the government and public institutions cannot discriminate against or grant any preferential treatment to persons on the basis of race, sex, color, ethnicity, or national origin in public employment, public education, and public contracting. (An effort to repeal Proposition 209 was defeated at the polls in 2020.) As a result of Proposition 209 and other laws prohibiting certain types of affirmative action, some admissions systems have adopted race-neutral alternatives that encourage diversity without explicitly favoring a particular group. For example, some schools create tiers based on socioeconomic status and reserve an equal number of places for students in each tier. These tiers can be filled using matching algorithms. As another example, 40% of the Harvard University class of 2018 went to private high schools, compared to 7% of U.S. high school students overall, so attending a private high school is a huge advantage in admissions that many families cannot afford. A deferred-acceptance algorithm could decrease the benefit of private high school for students, which would most likely cause Harvard to admit more students from disadvantaged backgrounds.

Mathematicians have the ability to create, to suggest, and to analyze fair division procedures to address real-world problems and advance real-world justice. Furthermore, mathematics educators can introduce the mathematics of fair division and its application to solve societal problems in the classroom, demonstrating to students both that considerations of ethics and fairness are integral to the practice of mathematics and that they can contribute to societal good by becoming a mathematician.

About the presenter

Dr. Michael A. Jones is the Managing Editor and an Associate Editor at the American Mathematical Society's Mathematical Reviews, where he has worked since 2008. His full biography appears on page 81.

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PART 6

CLOSING THOUGHTS

We take time to plan actions based the lessons of the workshop, to celebrate our place in the struggle, and to honor those who have come before us.

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THE POWER OF THE COLLECTIVE: UBUNTU

"UBUNTU" IS A ZULU WORD WHICH REPRESENTS two central principles of (broader) African philosophy: connection and community. Ubuntu connotes the power of the collective in contrast to any one individual; in essence, it can be understood as, "I am because you are." This principle of the power of the collective was experienced by all participants in the final breakout sessions and the workshop's closing libation ceremony.

Sessions for the Future

The organizers invited participants to use the workshop's discussions as jumping-off points for making changes in their classrooms or departments and for starting new research projects. On the final day of the workshop, participants joined breakout sessions based on the four thematic areas in order to meet colleagues at different institutions who were interested in focusing on similar areas and to exchange ideas and resources for taking the next steps in advancing racial justice in their work.

Bias in algorithms. Participants in the breakout session centered on bias in algorithms and technology discussed three primary areas for combatting algorithmic bias as mathematicians: in the classroom, through regulation and legislation, and through collaboration in research.

- In their classrooms, math educators can use their platforms to teach students about algorithmic bias, which may include developing classroom materials at various levels.
- At the policy level, mathematicians can develop and advocate for changes such as a digital bill of rights or the creation of an "FDA for algorithms," an agency that would evaluate and regulate algorithms to prevent harmful algorithms from being used. Both of these approaches have their strengths and weaknesses, and interested mathematicians should talk not only with each other but with other existing organizations, including European Digital Rights (EDRi), about potential pitfalls and the best ways to

advocate for change. Locally, mathematicians can push businesses, organizations, and local governments to adopt their own digital bills of rights.

- Individuals and small research groups can also start working on research collaborations to identify biased or harmful algorithms and quantify their effects.

Public health disparities. This breakout room focused on using mathematical models to address race-related public health issues. One important topic was addressing the problem of race as a confounding variable and how to build models that predict health outcomes regardless of race. If mathematicians want to use their backgrounds to help with public health-related research and policy, they also need to consider how to interface with other organizations, such as hospitals and NGOs, that are addressing imbalances in healthcare. For instance, researchers at a university could create a modeling consulting firm offering services to organizations that are studying public policy solutions to health disparities. Participants are eager to look more seriously into this possibility.

Racial inequities in mathematics education. Participants in this session grappled with the idea that diversity/equity/inclusion can be a facade that serves the status quo rather than offering a truly liberatory education to students. Danny Martin's talk asks mathematics educators to refuse to push students through a dehumanizing system, but educators are part of that system. They must try to exist both within and outside of the system at the same time. Some participants noted that education reform wars have been fought several times: mathematicians need to look at past efforts at reform and learn from them. They also need to separate, both for themselves and in the minds of the general public, the curriculum from the way it is enacted. For all of these concerns, there are people who have been doing the work for a long time, and mathematicians would do well to work with them rather than starting from scratch. Mathematicians have a lot of options for rehumanizing their classrooms, including rethinking their approach to grades and standardized testing, assessing their own classroom culture using a tool such as Niral Shah's and Daniel Reinholz's EQUIP (Equity QUantified In Participation) app, and involving students directly in course design.

Fair division, allocation, and representation. This breakout session raised several issues related to the clashes of real world and theoretical mathematics involved in these topics. Some participants who had attempted to get involved in local politics related to voting rights and gerrymandering had felt deflated after their efforts did not get results. One participant suggested that officials may feel less defensive or standoffish if someone approaches them looking for data to use in the classroom. Teaching is a good way to get started wrestling with issues related to fair division and allocation because it is compelling to students, many potential data sets exist, and students can decide how far they want to take the idea. Teachers who use these ideas in their classrooms may expect some pushback from politically conservative students, but much of that can be mitigated by leaving one's own politics out of the lesson as much as possible. Some participants mentioned that they are interested in creating a repository of curated data sets and learning modules to make it feasible for people to incorporate these lessons into their classrooms. For those interested in starting new research projects in these areas, interdisciplinary collaboration is crucial. Interested mathematicians can attend seminars outside of the math department to see what mathematics might be relevant and useful in other fields.

Libation: Acknowledging One's Place in the Collective Struggle

As a way of inviting all participants to collectively celebrate their place in the long struggle for racial justice and acknowledge this struggle itself as a part of the cultural continuum of the African Diaspora, the workshop closed with a libation ceremony.

In their book *Legacy: Treasures of Black History*, Thomas C. Battle and Donna Marcia Wells write, "The African Diaspora has three dimensions — the dispersion from Africa, settlement and adjustment abroad and the physical and psychological return."

The bonsai tree watered as part of the ceremony

A libation ceremony can be understood as a sacred sojourn within the African diaspora itself, described by this trichotomy. In particular, a libation is a way of giving homage and paying respect to those who have come before us and who now mediate on our behalf beyond this world, in the spiritual realm, understood very tangibly as not being wholly separated from the physical world: the divine energies within the trees, the rivers, the ocean, and the mountains, and the power behind the sun. The invitation to participate in our libation emphasized the perspective that while the ritual of libation is central in African spiritual traditions, it certainly has commonalities with practices of many world religions and faith traditions. South African racial justice activist and martyr Steve Biko wrote, "We [Africans] had our own community of saints. We believed — and this was consistent with our views of life — that all people who died had a special place next to God…. We [Africans] did not believe that religion could be featured as a separate part of our existence on earth. It was manifest in our daily lives. We thanked God through our ancestors before we drank beer, married, worked, etc."

“Concluding the workshop with this libation is representative of African Diaspora cultural traditions that do not separate the sacred from the mundane parts of life. Libation is a kind of anointing. We experience a similar anointing via tears: tears of grief, tears of joy, tears of hurt, tears of triumph, tears of frustration, tears of rage, tears of loss, tears of love. Tears come to us at healing moments, at moments of growth, and at moments of giving thanks. Tears should never be denied. Like tears, libation allows us to collectively: remember, give thanks, and consecrate ourselves to our collective purpose.”

Our libation ceremony had two cycles. The first cycle allowed participants to speak the name of a forerunner of racial justice, and the second cycle allowed participants to speak an intended action they planned to take in service to racial justice in their classrooms, departments, research fields, or communities. Each cycle of the libation followed a call and response cadence: first one member of the community spoke, then all other members responded in unison, “Aśe,” from a Yoruba word that can be translated as “so let it be” or “the power to make things happen.” Simultaneously, with each collective "Aśe," water was poured out of a glass onto a bonsai tree, by Dr. Caleb Ashley who was given the honor of leading the ceremony.

“Gathering together the memories of the forerunners of racial justice and the intentions of all the participants, we say asé.”

Suggested References

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AFTERWORD: A TRIBUTE TO BOB MOSES

> "*Like it or not, history has thrust mathematicians and specialists in mathematics education into the middle of a central American dilemma: the reconciliation of the ideals in the Declaration of Independence and the United States Constitution with the structures of race and caste and the legacies of slavery and Jim Crow.*"
>
> —Bob Moses and Ed Dupinsky

Bob Moses at MSRI in May 2005.

CIVIL RIGHTS LEADER AND MATH EDUCATION ACTIVIST Robert Parris Moses passed away July 25, 2021, a month after this workshop took place. Moses's work has inspired generations of mathematicians and mathematics educators to understand mathematics literacy as a civil right. From this understanding, they work to make their classrooms into places of, in workshop speaker Brittany Mosby's words, liberatory mathematics education.

Moses was born in 1935 in Harlem, NY. He earned his bachelor's degree from Hamilton College in upstate New York and master's in philosophy from Harvard before moving to Mississippi, where he worked on difficult and dangerous voter registration efforts. He helped to organize the Mississippi Freedom Democratic Party, which challenged the Democratic Party in the state, which at the time only allowed whites to vote in primaries or be elected.

Moses left the U.S. for Canada in 1966 after being drafted, although at the time, he should not have been eligible due to his age. He taught in rural Tanzania with his wife from 1969–1976, returning to the U.S. to earn a Ph.D. in mathematical philosophy from Harvard University after President Carter granted unconditional pardon to Americans who had left the country to avoid the draft. After earning his doctorate, he became increasingly interested in advancing social justice through mathematics education.

Moses saw firsthand the way whites in the Jim Crow South had kept African Americans uneducated and then used their lack of education to justify denying them rights. "There is an inextricable link between the denial of the right to vote for African Americans and sharecropper education," he said in an interview. The separate and unequal education of Black and white students was for decades a pillar upholding a racial caste system in the American South.

The Algebra Project

Mathematics specifically opens doors to careers in STEM fields that can provide economic opportunities and security. For better or worse, algebra and calculus are gatekeeping courses in high school and college that determine whether students can take upper-level science classes and access these opportunities. But apart from career training, quantitative literacy is vital for an educated citizenry to be able to understand and evaluate claims made by politicians, public health officials, salespeople, and others who use statistics to make their arguments.

In 1982, Moses was awarded a MacArthur Foundation Fellowship, colloquially known as a "genius grant," in recognition of his civil rights work, teaching, and writing. He used the prize money to found the Algebra Project, a nonprofit organization that aims to prepare students, especially those from low-income households or other underserved groups, for advanced mathematics classes in high school. The catalyst for Moses's focus on the transition time from pre-algebra to algebra came from his own daughter. Her school did not offer algebra to eighth-graders, and Moses knew that without algebra in eighth grade, she would not be able to get through her high school math curriculum and be prepared for calculus in college. Moses started teaching algebra to her and a small group of her peers; eventually, the project snowballed into a program that has served tens of thousands of students over the past four decades. He wanted to use the Algebra Project to "raise the floor" for mathematics education for students. "Can a culture be created," he asked, "where every child is expected to get ready for and do algebra in the eighth grade?"

Moses had a gift for finding ways to connect mathematics to students' lives. In an article for the *Notices of the American Mathematical Society*, he describes using Boston's public transportation system to help a student understand the importance of both magnitude and direction when reasoning about the number line. Moses looked to philosopher W.V. Quine's work on the experiential generation of mathematical concepts to build a five-step pedagogical method that allowed students both to develop mathematical concepts for themselves and understand their own reasoning and explain it clearly to others. In 2001, Moses and journalist Charles E.

Cobb, Jr. published *Radical Equations*, a book that described the motivation and methods of the Algebra Project, making it accessible to mathematicians and teachers throughout the country.

The Algebra Project method has proven effective in increasing the proportion of students who take advanced math classes in high school. It has also given rise to other organizations working on related efforts, including the Young People's Project, which trains math tutors for elementary through high school students; Quality Education as a Civil Right, which organizes more generally for high-quality public education for all students; and the "We the People" National Alliance, which organizes equity-focused math education organizations on a national level.

Mathematics to Advance Social Justice

Although mathematicians may sometimes feel that their work is orthogonal to the social, political, and technological problems of the day, Moses's example illustrates the fact that mathematicians can and should use their interests and strengths to advance social justice and nurture the mathematical literacy all citizens need to thrive in modern society. As Alice Walker wrote in a tribute poem following his death,

> "*The world is filled*
> *With many vineyards*
> *In which to labor.*
> *Look deeply in the valley*
> *In which your talent and spirit*
> *Have found rest*
> *And energy.*"

Moses's insightful and incisive speaking and writing about the importance of mathematics literacy as a civil right has had a broad and deep influence on many people, including the organizers of this Workshop on Mathematics and Racial Justice. We hope that the ideas discussed and new partnerships formed during this workshop will continue to advance his goal of high-quality, liberatory mathematics education for all students.

Reference

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