MSRI
Mathematical Sciences Research Institute

Mathematics of Climate Change

A new discipline for an uncertain century

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FOREWORD

In the spring of 2007 MSRI organized a large public event, "Climate Change: From Global Models to Local Action," to examine some political and economic aspects of climate: what we know, what we guess, how and how much our society can and should respond to what we are learning. We chose the date to coincide with a visit by Congressman Jerry McNerney, whose background, as both a mathematician and an advocate for alternative energy sources, made him a model participant for an event that would combine the perspectives of several disciplines.

The public panel discussion was followed on the next two days by a scientific symposium, in which mathematicians from many different fields mixed with economists, climate modelers and others who have already been working on the many questions involved. This booklet is a record of some of the discussions and ideas in those meetings.

Inez Fung (left) and David Eisenbud (right), co-organizers of the MSRI symposium on climate change.

The purpose of these events was to connect the mathematical community with the best current research and thinking about climate change, and to point out the many different kinds of mathematical challenges that are presented by this issue. Society needs to know more, and more accurately, about what is happening with the earth's climate — and to prepare for whatever action is necessary and practical to undertake. Mathematics and statistics already play a central role in this as in any sort of modeling effort. Likewise, computer science must have a say in the effort to simulate Earth's environment on the unprecedented scale of petabytes. With a problem of this complexity, new mathematical tools will undoubtedly be needed to organize and simplify our thinking. Thus it seemed to us at MSRI important to encourage direct discussions between those already in the field and the many mathematicians whose skills, and whose students' skills, can bring new insights.

As Director of MSRI I organized the conference, but as a non-expert I relied on a number of others to make sure that the important scientific aspects were well-covered, and to make sure that the conference would represent the best current science in the field. I am particularly grateful to Inez Fung, Bill Collins and Chris Jones for their scientific advice, and to Orville Schell for his advice and help in arranging the public event. Nat Simons provided expert suggestions as well as enthusiasm and great support throughout — without him the event could never have happened.

David Eisenbud

David Eisenbud
Director, Mathematical Sciences Research Institute, 1997-2007

CONTENTS

MATHEMATICS OF CLIMATE CHANGE

A NEW DISCIPLINE FOR AN UNCERTAIN CENTURY

Case Studies

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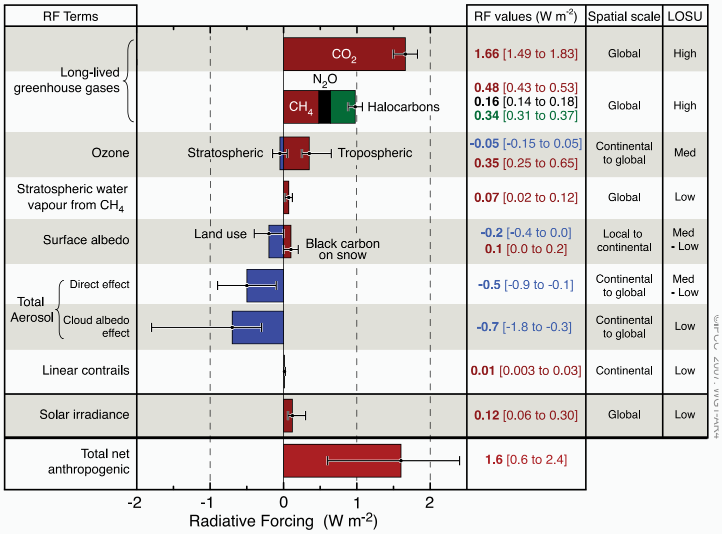
INTRODUCTION

WHEN THE HISTORY OF CLIMATE CHANGE is written, the years 2006 and 2007 may be seen as a turning point—a time when climate change ceased to be seen as a "green" issue and became an "everyone" issue. In 2006, Al Gore's movie "An Inconvenient Truth" placed global warming on America's movie screens. In October 2006, the British government released the Stern Review, a first attempt to quantify the economic costs of climate change. Over a period of four months, from February to May 2007, the Intergovernmental Panel on Climate Change (IPCC) released its fourth report on climate change, which attracted much more publicity than the previous three. In April 2007, the United States Supreme Court ruled that the Environmental Protection Agency has the authority to regulate carbon dioxide and other greenhouse gases. In October 2007, Gore and the IPCC shared the Nobel Peace Prize "for their efforts to build up and disseminate greater knowledge about man-made climate change, and to lay the foundations for the measures that are needed to counteract such change."

The increase in public discussion may reflect an increasing comprehension that the scientific debate over the reality of global warming has ended. (See sidebar, *How Do We Know?*) The IPCC's fourth assessment stated that warming of the climate is "unequivocal" and that it was "very likely" (meaning more than 90 percent likely) that most of the warming is anthropogenic. (See Figure 1.)

FIGURE 1: Radiative Forcing Components



Anthropogenic (human-induced) contributions to global climate change are measured in watts per square meter—in other words, the increase in solar radiation that would produce an equivalent warming effect. Some contributions (e.g., the greenhouse effect) are positive, and others (e.g., aerosols) are negative. However, the net anthropogenic effect since 1750, 1.6 watts per square meter, is unambiguously positive, and also significantly greater than the amount of warming due to natural fluctuations in the sun's brightness (0.12 watts per square meter).

Image from *Climate Change 2007: The Physical Science Basis*, Intergovernmental Panel on Climate Change.

There are many uncertainties, however, in the specifics of climate change and its impact. Climate models tend to agree on the twenty-year projections, both in regard to their sensitivity to variations in model physics as well as different emissions scenarios[1]. Disagreement arises when projections are carried out to the end of the century. For example, the equilibrium response to a hypothetical scenario, involving an immediate doubling of carbon dioxide, leads to varying predictions of warming from 1 degree Centigrade to a truly staggering 12 degrees. (Note that these should not be interpreted as literal forecasts, because an overnight doubling is impossible.) The difference arises primarily from uncertainties in the climatic feedback processes represented in the models, which tend to amplify the direct effects by two or three times.

Other aspects of climate change are even harder to predict accurately than temperature. We can be certain that precipitation patterns will change, and all the models indicate that some subtropical and tropical regions will experience severe droughts. But the models give contradictory predictions of where the droughts are likely to occur. As another example, scientists reported in early 2007 that glaciers in Greenland are melting faster than any of the models in the IPCC report had predicted. Clearly, there are processes going on that we do not understand. Yet the extent of the polar ice caps is a critical variable in climate models, because it triggers a feedback loop: the more the ice melts, the more sunlight is absorbed by Earth (instead of being reflected into space by the ice). This leads to an increase in temperature, which in turn stimulates more melting of the ice cap. This melting is of concern even to people who live far away, because the melting of glaciers on land is a major contributor to rising sea levels.

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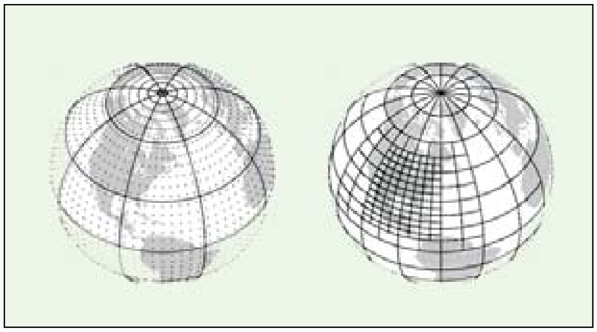
[1] Different assumptions about emissions of greenhouse gases can be used as inputs into the models.

The climate models used to make the IPCC projections are very complex systems of nonlinear equations solved on large computers. The IPCC relied on 24 different climate models in its report. Many of them have been developed under the auspices of national meteorological offices. Though some models are superior to others, identifying them publicly is a ticklish political issue. Because the models have different assumptions, we run the risk of comparing apples to oranges. In many of its scenarios, the IPCC report simply averages all the models together equally. It is not at all clear that this methodology is an optimal or even sound way to integrate the data.

A serious limitation of the current models is their coarse scale. At present, even the highest-resolution models chop up the world into pieces that are 10 to 50 kilometers wide. This resolution is not fine enough to capture important details of topography, such as mountain ranges, and it is also not fine enough to model individual clouds, which play a complex and important role in the climate system.[2] (See Figure 2.) In practice, model parameters, especially those that represent turbulent or fine-scale processes, are optimized or "tuned" in order to match available observations. For example, the effect of clouds has to be added to the model as an aggregate term, with all the uncertainties that implies. If the climate models could be improved to 1-kilometer resolution, then clouds and finer topography could be built into them; however, it has been estimated that this would require a 10-petaflop computer with 20 million core processors. That kind of computing power is on its way, but it is not here yet. Even when it arrives, it's questionable whether climate modelers can take full advantage of it. Many models are "legacy codes" of a half million lines or so that are not optimized for massively parallel computation.

FIGURE 2: Gridding



Climate models divide the world's atmosphere and oceans up into a very coarse grid. At present, even the best models do not have meshes fine enough to simulate individual tropical cyclones or the effect of mountain ranges. Future models may incorporate adaptive refinements of the mesh size, as shown on the right.

Finally, the mathematics of dynamical systems has taught us that uncertainty is an inevitable part of predictions based on nonlinear physical models. This "irreducible imprecision" requires us to use a variety of models, and run them with a diverse set of parameters, in order to capture the real range of uncertainty in the climate system. It also means that climate modelers must take care to communicate to policy makers that uncertainty is part of the story. As models improve and more information becomes available, the model forecasts may change, and this could lead to frustration among those needing to make decisions based on their predictions. This frustration might be avoided if the original predictions are presented as a range of possibilities rather than a single magic number.

Be that as it may, accurate and reliable prediction of global climate change is a key to policy making. It is clear that policies should be based on predictions that are built on a sound foundation. Mathematical scientists need to get involved, because the central questions facing this research are mathematical in nature.

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[2] For example, clouds provide an important negative feedback mechanism that could reduce global warming. As the moisture in the atmosphere builds up due to warming, it could create more clouds, which would reflect more sunlight back into space. However, this effect is by no means automatic; it depends on where the cloud is. High-altitude clouds radiate to space at a colder temperature and actually produce a net warming.

THE MSRI SYMPOSIUM ON CLIMATE CHANGE

How Do We Know?

THE EVIDENCE FOR CLIMATE CHANGE

Climate models and their projections for the future—especially extended out to 2100—are subject to a variety of uncertainties. These include imperfections in the climate models, the limitations of our computing power, and the inherently unpredictable nature of nonlinear equations. These uncertainties must not be allowed to obscure the central facts emphasized in this year's IPCC report: Climate change is happening, human activities are responsible for most of the change, and the evidence indicates that it is accelerating.

The basic facts that lead to this conclusion are the following:

1. Carbon dioxide levels (and levels of other greenhouse gases, such as methane) have been rising for at least half a century. In fact, they have risen by as much since 1960 as they did between the last Ice Age and 1960. (See Figure 1.1.) The current concentration of carbon dioxide in the atmosphere, 380 parts per million, is greater than it has been at any time in the last 650,000 years, according to ice cores that contain trapped bubbles of earlier atmospheres.
2. Carbon from fossil fuels is being added to the atmosphere. We know this because fossil fuels contain a lower ratio of the isotope carbon-13 to carbon-12 than the atmosphere as a whole does, because they are derived from plant matter and plants have a preference for the lighter isotope of carbon. Tree-ring and ice-core data show that the ^{13}C:^{12}C ratio began to decrease just at the same time the overall levels of carbon dioxide began to increase.

From April 11 to April 13, 2007, the Mathematical Sciences Research Institute (MSRI) convened a symposium, sponsored by the Sea Change Foundation, to assess how mathematicians can address the broader issues of climate change and the narrower issues of methodology lying behind the climate models. The symposium consisted of two parts. Several leading politicians, business people and academic experts on energy and climate convened for a panel discussion (see Figure 3) at San Francisco's Palace of Fine Arts Theater on April 11, which drew a crowd of more than 300 people. On the following two days, approximately 80 mathematicians and scientists attended a scientific symposium at the MSRI headquarters in Berkeley. (See Appendix B.)

FIGURE 3

The panel at MSRI's public symposium on climate change. Front row, left to right: Nancy McFadden, Doug Ogden, Michael Peevey, Inez Fung. Back row, left to right: Daniel Kammen, Severin Borenstein, Jerry McNerney, Ira Ruskin, David Eisenbud.

Inez Fung, the co-director of the Berkeley Institute for the Environment and one of the authors of the IPCC report, started off the public event with a brief overview of the evidence for global warming and the current state of knowledge about what will happen next. She characterized the IPCC report, which acknowledges that climate change has been caused by anthropogenic effects, as "a bittersweet victory, because we've been saying the same thing for 20 years." She outlined the reasons why we know that the climate is warming (see Sidebar, *How Do We Know?*), and she discussed the main forecasts from the IPCC report (see Sidebar, *A Bleak Future*).

After Fung's introduction, MSRI director David Eisenbud introduced Congressman Jerry McNerney (see Figure 4, page 7) and California Assembly Member Ira Ruskin (see Figure 5, page 7), who represents Silicon Valley. After brief remarks by McNerney and Ruskin, Eisenbud summoned onto the stage a panel of experts, which consisted of Daniel Kammen, professor of energy at the University of California at Berkeley; Severin Borenstein, director of the University of California Energy Institute; Nancy McFadden, senior vice president of public affairs for PG&E Corporation; Doug Ogden, executive vice president of the Energy Foundation in San Francisco; Michael Peevey, president of the California Public Utilities Commission; and Inez Fung, who had already been introduced. The legislators were given an opportunity to pose questions to the experts, and then the floor was opened to questions from the audience. The following section is based in large part on the questions and answers that ensued.

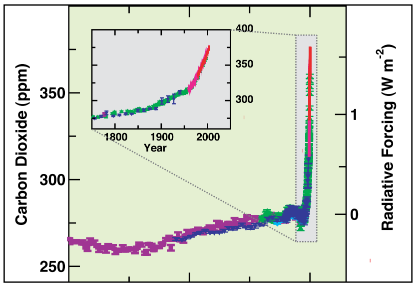


Figure 1.1 The concentration of carbon dioxide in the atmosphere over the last 10,000 years (main figure) and over the last 250 years (inset). Data from the last 50 years (pink and red) are based on direct measurement, and earlier concentrations are inferred from ice cores. At right, the concentrations are converted to an equivalent increase in solar radiation (using the year 1750 as a baseline).

Image from *Climate Change 2007: The Physical Science Basis,* Intergovernmental Panel on Climate Change.

FIGURE 4 (left). U.S. Congressman Jerry McNerney at the MSRI symposium.

FIGURE 5 (right). California Assembly member Ira Ruskin speaking at the MSRI symposium.

What actions is Washington taking to reduce global warming?

At present, the U.S. government is trailing both public opinion in the U.S. and many other world governments in addressing the climate-change problem. Nevertheless, there are some grounds for optimism. At the United Nations Climate Change Conference in Bali, in December 2007, the U.S. agreed to a "roadmap" for future negotiations that did not set specific emissions targets. Also, in that same month Congress passed and President Bush signed into law the Energy Independence and Security Act of 2007, which will increase automobile mileage standards to 35 miles per gallon by 2020 (the first change in the standards in more than 30 years).

As of July 2007, one hundred bills related to climate change had been introduced in the current session of Congress. For example, H.R. 2809, the New Apollo Energy Act, would set a target of decreasing greenhouse gas emissions to 80 percent below 1990 levels by the year 2050. It would institute a carbon cap-and-trade program, commit $49 billion in Federal loan guarantees for the development of clean energy technologies, offer tax incentives for consumers to purchase plug-in hybrid vehicles, increase funding for research and development of clean energy technologies, and create a venture capital fund to help move new technologies to market.

It is also important for the U.S. government to make climate change a part of its foreign policy, because climate change is an problem of unprecedented international scope. H.R. 2420, the International Climate Cooperation Re-engagement Act, would create an Office on Global Climate Change within the State Department and commit the U.S. to sending high-level diplomats to future international conferences on climate change. Though proposals like H.R. 2809 and H.R. 2420 did not become law this year, they represent an increased awareness of the climate change issue on Capitol Hill.

How is Sacramento addressing global warming?

The panelists emphasized that California can do little on its own to solve the climate change problem, because it is global in scope. Nevertheless, in the absence of concerted Federal action, California has played and can continue to play an important role as a model for other states and even other countries.

3. Evidence from ice cores shows a very strong correlation between carbon dioxide levels and global temperatures. (See Figure 1.2) When carbon dioxide levels go up, so does the temperature.

4. The physics behind the "greenhouse effect" is not in dispute. It has been known for more than a century that gases such as carbon dioxide, methane, and water vapor absorb infrared radiation coming from Earth (which would otherwise escape to space) and re-radiate some of its energy back toward Earth. Therefore an increase in greenhouse gases must lead to an increase in temperature, unless some other process comes along to prevent it.

5. Finally, Earth's surface temperature has increased sharply in recent years, just as one would expect. The observed warming trend over the last 100 years was 0.74 degrees per century, but over the last 50 years the rate of increase has nearly doubled, to 1.3 degrees per century. The six hottest years on record occurred in 1998 (an El Nino year), 2002, 2003, 2004, 2005, and 2006. The warming effect is now too large to be explained as a statistical aberration. (See Figure 1.3.)

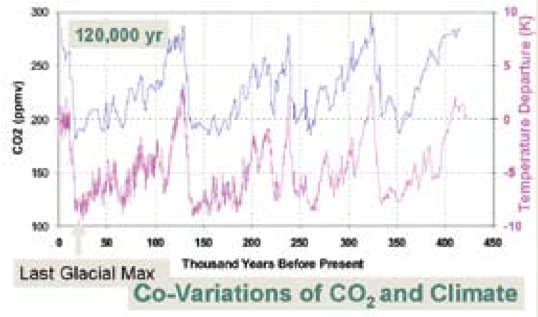


Figure 1.2 Over the last 400,000 years, global greenhouse gas concentrations (top) and estimated temperatures (bottom) have been extremely tightly synchronized.

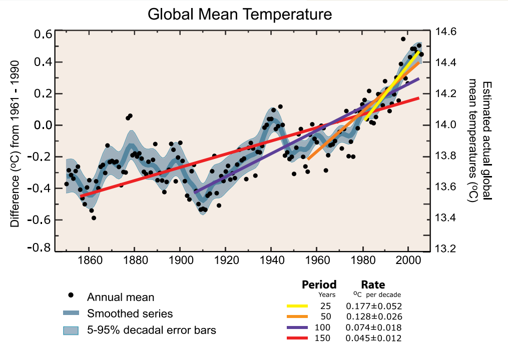


Figure 1.3 Direct measurements of global mean temperature leave little doubt that a warming trend exists, and is accelerating. The average trends over the last 150 years (red), the last 100 years (blue), the last 50 years (orange) and the last 25 years (yellow) have gotten progressively steeper.

THE IPCC REPORT: A BLEAK FUTURE

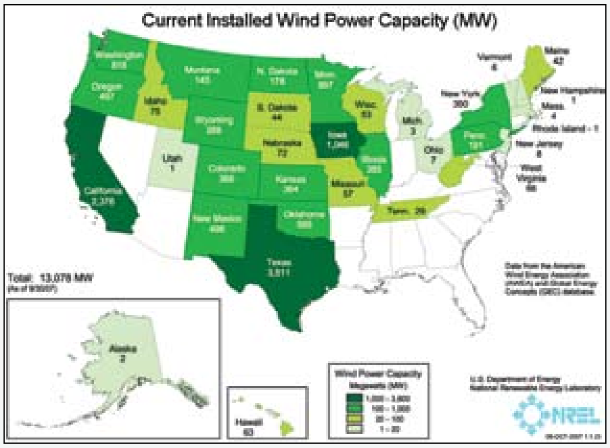
In 2007, the Intergovernmental Panel on Climate Change (IPCC) released its Fourth Assessment Report on the world's climate. The report has been released in three sections, produced by three separate working groups. A final "synthesis report" was released in November.

Working Group I focused on the physical indicators of climate change and projections of key climate variables into the future. This group used the combined output of 24 climate models to project surface temperatures, precipitation, and sea level changes out to the last decade of this century, under six different emissions scenarios. The best estimates of the temperature change range from 1.8 degrees Centigrade, in the most optimistic case (a world with high priority on sustainable development) to 4.0 degrees in the most pessimistic case (a "business-as-usual" world with intensive fossil-energy use). For comparison, the report also includes one "too good to be true" scenario, in which carbon emissions stay constant at 2000 levels. This scenario represents the minimum amount of climate change to which we are already committed: about 0.6 degrees Centigrade. The IPCC report specifically avoids any sort of "doomsday" scenario involving a widespread breakdown of social institutions (though such a scenario might have made for juicier headlines).

As explained elsewhere in this report, individual numbers do not adequately summarize the complexity of climate models. For instance, the "likely" range for the business-as-usual scenario is from 2.4 to 6.4 degrees Centigrade. This translates to a $^2/_3$ probability that the actual temperature increase would lie within the stated range, and a $^1/_3$ probability that it would be greater or less. The temperature increase is

In particular, the California assembly last year passed Assembly Bill 32, the Global Warming Solutions Act of 2006, which committed California to reducing its greenhouse gas emissions in 2020 to 1990 levels. The governor has proposed an allocation of $36 million to create the new positions required to implement the act—for example, to determine what exactly is meant by "1990 levels." Panelist Borenstein commented that we should focus on ways of meeting this target that are not "idiosyncratic to California," but can be exported to the rest of the world.

FIGURE 6



Map of installed wind-energy capacity in the United States in 2007, in megawatts. Wind energy potential is greatest in the Great Plains and Rocky Mountain states, but so far the exploitation of this resource has been very uneven.

On this year's docket, the California legislature is considering bills to create "green building" standards for the state government (A.B. 35); to provide funding for alternative fuel research (A.B. 118); and to create rebates on the cleanest new cars and surcharges on the dirtiest ones (A.B. 1493). The latter bill was voted down between the time of the symposium and the writing of this document.

What are the most promising technologies for mitigation of climate change?

Panelist Kammen commented that we should not look for a single "magic bullet," but should look to a variety of technologies. At present, Germany, Spain, and Denmark are leading the way in wind energy, but the U.S. has a large untapped potential (see Figure 6) and placed more new wind generation capacity in service than any other nation in 2006. The photovoltaic industry is still trying to reduce costs, but on a hot summer day solar power can be generated more cheaply than the spot market price of energy. Recently, scientists invented "spray-on" solar panels, a material like spray paint that can generate electricity from infrared light. Biofuels continue to be a major area of research, as scientists try to find crops that can be harvested to produce fuel more efficiently than corn. Energy efficiency is also an important field of research, with hybrid vehicles, plug-in hybrids, and all-electric vehicles leading the way. (See Figure 7.)

FIGURE 7

The Tesla Roadster has been cited often as a model for high-performance all-electric vehicles. As of 2007, no Roadsters are yet available for purchase, but reservations are being taken.

A serious issue for these new technologies, Kammen said, is how to move past the so-called "valley of death" of tiny market share and high cost. Policy-makers need to set aside money for these emerging technologies not only in the research stage, but also in the stage of moving them to market. The New Apollo Energy Act would be a step in that direction.

Will our efforts to reduce greenhouse gas emissions be overwhelmed by the increasing emissions from China?

Yes, but that doesn't give America an excuse for inaction.

China is very dependent on coal energy, and built 92,000 megawatts of new coal-fired plants in 2006—enough in one year to cancel the entire greenhouse gas reductions pledged by European nations under the Kyoto protocol. As long as the U.S. does not observe the Kyoto treaty, China will be able to hide behind America.

Even so, China has made major commitments to improve its energy efficiency. The eleventh Five-Year Plan calls for a 20 percent increase in energy efficiency by 2010. All of China's leading 1000 enterprises, which together account for one-third of the country's energy usage, have made a commitment to increase their energy efficiency by 20 percent. China has also pledged to derive 15 percent of its energy from alternative fuel by 2020.

Although China has recently passed America as the world's largest emitter of greenhouse gases, panelist Ogden said that it is important to realize that it also has a much larger population base. China's *per capita* energy use is still only an eighth of ours. It is difficult to tell the Chinese that they must cut back when they have not yet reached the standard of living and energy use that Americans enjoy.

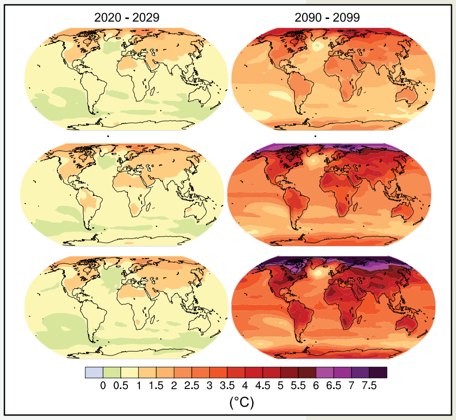


Figure 2.1 The IPCC's climate change simulations for the decade 2020-2029 (left) and 2090-99 (right), under three different scenarios. Note the uneven distribution of temperature change, with especially dramatic increases in polar regions.

Image from *Climate Change 2007: The Physical Science Basis*, Intergovernmental Panel on Climate Change.

Can renewable energy sources be integrated with the rest of the power grid even though several of them are "intermittent," i.e., not always available?

Wind and solar energy, of course, are not under our control. The wind doesn't blow when we tell it to, and the sun shines only during the daytime (and even then it may be obscured by clouds). Fortuitously, the time of peak availability of solar energy coincides with the time of peak demand. Wind energy can be "load-shaped," by using natural gas, for example, to fill in gaps in availability. Also, pricing schemes called "demand response programs" can help shift the demand from peak hours to other times of day. Battery storage may make it possible to distribute energy availability more evenly. Finally, some alternative energy sources, such as geothermal and biomass, do not have any intermittency problems.

Panelist Peevey noted that the California Public Utilities Commission is "committed by statute to obtain 20 percent of our energy from renewable sources by 2010, and committed by policy to obtain 33 percent from renewables by 2020." He expects that the state will in fact meet or come very close to the former target. Recent news reports, however, indicate that California is still well below the target, with 12 percent of its energy coming from renewables.

What kinds of governmental regulation would PG&E like to see?

Panelist McFadden said there was "no doubt that we need caps on carbon production." However, she felt that it would be a "heavy lift" to get such caps passed at a national level in the short term. As an intermediate target, she suggested that the rest of the country should improve its energy efficiency as much as California has. The per capita usage of energy in California has remained constant in recent years, while increasing 50 percent in the United States as a whole. Because California is a bellwether for the nation, California should continue doing more to improve its energy efficiency.

not uniformly distributed (see Figure 2.1) but is greater over land and much greater in the Arctic. Increases in precipitation are very likely in polar regions and droughts are likely in subtropical regions.

The consequences of these climate changes were explored in the Working Group II report. Many biological effects are already apparent, such as earlier spring blooming and shifts in the range of species. Under even the optimistic scenario, the report states that about 20 to 30 percent of plant and animal species are "likely to be at increased risk of extinction." A modest amount of global warming—less than 3 degrees Centigrade—would be favorable for global food production. However, greater temperature increases would have a negative effect, and in arid and tropical regions, even a small rise in temperature is expected to decrease crop productivity. Extreme weather events, such as floods and hurricanes, will become more common.

Finally, Working Group III reported on the potential of mitigation efforts to reduce greenhouse gas emissions. It concluded that options with "net negative costs"—those that save more money over the long run than they cost—can already reduce emissions by 7 to 10 percent, compared to the "business as usual" scenario. Further reductions, up to 46 percent, can be achieved with strong enough incentives, in the form of carbon trading and carbon taxes or fees.

Is ethanol from corn a boondoggle?

This question elicited some disagreement. Panelist Kammen said that his studies show that ethanol derived from corn is marginally more efficient than gasoline. Kammen went on to say, though, that it would be surprising if corn, which has been developed for generations as a food crop, happened to also be optimal for use as a fuel. (See Figure 8.) Other options, including switchgrass or landfill waste, will probably turn out to be better. Also, the net emissions effect of any biofuel will improve dramatically if the distillery runs on a cleaner energy source. There would be no point in building an ethanol distillery and powering it with a dirty coal-fired generator.

Panelist Borenstein remained skeptical. "Better ways to produce ethanol also cost a lot," he argued. He felt that the excitement over ethanol is motivated primarily by the economic self-interest of the Midwestern states.

FIGURE 8

Photo: John Cross, Minnesota State University.

"E85" gas pumps sell a blend of 85 percent ethanol and 15 percent gasoline. Ethanol has been highly touted in some places as a fuel that can reduce greenhouse gas production. However, the technology is problematic at present, because the distillation of ethanol itself requires energy that may come from a greenhouse gas-producing power plant.

Besides carbon dioxide, methane has also been implicated as a greenhouse gas. How serious a problem is it?

Molecule for molecule, the greenhouse effect of methane is 20 times stronger than that of carbon dioxide, but it is not as big a problem for several reasons. The absolute levels of methane in the atmosphere are much lower than those of carbon dioxide (though they, too, are rising fast). Second, methane remains active as a greenhouse gas for only about ten years before chemical reactions in the atmosphere break it down. Carbon dioxide, on the other hand is effectively immortal.

Finally, methane is harder to regulate than carbon dioxide, because much of it comes from agricultural sources, such as cattle and rice paddies. However, in this country, the main sources of methane are landfills and leakage from coal mines and gas pipelines. Therefore, an opportunity exists to control it, simply by reducing the amount of leakage and the amount of waste we put in landfills.

What are the prospects for nuclear power?

Surely one of the most controversial outcomes of climate change has been the rehabilitation of nuclear power. In the audience for this symposium, opinions were deeply divided, reflecting the ambivalence of society as a whole toward nuclear power. (See Figure 9.)

In California, at least, the legal status of nuclear power is clear. Under state law, no new nuclear power plants can be built in California unless and until the state government certifies that there is a safe way to dispose of the waste. With the fate of the Yucca Mountain nuclear repository still in limbo, it is clear that there will be no new investment in nuclear power in California for the foreseeable future.

Economically, nuclear power is less well understood than any other energy source. The costs of waste management and protection against terrorism need to be factored into the

ENERGY ECONOMICS

Electric fuel costs for different energy sources are difficult to compare. For natural gas, the greatest expense is the fuel itself. For nuclear power, the cost of fuel is relatively small but the cost of building the plant, running it safely, and decommissioning it is much higher. In addition, the costs may depend on location; natural gas, for instance, is cheaper in Texas, and geothermal energy is not even available in New England. The capital costs for nuclear power are particularly uncertain because no new plants have been ordered since 1977.

Nevertheless, it can be useful to compare the "levelized" cost of electricity, which amortizes the cost of an electric plant over its entire (estimated) lifetime. The Department of Energy estimates the following costs for new plants that would come online in 2015 and in 2030 (costs are given in cents per kilowatt-hour):

Year	*2015*	*2030*
Coal	5.6	5.4
Natural gas	5.5	5.7
Wind	6.9	6.3 (*)
Nuclear	6.3	5.9
Biomass		6.4 (*)
Solar Thermal		13.1 (*)
Geothermal		6.1 (*)

(*) These figures are specifically for a plant located in the Northwest.

Source: *Annual Energy Outlook 2007*, Figures 56 and 62.

As these figures show, the costs of several renewable energy sources are expected to come down to the

FIGURE 9

The Three-Mile Island nuclear power plant in Pennsylvania is a symbol of nuclear energy's troubled past. For many years, no new nuclear plants have been built in the U.S. because of concerns about safety and storage of spent fuel. With climate change now looming as a greater threat, even some former opponents of nuclear power are beginning to reconsider this carbon-neutral energy option.

cost of nuclear power, but no one has any idea how large these costs will be. Also, the nuclear industry gets a subsidy from the government, in the form of protection from insurance claims resulting from a catastrophic accident. Depending on your point of view, the value of this subsidy may be anywhere from zero to infinity.

Even putting aside these unknowns, nuclear energy has an uncommonly large range of costs. (See Sidebar, *Energy Economics*.) The cost of nuclear power presently ranges from 3 cents to 12 cents per kilowatt-hour. If America is going to embark on an ambitious new program of nuclear construction, we need to understand the reasons for this broad range of economies, standardize the designs, and choose designs that are cheaper and safer.

All in all, nuclear power is back on the table. But it seems unlikely that America, after shunning it for more than 20 years, is ready for the kind of huge ramp-up that would be required to have a significant impact on greenhouse gas emissions. The problems of safety and waste disposal are not mere public relations.

What is the status of carbon sequestration?

Sequestration refers to the process of burying carbon in the ground, the ocean, or in vegetation and soils. One method of sequestration involves injecting pressurized carbon dioxide into an oil field. This procedure can help companies extract more oil from it, so the process is sometimes called "enhanced oil recovery." Once injected, the carbon dioxide will—hopefully—remain isolated indefinitely from the atmosphere. Whether this is true in fact remains an open scientific question.

Carbon sequestration is attractive to large oil companies because it requires a minimal change from "business as usual" (and, in fact, can be seen as improving business). Several sequestration projects are already in place, in Texas, in the North Sea, and in Norway. BP has recently announced plans for a new clean energy plant in California, which would separate petroleum coke—a very dirty fuel that is currently shipped to China for burning—into hydrogen compounds and carbon dioxide. The hydrogen compounds would be burned cleanly, while the carbon dioxide would be sequestered.

What are the prospects for carbon cap-and-trade agreements and carbon taxes?

Although carbon cap-and-trade agreements may be a useful and even essential mechanism, panelist Borenstein said that he does not consider them a solution by themselves to the problem of greenhouse gases. Somebody, somewhere, has to cut back on the production of carbon. Another unresolved question is how to enforce the agreements so that no one can cheat on them.

point where they are nearly, but not quite, competitive with conventional sources. However, solar energy remains prohibitively expensive.

In the "reference" scenario of the Annual Energy Outlook report, renewable energy sources will not gain any ground as a percentage of the market between now and 2030. (See Figure 3.1.) They provided 9 percent of the U.S. overall output of energy in 2005, and they are forecast to provide 9 percent in 2030 as well. Coal power is projected to increase from 50 to 57 percent of the market. Meanwhile, the total amount of energy sold will increase from 3660 billion kWh to 5168 billion. Thus the total output of energy from coal will increase by 60 percent—an especially worrisome outcome for the climate, because coal plants produce the most greenhouse gases.

As noted in the report, changes in fuel prices or in environmental policies could affect all of these projections.

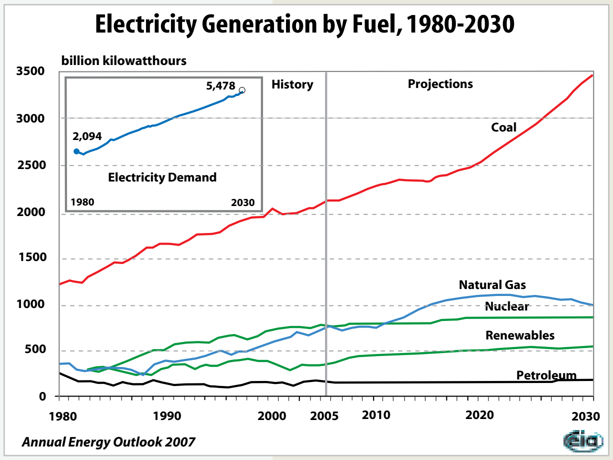


Figure 3.1 Sources of United States electric power, historically and projected through 2030. (Note that energy used for transportation or for heating does not appear in this figure.)

MATHEMATICS AND RENEWABLE ENERGY

How can mathematics contribute to the development of renewable or alternative energy sources?

This question was not discussed specifically at the symposium. However, some areas where mathematicians are currently making contributions include:

Fuel cells. The membranes in a fuel cell are made of a porous Teflon-like material, which allows ions to pass through. The process of pore formation involves differential equations that have not been solved before. A good mathematical model of the pores might bring down the cost of fuel cells, by reducing the amount of platinum required as a catalyst.[1]

Wind energy. The mathematical problems in designing a wind turbine are similar to those in designing airplane wings. However, to maximize energy efficiency, these “wings” have to push the limits of size and weight. Large, lightweight wings tend to flutter, so engineers need methods to predict and automatically compensate for this behavior.[2]

Carbon sequestration. Mathematical models of porous media are used to predict how long carbon dioxide will remain underground. One recent study showed that abandoned oil wells may compromise the ability of an oil field to store carbon dioxide.[3]

Nuclear energy. Mathematicians are helping to design the next generation of reactors. For example, researchers use computational fluid dynamics to model the flow of coolant past a fuel pin. They have showed that wrapping wire around the pins, like a stripe on a barber pole, can improve the mixing of coolant and bring down the temperature of the pins.[4]

Wave energy. Harnessing energy from ocean waves is still a technology in its infancy. Engineers used nonlinear optimization, a mathematical technique, to design a generator that produces energy from the relative oscillation between two floats. The product is expected to go on the market in 2010.[5]

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[1] Keith Promislow, NSF Award Abstract # 0708804.

[2] A. Balakrishnan, NSF Award Abstract # 0400730.

[3] Barry Cipra, “Geosciences Conference Tackles Global Issues,” SIAM News, June 2007.

[4] P. Fischer et. al., Large Eddy Simulation of Wire-Wrapped Fuel Pins I: Hydrodynamics in a Periodic Array. Joint American Topical Meeting on Mathematics and Computation and Supercomputing in Nuclear Applications, 2007.

[5] Scott Beatty, “Capturing wave energy off the coast of BC – a profile of an intern,” MITACS Connections, May 2007. The product is the SyncWave Power Resonator.

Nevertheless, carbon cap-and-trade agreements are popular in Washington because they use market forces. In the present political environment, Congressman McNerney said, it is simply impossible to talk about carbon taxes, even if they are called “fees.” The minute he arrived in Washington, his opponents began painting him as an advocate of carbon taxes, even though McNerney had never advocated them. Assembly Member Ruskin strongly echoed this last point. He recalled a conversation with an environmentalist in Europe who said that his country had wasted ten years debating a carbon tax. “We need to debate things that are possible,” Ruskin concluded.

How will climate change affect developing countries?

It seems certain that some of the effects of climate change will hit developing countries hardest. For example, subtropical and tropical regions are more likely to be subjected to drought. Low-lying island nations will be threatened by rising sea levels. Most importantly, poorer countries will not have the resources to adapt to climate change, while wealthier countries will. For all of these reasons, plus simple cost-effectiveness, investing in energy efficiency is the fairest and most universal approach to mitigating climate change.

What can individuals do about climate change?

For individuals as for countries, the most cost-effective solution is to reduce consumption through energy efficiency—for example, changing from incandescent to compact fluorescent lightbulbs. Several California communities provide good examples of action a local level. For example, Palm Desert has reduced its energy usage by 30 percent.

While individual and local conservation efforts are important, panelist Fung noted that there is one other remedy that citizens should be ready to use: the vote. “The problem is so large that we need state and government-level action. That means voting,” Fung said. Congressman McNerney noted that the League of Conservation Voters’ “Dirty Dozen” list had proven very effective in the 2006 election. Assembly Member Ruskin added that McNerney was being too modest, because he had personally defeated one of the “Dirty Dozen” incumbents in 2006.

CLIMATE CHANGE MODELING

THE SCIENTIFIC WORKSHOP PORTION OF THE MSRI SYMPOSIUM ON Climate Change convened in Berkeley on April 12 and 13. (See figure 10.) In this session, the focus shifted from local action to global models, and from energy policy to climate issues.

The symposium was organized into six groups of lectures (see Appendix A), which provide the source material for this section. In addition, discussion groups were formed to identify research problems in climate models that would be amenable to mathematical research. The following two sections, "Research Topics in Climate Change" and "Opportunities and Challenges for Mathematical Sciences," are based in part on the reports of the discussion groups.

FIGURE 10

MSRI's scientific symposium on climate change brought about 80 mathematicians and climate researchers to Chern Hall. Christopher Jones (far left, facing camera) was instrumental in organizing the two-day symposium.

The questions below give a representative, though not exhaustive, sample of the issues discussed in the lectures.

What goes into a climate model?

The main components of a climate model are the atmosphere, the ocean, land, and ice. As shown in Figure 11, the atmosphere model incorporates four main differential equations, which relate the motion of air to the physical inputs. First, the momentum equation relates the acceleration of any parcel of air to the forces on it: the pressure gradient, gravity, and friction. This equation also includes the Coriolis force, from Earth's rotation, and a nonlinear inertial term. The conservation of mass equation says that matter is neither created nor destroyed. The energy equation says that the energy of a unit of atmosphere can change in two ways—by changing the temperature or by advection (conveying the warm or cold air somewhere else). The net of these two effects is governed by four energy inputs: short-wave radiation from the Sun, long-wave radiation from Earth, sensible heat, and latent heat (the heat stored or released in water when it changes phase). Finally, a separate water vapor equation says that the amount of water in the atmosphere changes by advection as well as by evaporation or condensation. This equation determines the water vapor content of the atmosphere, which in turn affects its density and pressure, and in this way feeds back into the momentum and mass equations.

FIGURE 11: ATMOSPHERE

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \bullet \nabla \vec{u} + 2\Omega \times \vec{u} = -\frac{1}{\rho}\nabla p + g\hat{k} + \bar{F} + \boxed{\Im(\vec{u})}$$

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{u}) = 0$$

$$p = \rho RT;\ \rho = f\ (T,q)$$

$$\frac{\partial T}{\partial t} + \vec{u} \bullet \nabla T = SW \updownarrow + LW \updownarrow + SH + LH + \boxed{\Im(T)}$$

$$SW = f\,(clouds, aerosols \ldots)$$

$$LW = f\,(T,q,CO_2, GHG \ldots)$$

$$\partial q + \vec{u} \bullet \nabla\ q = Evap - Condensation + \boxed{\Im(q)}$$

$\Im$ convective mixing

Equations of a typical climate model. The first differential equation reflects conservation of momentum, and the second expresses the conservation of mass. This equation is coupled to the first by the ideal gas law (line 3). The third differential equation (line 4) models the energy flux, with short-wave radiation (SW) and long-wave radiation (LW). The latter term includes the effects of carbon dioxide (CO_2) and other greenhouse gases (GHG). The final differential equation tracks the motion of water in the atmosphere, and must be coupled to an ocean model. Several of these equations also include terms (red boxes) that model convection in the atmosphere, which is still poorly understood because it occurs at such a small scale (the scale of individual clouds).

Uncertainties in these equations enter on the physics side. How much energy is coming in from the sun? How much is reflected into space by clouds or aerosols? What is involved in the turbulent mixing in the atmosphere, which governs the formation of clouds? The effect of convective mixing is added in as an extra term in the momentum, energy, and fresh water vapor equations. Every climate model does this differently.

The ocean models likewise contain equations for momentum, mass, and energy, plus a fourth equation describing the salinity. The ocean exchanges momentum, energy, and water with the atmosphere, so these equations are linked to the previous four. Salinity affects the ocean in much the same way that water content affects the atmosphere: it changes the water's density, which in turn changes the pressure gradient. Two very important parts of the ocean model are the wind-driven ocean currents at the surface, and the thermohaline circulation, which takes place deep in the ocean. (See Figure 12.) The "thermo" part of this word reflects the fact that cool water tends to sink, and warm water tends to rise. The "haline" part refers to salinity, and the fact that saltier, denser water tends to sink, while less dense fresh water tends to rise. The interplay of these two factors creates a worldwide "conveyor belt" of water that redistributes heat from the equator to the poles, and is believed to have a strong moderating effect on our climate.

Like the atmosphere models, the ocean models are complicated by convective mixing. They also have to deal with the complicated geometry of coastlines and the ocean floor. The rearrangement of continents has had a huge effect on ancient climates. However, on a time scale of hundreds or thousands of years, the arrangement of land masses can be assumed constant.

THE SEA ICE CONUNDRUM

One of the most dramatic yet least understood effects of global warming is taking place in the Arctic Ocean, where both observational data and climate models point to a rapid melting of the polar ice cap. The extent of the ice cap at the peak of its summer melting has been decreasing by 8 percent per year since 1979. The area covered by sea ice in the winter has not decreased as rapidly, because the ice pack tends to recover during that season. However, the thickness of the ice cap in winter is decreasing. As the amount of recovery during the winter decreases, the extent of the ice pack in summer will also tend to decrease.

It is well known that melting sea ice causes an amplifying feedback loop, called the ice-albedo feedback, which tends to exacerbate global warming. Melting ice leaves more open water exposed, which in turn absorbs more solar energy rather than reflecting it into space. All of the climate models incorporate this feedback loop, and as a result they predict much steeper temperature increases in the Arctic than worldwide (See Figure 4.1).

Unfortunately, sea ice is also one of the least well-understood ingredients in the climate change puzzle. Not only is the amount of warming expected in the Arctic greater than the rest of the world, but the uncertainty in this forecast is also greater. In the IPCC climate models, while the equatorial regions face a 2 to 4-degree increase by the end of the century, the North Pole region is predicted to warm up by 4 to 12 degrees. And the different models for the extent of sea ice vary extravagantly (see Figure 4.2). Some of them show the summer ice pack virtually disappearing in the Arctic Ocean by mid-century, while others predict only a moderate decrease. This figure is more a confession of our ignorance than a reliable prediction. (Actual observations in this figure are shown by the heavy red line.)

Figure 13 represents in schematic form the various processes that enter into a climate model. As described above, of the processes illustrated here, the most challenging for modelers to get right are the clouds (which are too small-scale for the current generation of models to describe accurately) and the ocean currents, including the vertical motions. The modeling of the solid phase of water presents its own peculiar problems. (See Sidebar, *The Sea Ice Conundrum*.)

Finally, there are significant unanswered questions about the amount of incoming solar radiation—the "solar constant," which is currently estimated at 1362 watts per square meter—and how constant it really is. The total amount of anthropogenic forcing of the climate since 1800 is estimated at 1.6 watts per square meter. Thus, even a tenth of one percent variation in the "solar constant" would equal the entire human impact on the world climate. At present, there is no evidence that the solar constant has varied that much in the last 200 years. However, it may have varied by that much in the past. What will happen to it in the future is beyond the expertise of climate modelers, who have to ask solar physicists for the answer.

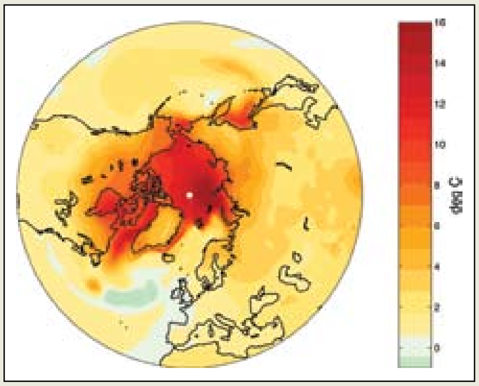


Figure 4.1 Predicted winter temperature increase by mid-century (2040-59 against 1980-99). Winter warming in the Arctic is at least double the global mean and peaks at more than 16 degrees Centigrade in some places.

What is left out of climate models?

Figure 13 omits one important ingredient in the climate: the entire carbon cycle. In fact, this figure represents the status of climate models about 5 years ago, when the work behind the fourth IPCC Assessment was being done. At that time, the concentrations of greenhouse gases like carbon dioxide and methane had to be added in as an exogenous forcing term. Newer models are beginning to incorporate the carbon cycle: the effects of plants and animals, the effects of fossil fuel burning, and the dozens of chemical reactions that convert one form of carbon to another in the ocean.

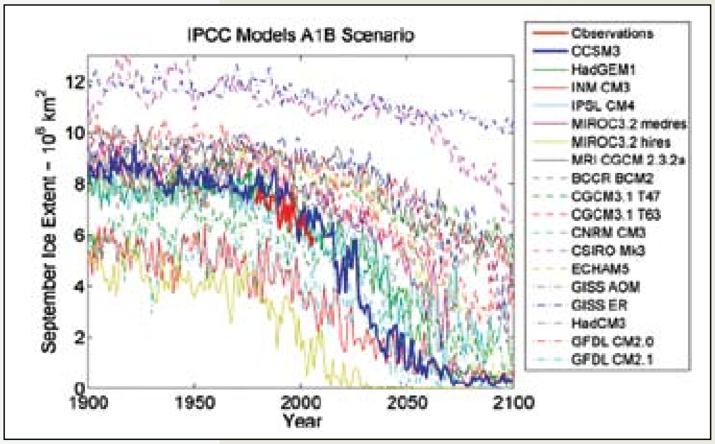


Figure 4.2 Projected summer ice extent in the Arctic, in the business-as-usual scenario. The heavy red line represents observed data; the remaining lines represent 18 different climate models used by the IPCC. Different models disagree widely, due to different assumptions about sea ice physics. However, the rate of retreat of ice in the 21st century is significantly correlated with the mean ice extent in the late 20th century.

Image from *Climate Change 2007: The Physical Science Basis*, Intergovernmental Panel on Climate Change.

Another omission will be even more challenging to repair: None of the models contain any humans. Again, in the IPCC simulations the results of human activities (primarily the production of greenhouse gases) are simply added in by fiat. However, such an approach is not completely satisfactory. Even in the absence of deliberate governmental policies, the change in climate will produce changes in human behavior. Different crops will be planted, different regions of the world will become suitable or unsuitable for agriculture, and so on. A truly integrated model should include these effects (see Sidebar, *Climate and the Indian Rice Crop*, page 16).

Furthermore, climate modelers realize that in the new environment, decision makers will be consulting their models more and more frequently, and they will ask different sorts of questions. Instead of "How much will the temperature rise?" they will ask "How much will it cost?" or "What are the impacts?" In other words, climate variables will eventually have to be restated in economic or social-justice terms. Some preliminary efforts to do this have been made. For example, the Stern Review in the U.K. was an attempt to delineate the economic impacts of climate change. Another example, presented at this meeting, was Max Auffhammer's study of the effects of climate change on agriculture. Nevertheless, a true integration of climate and economic models remains in the future.

FIGURE 12

The thermohaline circulation of sea water, driven by differences in temperature and salinity, has a major impact on world climate. The shutdown of this circulation is often cited as a tipping point that could lead to dramatic global cooling.

How might economics enter into climate change models and strategies?

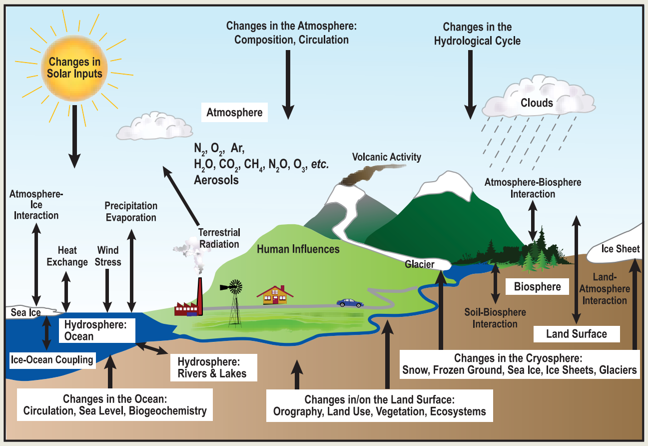
In order to formulate the results of climate change in economic terms, modelers will have to learn from the great advances economists have made in quantifying risk. However, some conference attendees expressed concern at a too narrow, market-centric approach to defining risk. First, such an approach might not give adequate weight to the interests of people who do not participate in financial markets, such as native peoples, developing countries, or unborn generations.[3] Also, a purely economic approach might downplay the importance of outcomes such as species extinctions.

Possibly a separate issue, but nevertheless important, is the question of how we can economically reach a desired emissions target. Can we get there using market forces and cap-and-trade agreements? Do we need a carbon tax? Some attendees suggested using game-theory approaches to design agreements that would be self-enforcing—in other words, to give both parties an economic incentive to abide by the agreement.

How does a climate model differ from a weather model?

The physical equations in a climate model are similar to those in a weather model, and some speakers argued that there is no real difference between them. However, the time scales involved are vastly different, and the nature of the questions asked of them is different as well. Weather models track the evolution of weather systems, and lose

FIGURE 13



This diagram illustrates the variety and complexity of interactions that enter into the current generation of global climate models. Note that the current models omit one key ingredient: the feedback between the climate and human activities.

The models perform so erratically for several reasons. First, they vary up to 50 percent in their estimates of cloudiness. That translates to a variation of 40 watts per square meter in the amount of solar energy reaching the surface—an uncertainty that swamps the greenhouse gas effect. (Remember that the anthropogenic change in carbon dioxide accounts for 1.6 watts per square meter.) In addition, none of the models treat sea ice in a physically realistic way. They have just begun to incorporate the thickness of ice as well as its extent, and they do not yet include an estimate of the floe size distribution. As floes get smaller, there is more contact between ice and water and hence more rapid melting in summer (and freezing in winter). In general, climate models treat sea ice as a homogeneous and continuous medium, but both assumptions are wrong. Sea ice varies in thickness and composition, and it is highly fractured.

Why is it important to model sea ice correctly? First, the regional impacts are huge. The opening of the long-sought Northwest Passage in the Arctic Ocean could be an economic boon to Canada, or an ecological nightmare. The lifestyles of native populations would be threatened by the retreat of the ice. Species like the polar bear, which depends on the ice, would suffer even more.

In addition, the extent of sea ice has global ramifications. No, the melting of sea ice does not raise the sea level (a popular misconception), because the ice and water are already in hydrostatic balance. But the melting of land ice would cause sea levels to rise. The increase in temperatures caused by the ice-albedo feedback affects glaciers on land, too—and indeed, observations show that the ice on Greenland is melting even faster than predicted. Finally, melting of sea ice also reduces the salinity of the ocean, an important ingredient in all climate models. An extreme possibility would be a shutdown of the oceans' temperature and salinity-driven circulation. This is the circulation that brings warmth from the tropics to the mid-latitudes, by powering ocean currents such as the Gulf Stream. Such a shutdown would produce a negative feedback, and would alter the climate dramatically.

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[3] The Stern Review took a very hard-line position on this issue, arguing that all generations should be treated equally, which implies a "discount rate" of 0 percent. Other economists have questioned this assumption and argued that it leads to an unrealistically high estimate of the current cost of climate change.

accuracy after four or five days, and there is no real point in running them beyond a few months. On the other hand, for climate predictions we are interested in running the models decades, even 100 years into the future.

Given the difference in time scales, an uncharitable observer might wonder whether a climate model is anything more than an expensive random number generator. The answer is yes and no, because the type of question one asks of a climate model is different.

Climate is, by definition, the statistics of the weather of an area over a long period of time, including the long-term mean, the variability, and the extremes. The mean climate is what remains of weather after you average out all the fluctuations. Numerical weather forecasting is, by contrast, all about the fluctuations—how the weather yesterday is going to change today, tomorrow, and so on. The goal of a mathematical model is to capture the average state, and even a probability distribution of deviations from the average, and is not to predict, say, the wind in Edinburgh on December 13, 2080. If one imagines boiling water in a pot, weather prediction is analogous to describing the location of the bubbles, while climate describes the temperature in the pot. From another perspective, weather prediction is an initial-value problem whereas initialization is less important in the climate. Predicting tomorrow's weather is based on information about today's. But in climate change the predictions are about seeing what happens under different forcing scenarios. For instance, how will the climate system respond to the doubling of CO_2, or a change in the amount of energy from the Sun?

However, there remain some serious issues with climate models that make them a good deal less predictable than the temperature of the heated water (in the analogy above). First, climate models cannot be completely tested and validated, while weather models are validated every day. The climate forcing of a century ago is poorly known, and observations of what actually happened are sparse. Climate models are assessed by plugging past forcing data (e.g. aerosols from volcanic eruptions) into them and comparing the predicted climate states with available observations.

Unfortunately, though, some models are already using the observations to estimate model parameters. That makes it impossible to validate the model independently with past data. Finally, even if a model tested out satisfactorily against the more static, pre-1950 climate, it would not necessarily give correct answers for the changing climate of today because the processes that are important for the future climate, such as sea ice and glacial dynamics, may not be operating in the same way in the static early 20th century. Another difference between weather and climate models is that weather models are constantly assimilating new data to update the initial conditions for the next prediction. Tomorrow's forecast will be based on a combination of today's forecast and the new observations accumulated over the next 24 hours by weather instruments and satellites. This allows the weather models to get back on track quickly after an unsuccessful prediction. On the other hand, if a climate model gets off track by 2030, its predictions for 2100 may be completely invalid.

Why do different climate models disagree?

First, it is worth pointing out that there are significant areas of agreement. All the climate models agree that global warming is a reality, and their predictions for 2030 are also in rough agreement. Their predictions for 2100, however, span a wide range.

One reason for the wide range is that the models prioritize differently the processes on the physical side of the equations—particularly the processes that are not well understood, such as convective mixing in the atmosphere and ocean, and the formation of clouds, and hence represent them differently. To some extent, this divergence among models is a good thing. Most

CLIMATE CHANGE AND THE RICE HARVEST IN INDIA

A recent paper published in the *Proceedings of the National Academy of Sciences*[1] exemplifies the insights that can be obtained from an integrated model that combines climate and economy. Global climate models show that the effect of greenhouse gases is reduced, to some extent, by industrial haze in the atmosphere. Aerosols absorb solar radiation and release it back to space, thus reducing the energy that reaches Earth's surface from the sun.

Figure 5.1 Rice harvest, Kashmir, Pahalgam, India.

The PNAS study highlights an economic system where greenhouse gases and aerosols have a complementary, not offsetting, impact: the Indian rice market (see Figure 5.1). Rice grows better when nighttime temperatures are cool, which suggests that greenhouse gases would reduce rice output, while the "Indo-Asian haze" would increase it. On the other hand, rice requires plenty of rain during the monsoon season. But the Indo-Asian haze tends to reduce rainfall, by reducing the temperature gradient between the southern and northern Indian Ocean. Thus a purely climatic viewpoint leads to ambiguous conclusions for the effect of aerosols.

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[1] M. Aufhammer, V. Ramanathan, and J.Vincent. Integrated model shows that atmospheric brown clouds and greenhouse gases have reduced rice harvests in India, *Proc. Natl. Acad. Sci.* 103 (2006), no. 52, 19668-19672.

climate modelers agree that there is no such thing as a “best model,” and it is useful to have a variety of models to sample the space of different possibilities. In fact, when weather models are run on the time scale of months, to make seasonal predictions, an ensemble of several models will usually perform better than any individual one.

In addition, a certain amount of “tuning” of the models is standard practice. Ideally, this is justified on the grounds that climate scientists can use the observations of different climate variables (e.g. cloud top height and sea surface temperature) to deduce the best parametric relationships linking them. But in practice, tuning is, as one participant said, “a subterranean process where all that’s reported is the outcome.” Some models are tuned to the point where they actually violate well-known laws of physics. It might be desirable to discard or discount models that are known to be less trustworthy, but politically this is hardly feasible.

Finally, another reason that models differ is that the climate system itself is inherently unpredictable. Precipitation is especially difficult to forecast accurately (see Sidebar, *Rainfall: Beyond “It’s Warmer, So It’s Moister”*). Even a mathematically exact model started from two slightly different initial conditions may not be able to issue similar precipitation forecast a season ahead, because precipitation processes, such as evaporation and condensation, are inherently non-linear. At best, it would offer a range of possibilities and a most likely case—and indeed, this is the way that the IPCC presents its model results. The chaotic dynamics within the climate system make it impossible to do better.

This inherent uncertainty may explain why a suite of models will outperform a single model. A well-designed ensemble might be able to sample different parts of “parameter space” or “model space” and in this way more clearly outline the uncertainties in the climate forecast.

There was a very strong consensus at the symposium that communicating the uncertainty in the model predictions was a difficult and important challenge for climate modelers. One speaker worried that as the models improve, they will inevitably give slightly different answers from the old ones, and it will look to the public as if the climate modelers are changing their minds—when in fact the new predictions may lie within the error bars of the old predictions.

This is not merely an academic concern, as proved by some of the press coverage of the fourth IPCC report. The media made a fuss over the fact that the predicted rise in sea levels was not as great as in the third IPCC assessment. Did this mean that global warming was not going to be as bad as predicted? Not at all. It meant that the uncertainty had been improved, and in fact the modelers had been more honest—they had no longer attempted to quantify the uncertainty in sea ice melting, because the process is not well enough understood. An improved product turned into a black eye for the modelers, as they were forced to explain that they weren’t backing down on the dire consequences of global warming.

How can we combine the results of different models?

In general, the IPCC averages the outcomes of the different models and reports an ensemble mean, along with error bars representing a 66 percent confidence interval. Such an approach would be statistically valid if the models represented independent random samples from a

However, a combined climatic-economic analysis tells a different story. When early-season rainfalls fall short, farmers respond by shifting the acreage planted in rice to other crops. In this way, economic factors enhanced the impact of the aerosols. The article concluded that, over the period from 1985 to 1998, aerosols led to a 10.6 percent reduction in the rice harvest, compared to the harvest in a simulated climate without aerosols. The combination of aerosols and greenhouse gases reduced the rice harvest by 14.4 percent over the same period of time. These results coincided with a period when India’s rice production, which had grown rapidly in the 1970s and early 1980s, began to grow more slowly and eventually leveled off. The study suggests that the increasing levels of aerosols and greenhouse gases in the atmosphere were responsible.

The interaction between climate and human behavior, driven by economic factors, was crucial for understanding the effects of the aerosols. “Most of the effect isn’t on the plants themselves, but on the farmers shifting to other crops,” Auffhammer said. In spite of the title’s description of an “integrated model,” the interaction between climate and economy in his paper was fairly simplistic. Auffhammer simply took the outputs from a climate model and plugged them into a regression equation to predict the farmers’ response. In the future, he says, climate scientists and economists should work together on the same model. “Instead of merely downloading data, we need a spirit of true collaboration across disciplines,” he said.

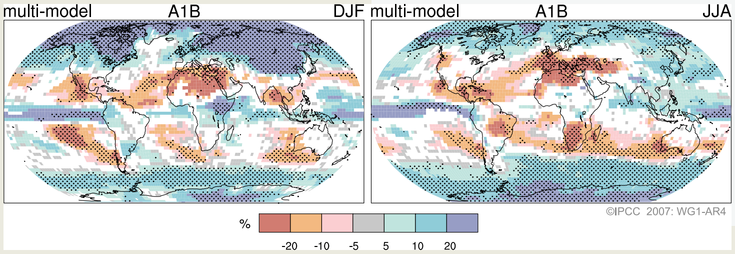

Figure 6.1 Precipitation changes for the decade 2090-2099, relative to 1980-1999. “Business-as-usual” scenario, December-February (left) and June-August (right). White regions indicate where fewer than two-thirds of the climate models used for the IPCC report agreed on the direction of change; shading indicates where more than 90 percent of them agreed.

Image from *Climate Change 2007: The Physical Science Basis*, Intergovernmental Panel on Climate Change.

single "model space." However, the existing 24 models used in the IPCC report are in no way a rationally designed, systematic exploration of model space. They are a sample of convenience. Moreover, as some participants pointed out, there is a danger of throwing out the physical baby with the statistical bathwater. Differences between models may result from physical phenomena that are represented correctly in one model and incorrectly in another. Obviously, it will be a challenge to modelers to try to distinguish chance effects from differences with real, physical causes.

One speaker illustrated the problem with averaging by a colorful parable. Three statisticians are asked whether it is safe to cross a river. They construct separate models of the river, and each one finds that the river is deeper than 6 feet in some place. But they disagree on where. So they average their models, and find that in the ensemble mean, the river never gets deeper than 3 feet. As one might guess, the parable ends with the statisticians drowning. (See Figure 14.)

All in all, there must be a better way than taking a mean. A weighted average, which takes into account each model's strengths and weaknesses, might be an improvement. Even better would be a Bayesian (machine-learning) approach, described by one speaker. In this approach, one model is omitted from the ensemble, and then treated as a "new" model that changes the *a posteriori* probability of various climate outcomes. Then a different model is left out of the ensemble, and the process is repeated. After this process is repeated many times, one can bootstrap up to a reasonable weighting of the different models.

How can we downscale global models in order to obtain local predictions? How can we upscale local effects to incorporate them in global models?

Several modelers felt that the issue of "unresolved processes" or "sub-grid processes" was crucial. They are besieged with questions like, "What will happen to this species?" or "How will this affect the water supply in that state?" For elected officials, what really matters is what will happen in their community or their constituency. If the climate modelers shrug their shoulders and say they don't know, they will lose credibility (even if that's the honest answer).

The one obvious solution is more computing power, in order to resolve the models down to smaller and smaller grid sizes. As computers have steadily increased in power, the resolution of climate models has improved as well. For example, as seen in Figure 15, the grid sizes in the IPCC's four assessment reports, over a period of less than two decades, have shrunk from 500 kilometers to 110 kilometers. Even so, the grids of all global models are too coarse to resolve individual clouds, or even to represent a hurricane realistically.

Besides increasing computer power, there are several other options for modeling subgrid processes. One is adaptive mesh refinement, in which the size of the grid is reduced in regions that require more detail—say, a storm system or a mountain

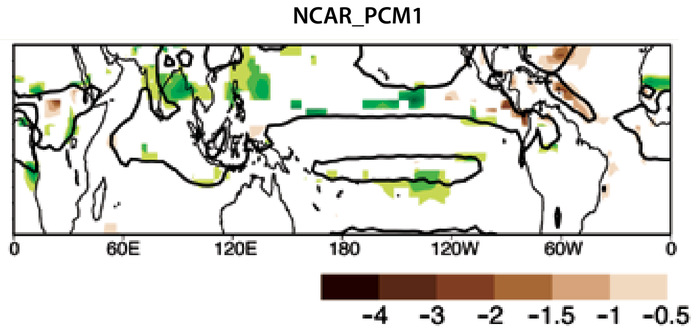

Figure 6.2 Two of the models that were used in the IPCC forecast disagree on the precise location and magnitude of precipitation increases or decreases. Nevertheless, the overall message of the models is fairly consistent, with increased precipitation in the tropics and decreased precipitation in the subtropics. (Units in the figure are 0.1 mm of rain per day, with increases in green and decreases in red.)

Image from *Climate Change 2007: The Physical Science Basis*, Intergovernmental Panel on Climate Change.

RAINFALL: BEYOND "IT'S WARMER, SO IT'S MOISTER"

The public's attention in discussions of climate change has always tended to focus on the increase in temperature. Indeed, the most popular term for many years was not "climate change" but "global warming." However, some of the most disruptive effects of climate change are likely to involve precipitation: severe storms, floods, or droughts.

It makes sense that an increase in global temperatures should lead to an increase in global precipitation. Warmer air can hold more water vapor, and with more water vapor in the atmosphere there should be more clouds and eventually more rainfall. However, common sense can be misleading. Where water vapor is concerned, it's not necessarily true that what goes up must come down. The warmer air could simply hold onto the extra water. For this reason, the IGCC report predicts only a 1 to 3 percent increase in global precipitation per degree of global warming. However, satellite observations disagree: Over the last 20 years, the precipitation increase has been closer to 7 percent per degree of warming.[1]

Precipitation also has a much more complex pattern of local and regional effects than temperature. Indeed, it is hard to find any place in the world that will have a decrease in temperature between now and 2100. But precipitation will decrease dramatically in some places, while increasing in others (see Figure 6.1, page 17). Even under the conservative assumptions of the climate models, many areas are predicted to have precipitation changes well over 20 percent.

Unfortunately, the different climate models used for the IPCC report disagree strongly on the regional details (see Figure 6.2). Given the extent of disagreement, can we say anything solid about rainfall?

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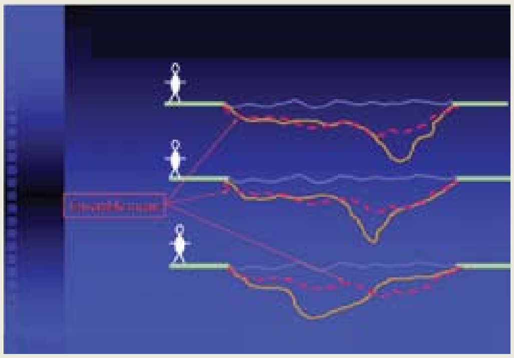
[1] F.J. Wentz et.al., *How Much More Rain Will Global Warming Bring?* Science 317 (2007), 233-235.

FIGURE 14



Parable of the statisticians (after Lenny Smith). Three statisticians independently forecast that the river is unsafe to cross, but average of their profiles of the river bottom (red dashes) indicates that it is safe. Smith told this story to illustrate the dangers of relying on "ensemble means," instead of critically examining each model on its own merits.

range. As one speaker pointed out, this has to be done with caution because it can lead to unrealistic artifacts along grid boundaries. Methods do exist for understanding what causes these artifacts and controlling them. Another speaker discussed "Levy noise," which would allow for a more realistic depiction of atmospheric turbulence.

Everybody is looking forward to "petaflop computers," which might bring cloud-scale processes into the picture for the first time. However, as explained next, Moore's Law has some caveats.

What kind of results can we anticipate from next-generation computers?

In a word, the world is going parallel.[4] Moore's Law (which says that the number of transistors on a chip doubles every year and a half) is still going strong, but clock speeds are not keeping up, because the heat density inside today's chips is getting too high. Parallel processing is a way to compensate for the lack of improvement in speed. The most powerful processors today are, ironically, made for computer games, and they typically have eight cores. Programming for these machines is not easy, and it may not become easy until new languages are invented.

It is not clear that climate modelers are ready for the new computing environment. Their programs typically have half a million lines of code, and it will be a non-routine task to convert them to work on parallel processors. Climate modelers will have to think about what algorithms can work efficiently on parallel processors. For example, adaptive mesh refinement, though it is desirable for other reasons, is very tricky to implement on a parallel machine. In all likelihood, it is not the climate modelers who will have to solve these problems but the postdocs and graduate students whom they hire. But this talent will not

In fact, according to David Neelin, the situation is not as bad as it looks. The predictions do follow a pattern that makes physical sense, which he calls the "rich-get-richer" model of precipitation. The regions that will see the greatest rainfall increase are precisely the ones that get the most rainfall now, the tropical latitudes. And the big decreases in rainfall will occur at the edge of those regions, where increased advection will bring dry weather in from the subtropics. Thus the models agree on the physical processes. They disagree on the precise location of the wet and dry spots because of differences in wind circulation from model to model.

In a few regions the models did produce consistent predictions. In the Caribbean, nine or even all ten of the ten models in Neelin's survey agreed that there will be a more than 20 percent drop in precipitation. And indeed, 50-year precipitation records in the Caribbean already show a pronounced decrease (See Figure 6.3). Neelin concluded that these regions need to take the climate forecasts very seriously.

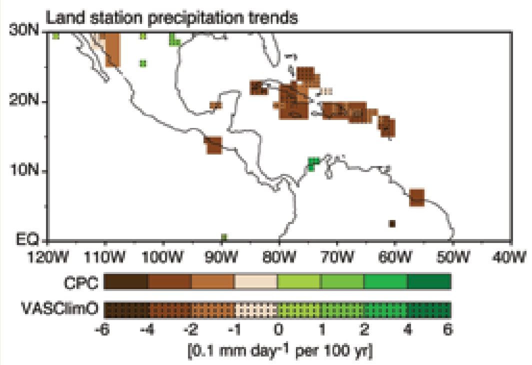
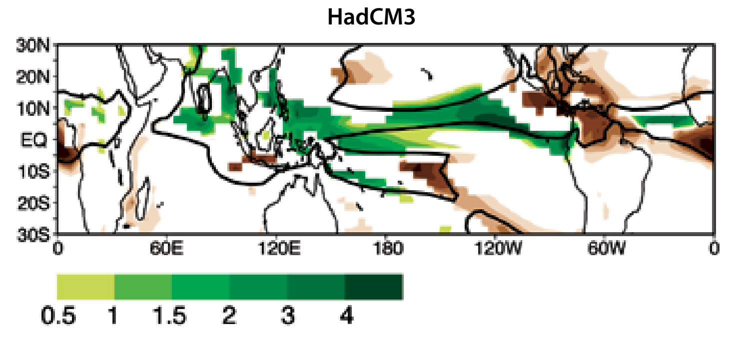


Figure 6.3 One region where the IPCC models substantially agree is the Caribbean Sea, where precipitation records over the last 50 years already show a significant decrease in rainfall, which is expected to continue. Red shading indicates the observed amount of decrease over the past 50 years, measured in units of 0.1 mm per day.

Climate modelers do need a better understanding of the convective threshold, the point where a moist column of air starts to precipitate. The onset of convection is usually described by "quasi-equilibrium" models, but according to Neelin, these make the process appear too smooth. The result is too many gentle rain showers and not enough extreme weather events. He presented an alternative model, developed in conjunction with Ole Peters, which describes convection in a similar way to other threshold phenomena in statistical mechanics. A rainstorm is like an avalanche, with a slow buildup and a fast release, so the statistical frequency of mild and intense rainfalls should resemble that of small and large avalanches. Though still relatively untested, Neelin and Peters' interdisciplinary approach might find a place in future climate models.



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[4] This section is based on a presentation by Kathy Yelick, "Architectural Trends and Programming Model Strategies for Large-Scale Machines."

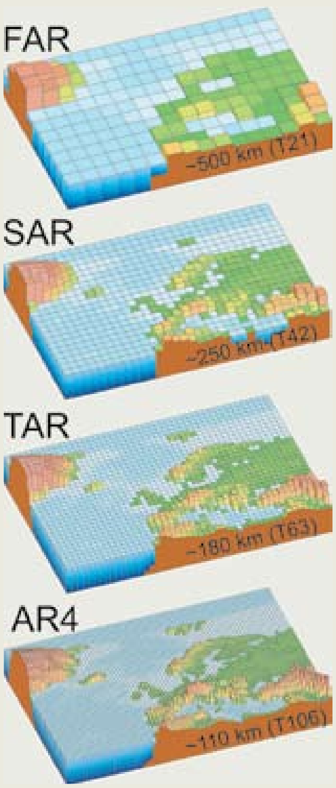
come cheap. Climate modelers will have to compete with the big money being offered to these programmers by game companies.

To run a global circulation model with a 1-kilometer grid size, which would be detailed enough to allow for the modeling of clouds, a "back-of-the-envelope" calculation suggests that climate scientists will need a 10-petaflop computer, with 100 terabytes of memory and 20 million processors. Both IBM and Japan have set targets of developing a 10-petaflop computer by 2012.

What ideas can mathematicians contribute that climate modelers don't even know about yet?

The main purpose of this workshop was to inform mathematicians about climate modeling—not vice versa. It is hoped that the active participation of mathematicians will lead to new insights or new ways of doing business that the climate scientists have not anticipated.

FIGURE 15



Each generation of climate models has used finer and finer grids. (The labels FAR, SAR, TAR, and 4AR refer to the first, second, third, and fourth assessment reports of the IPCC.) The next generation may finally be able to model individual storms. However, to achieve this level of refinement, climate scientists will have to adapt their programs to run in a new, massively parallel computing environment.

The two following sections contain a much more thorough discussion of the possible role of mathematicians. However, two points may be worth mentioning here because they came up repeatedly at the symposium:

- Mathematicians like to work from simpler models to more complex ones. Many of the mathematicians in the audience expressed serious reservations about being able to carry out serious mathematical investigations on models of such complexity. Mathematicians should not try to re-invent the wheel by designing their own models. However, climatologists do use smaller models, so-called "Earth models of intermediate complexity," for intuition-building. They also use simpler models or "process" studies aimed at revealing phenomena and cause-effect relations in more localized settings. For mathematicians interested in working on climate change, these models may make a good entry point.
- Informal discussions seemed to show that climate modelers have a few misconceptions about dynamical systems. Even if a dynamical system is inherently unpredictable because of chaos, some aspects of its behavior—the probability distribution of its states—may in fact be tractable. The "technology transfer" of ideas from stochastic dynamical systems to climate models has not happened yet.

Climate modeling

A large amount of effort continues to go into modeling of climate. The

RESEARCH TOPICS IN

A partial list of active research

motion of fluid is described by partial differential equations (PDEs), using the framework of continuum mechanics. Forcing terms, or external inputs, for the fluid equations come from the physics, chemistry, and biology of the atmosphere, ocean, land, and cryosphere. The main questions are what effects to include in the model, how to include them accurately and efficiently (when their effects range over several orders of magnitude), and how much of an impact they will have on the prediction. It is important to note that there is no clear consensus of what needs to be included in the model.

It is not out of the realm of possibility to remove Newtonian physics from the equations entirely. This radical proposal has a precedent in molecular biology, where the most successful models of protein folding do not model the protein molecules from first principles, but instead use an empirical approach based on learning from data.

A particular challenge lies in the fact that the continuum model response, and the forcing terms going into the model, vary in time scales of hours and days, while the predictions we need involve the coarse behavior in time windows of decades. Many established climate models have their origin as numerical weather prediction models, which do remarkably well at short-term prediction. However, models for long-term prediction need to include slow processes, for which there are few observations.

Analysis of data

An enormous amount of climate data continues to be collected at a wide range of locations, from diverse platforms, and using different methods. They need to be synthesized into coherent frameworks and linked to standard climate variables. For example, work needs to be done to determine how satellite measurements, which integrate over a column or "noodle" of atmosphere, correspond to events at the surface. The data should guide modelers in deriving the proper representation of climate processes, and the models should indicate what other measurements should be collected to gain further insight into the system. In this way, a mutually beneficial feedback would occur between models and theory. Another active

areas in climate change is given below.

research area is to use data, possibly in conjunction with models, to “fingerprint” the different factors that contribute to climate change.

Computational methods and platforms

Once a model is chosen, and initial and boundary conditions are specified, numerical methods are used to simulate the climate. There is a wide variety of computational approaches to integrating time-dependent partial differential equations. This is an active area of research, as climate modelers strive to balance efficiency and accuracy, while being concerned with stability of computational schemes.

Because numerical climate models usually involve a very large number of operations, computational scientists need to design algorithms that exploit capabilities of available computing platforms. At present, high-end computing is moving more and more toward parallel and multi-core processors, and climate models need to take advantage of that fact. It seems certain that ever-increasing computational capability and resources will be required for climate modeling, as the models move toward finer and finer resolution. However, finer resolution and bigger computing platforms should not become an end in themselves, but instead should be guided by concrete needs as well as evidence that the increased power will actually improve model performance.

Predictions from models and quantification of uncertainty

Each climate model is based on slightly different assumptions and therefore requires different specifications of initial conditions and forcing terms. This fact, together with the fact that the forcing terms themselves are known only approximately, leads to predictions that can be quite different from model to model. Researchers need to be careful to distinguish between variations due to chance and those that have identifiable physical or parametric causes.

Statistical techniques are used to assimilate the information from various models and synthesize these projections. Reporting a standard deviation of model results, as in the IPCC report, is simple to describe but may not be the most informative technique. Better alternatives include weighted averages or Bayesian algorithms. This is an active area of research, but is potentially controversial if it is viewed as ranking the quality of the models.

Inverse problems and data assimilation

Available data can be used in conjunction with a model to extract information about model parameters that cannot be directly measured. This approach has been used very effectively in short term weather prediction. It is a research area that has the potential to contribute to development of better climate models, and in turn, better predictions. It can also be used as a platform for validating a model and for studying the sensitivity of various factors in a model.

Economic concerns and effective policies

Quantitative methods are being applied to study the economic impact of climate change, for example on crop yields. The approach is to use the prediction provided by the climate models together with a simple model of its impact on agriculture to understand the economic costs of warming. Such analysis could also be applied to risk assessment and the economic benefits of mitigation policies. At present, economic models are not well integrated with climate models, and this is a problem that requires attention. Furthermore, uncertainties in climate model projections should be included in economic or impact models, and metrics designed to compare the costs and benefits of various policy decisions.

Research is also being conducted into developing mechanisms for curbing emission of greenhouse gases that will be effective on a political level (e.g., cap-and-trade agreements). Agreements of this type are necessarily multinational, and each player will operate with very different objectives and constraints. The challenge is to develop a policy such that it will be to the benefit of each country to comply with the policy, and still achieve global reductions in greenhouse gas emissions.

OPPORTUNITIES AND CHALLENGES FOR THE MATHEMATICAL SCIENCES

HIGH DIMENSIONAL DYNAMICAL SYSTEMS

There is a pressing need to formulate a language and theory of transient dynamics for the classes of systems that arise in climate science, including notions of stability for transient dynamics. The current use of the language of chaos and attractors leads to confusion for transient dynamics because these terms are imprecise in that context, and therefore mean different things to different people. The existing vocabulary, designed for phenomena that occur over extremely long time periods, should be replaced by terms that describe the transitory response of a high-dimensional nonlinear system.

These may include transitions between local attractors, or transient dynamics due to external forcing. Computing bifurcations in very high-dimensional systems is likely to be helpful here. It will also be helpful to find criteria under which the dynamics of low-dimensional systems remain robust when translated back into the high-dimensional setting.

Relevant mathematics: Dynamical systems, nonlinear ordinary differential equations, nonlinear partial differential equations, global analysis.

INTERPRETING AND "TUNING" MODELS

Climate scientists more or less agree that there is no such thing as a "best" climate model. Given an ensemble of models with different strengths and weaknesses, we need tools to address the quality and relevance of models. For instance, one model may be better at representing a given variable under 20th century conditions, but another may represent the stratosphere better. It is an open question how well the current suite of climate models cover the space of models relevant to the Earth's climate system.

Methods need to be developed to evaluate the impact of missing processes, and to estimate the spatial and temporal scales over which we can make a meaningful interpretation of a particular model. Shadowing experiments have been suggested as a useful approach to the latter problem.

A very serious question of quality control arises from the "tuning" of climate models. One unintended result can be that models no longer obey the laws of physics that are supposedly programmed into them. Tuning needs, first of all, to become an open rather than clandestine practice. Second, the mathematical foundations should be clarified so that climate modelers can have an optimal, or at least systematic, way to explore parameter space.

Relevant mathematics: Dynamical systems, nonlinear differential equations, statistics, knowledge discovery.

MODEL REDUCTION

Climate models are inherently very complicated and difficult to analyze. Climate modelers themselves do not rely only on state-of-the-art models for gaining insight into the climate system. They also use simple conceptual models as well as "Earth Models of Intermediate Complexity," which can be roughly described as climate models that are a few generations old. Mathematicians can experiment with these models and attempt to determine how well they mimic the dynamics of larger models. A systematic and mathematically justified model reduction method is needed in order to simplify models so that they are amenable to mathematical analysis and computation. Using such an approach, mathematical ideas can be developed on relatively simple models and then tested in the full models. The use of a hierarchy of models is critical to the productive involvement of mathematics.

Relevant mathematics: Differential equations, model reduction, asymptotic analysis, mathematical and multiphysics modeling.

MODELING

While much of the modeling process is physically based and well understood, some components in the forcing terms are inherently stochastic. Therefore, there is a need for understanding the dynamics of climate model under stochastic forcing. In addition, some phenomena that are deterministic in the short term may become effectively stochastic in the long term. In the context of model reduction, it may be useful to replace the deterministic equations with stochastic ones.

Relevant mathematics: Stochastic processes, stochastic PDEs.

MULTISCALE COMPUTATIONS

There has been considerable progress in the development of computational methods that obtain coarse scale behavior when the problem involves multiple, finer scales. This strategy has been particularly successful in the area of mathematical material science. Climate modeling could also benefit from these developments. In climate modeling, multiscale phenomena occur both in space and time.

Any complete treatment of multiscale behavior also needs to address phenomena that occur at the sub-grid level, such as turbulence, ice dynamics, and clouds. There are also practical reasons for paying attention to sub-grid phenomena. Many communities working on subsystems in areas such as hydrology, agricultural production, and ecosystems need to predict how their subsystem will respond to a change in climate. Mathematicians must provide the tools to answer such questions. Just as important, they need to delineate the limitations of the models and what constitutes appropriate use of them.

Relevant mathematics: multiscale methods, asymptotic analysis, fluid dynamics, stochastic processes.

Climate change provides mathematical scientists with a broad range of challenging research problems whose solutions could have a large societal impact. Several mathematical research topics are listed below, along with the areas of mathematics that might contribute to resolving the problems.

NUMERICAL AND COMPUTATIONAL ASPECTS

There has been tremendous progress in the numerical solution of PDEs that has not made its way into climate modeling. A careful study of the trade-off between efficiency and accuracy for the purpose of climate modeling remains to be done.

This area of opportunity overlaps with multiscale computations, because one approach to multiscale problems is adaptive mesh refinement. Simple schemes for mesh refinement lead to ill-posed problems and to artifacts in the models. Therefore, careful thought should be devoted to developing algorithms that minimize these effects on a sphere. Furthermore, the appropriate meshes for the atmosphere, land, and oceans may differ, and techniques need to be developed to couple them.

The convergence of numerical methods should be investigated, because many climate models may be operating outside the domain in which approximations can be expected to converge to the real solution of the PDEs. This factor may contribute to the "irreducible imprecision" of climate models.

Relevant mathematics: numerical analysis, spherical geometry, computational science, computer science.

DATA ASSIMILATION

Data assimilation has been proven to be effective in short-term weather prediction. With the abundance of measured data and well understood models, an effort can be mounted to use techniques from data assimilation to better obtain estimates of model parameters and forcing terms.

The challenge lies in the complexity and type of models in climate science. Techniques for linear or near-linear systems are well established and relatively effective. Many of these Kalman-based filters and variational methods either break down or become computationally unfeasible in models with the high degree of nonlinearity typical of the climate.

The fusion of data into models that incorporate a diverse array of physical, chemical, and biological processes presents particular obstacles. The data can come in different forms, and targeted techniques that exploit the particular nature of the data will prove very useful.

Relevant mathematics: numerical analysis, optimization methods, filtering methods.

UNCERTAINTY QUANTIFICATION.

The validation of individual models involves questions of how to integrate observations into the model and how to translate the uncertainty in the observations to uncertainty of model output. Also, not all model error is due to chance, and thus a purely statistical approach may miss more fundamental issues. Tools need to be developed to understand the reasons why a particular model is good or bad, with a focus on the validation of physical processes.

Many climate predictions combine the results of several models that have undergone validation separately. The big question is how to combine the different predictions, along with their individual uncertainties, into a single prediction with confidence, given that each model may do particularly well in modeling different aspects of the climate. What statistical foundations and assumptions are necessary to make inferences from this diverse ensemble of models?

Attribution is a relatively novel procedure in climate science, by which data, combined with models, are used to identify causes that produce observed effects. Statistical inference has the potential to provide a framework for such analysis. In general, climate modelers and mathematicians should make an effort to understand and import approaches and methodology from other fields in the natural sciences that are both highly data- and model-driven, such as genomics, particle physics, and nuclear testing.

Relevant mathematics: statistics, inverse problems, learning theory.

ECONOMICS AND SOCIETAL ASPECTS

Research is needed to quantify the economic risks associated with various actions in response to global climate change. Mathematics from option theory, econometrics and game theory can have a role in this aspect.

On the international level, economic incentives play a big role in whether a country is motivated to enter into an agreement and then abide by it. Here game theory for multiple players with diverse objectives can be used. On a national level, work needs to be done on ways to use market mechanisms to curb carbon production. Economic modelers need to understand how cap-and-trade agreements will affect greenhouse gas production, and how that will in turn affect the climate. They should also investigate the ramifications of proposals that are not politically feasible in the United States at the moment, such as carbon taxes. The political climate could change, and such mechanisms could become feasible in the future.

Once policies to curb carbon dioxide emission have been put in place, their effects will feed back into the climate. Integrated economic and climate models, perhaps of an intermediate complexity, should be developed to make this feedback loop easier to understand. Modelers should strive to make these models easy to use, so that policy makers can ask specific "what if" questions and receive answers. Perhaps even more interesting, and challenging, is to view the process as a control problem, and thus to design controls, i.e., policies, that force the system to achieve a certain desirable state. At a smaller scale, the auction of emission allowances also poses interesting mathematical problems of what economists call mechanism design.

Relevant mathematics: probability, statistics, stochastic processes, econometrics, game theory, operations research, optimization, control theory, financial mathematics, intergenerational economics.

The Symposium on Climate Change has

demonstrated that there are many opportunities for collaboration between mathematicians and climate scientists. It is encouraging to see that climate modelers are aware of this fact and are soliciting help from mathematicians. On the other hand, the mathematical community has, for the most part, not awakened yet to the role it can play in informing climate models and policy. While the thrust of the motivation outlined here comes from the need to understand our changing climate better, a sentiment also emerged from the symposium that the application of mathematics in this area will likely result in new and unexpected mathematical ideas and thus will enrich the mathematical sciences.

Each of the "Opportunities and Challenges" identified in the previous section carries with it an implicit recommendation: These are areas and problems that the mathematical and climate modeling communities should devote their attention to. The question then arises of what concrete steps can be taken to encourage this result.

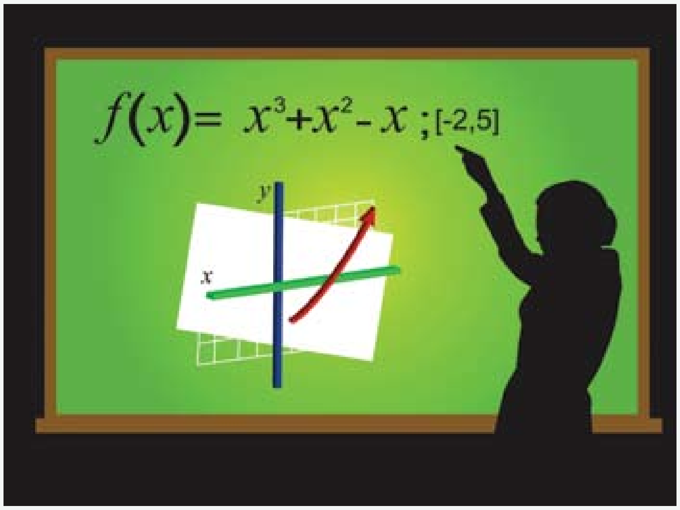
First, mathematical institutes and organizations need to engage their members in climate change research through targeted workshops. Already there are plans to build on the progress made in this symposium. The Joint Mathematics Meetings (January 2008) will include a plenary talk and several special sessions on climate change. Some of these sessions include economists and policy makers as well as geoscientists. As a follow-on to this symposium, MSRI may host a summer workshop in 2008. It has also been suggested that one of the mathematical sciences institutes, such as SAMSI (the Statistical and Applied Mathematical Sciences Institute) or IMA (the Institute for Mathematics and its Applications) might hold a special year on climate research. The Newton Institute in the UK has also indicated interest in a six-month or year-long program on the topic. The organizers of these events should work together to achieve a cumulative effect.

There has, as yet, been no formal discussion of establishing permanent institutions devoted to mathematical aspects of climate change (comparable to, say, the Mathematical Biosciences Institute at Ohio State). However, a permanent institute would be the logical culmination of three trends we expect to continue: the increasing severity of climate change, increasing public awareness and governmental and non-governmental support for research on climate change, and increasing legitimacy of climate change research within the mathematical community.

Efforts should be made to train a new generation of mathematical scientists to do research in climate change. Some possible mechanisms are summer schools, specialized postdoctoral programs (possibly jointly with organizations such as NCAR and NOAA), and visiting positions. Mathematical educators should be trained and encouraged to introduce climate-related examples into the classroom. Research experiences for undergraduates (REUs) should include realistic opportunities for climate research. This broad range of training could be facilitated by common digital repositories of data, models, and software that afford easy access to research tools. (See the web portal mentioned below.)

Much of the work of stimulating interest in this field will have to be done at the local level. Mathematics departments are encouraged to reach out to their colleagues in other disciplines to develop collaborative efforts in climate change. This could start out modestly by organizing joint seminars. Departments potentially ripe for engagement include environmental sciences, atmospheric sciences, geology, oceanography, economics, ecology, natural resources and behavior sciences.

Climate modelers, too, can play a role in making this field of research more hospitable to



mathematicians. First, they should realize that mathematicians generally work from simplicity to complexity, starting out with the simplest possible models that display the behavior that needs to be understood. Climate modelers could make readily available a hierarchy of models, ranging from the simplest to the most complex. This might be achieved by devoting one issue of an applied mathematics journal (SIADS, for example) to climate related models of the simplest and intermediate complexities. The hidden process of tuning the models should also be made more transparent -- a change that would, in any event, be beneficial to the integrity of the field.

Ideally, a web portal should be developed that would be useful to all participants in mathematical climate research. This could be a repository of mathematical problems; a source for simple and intermediate-complexity climate models; a match-making mechanism for interdisciplinary teams; and a resource that non-scientists (such as businesspeople or policy makers) could turn to for objective information on climate change. It is not clear yet who would host or maintain such a portal.

Mathematicians should explore existing funding opportunities for research on climate change, not only at the National Science Foundation (NSF) but also at the Department of Defense, the Department of Energy, and private foundations. One particular source of new money at the NSF is the Cyber-Enabled Discovery and Innovation (CDI) initiative, which will start in 2008 with a first-year budget of $52 million. The themes of CDI (knowledge extraction, interacting elements, computational experimentation, virtual environments, and educating researchers and students in computational discovery) seem to mesh well with the objectives of climate research.

At the same time, mathematicians should explore the possibilities for creating new funding opportunities. The NSF's Collaboration in Mathematical Geosciences (CMG) provides a model for such a funding mechanism. Research in climate change will involve several other disciplines with different cultures and languages, such as economics and behavioral sciences. Therefore it is imperative that collaborative mechanisms be well-designed.

Finally, both mathematicians and climate experts will need to communicate with the public, with elected officials, and with the media in a way that emphasizes the robust aspects as well as the uncertainties of climate predictions. They should be frank and forthright about the complexity of the climate system and the limitations inherent in any approximation of it. To the extent possible, they should educate the public to the fact that even the best model will produce a range of possibilities, not a single "forecast for the future." They should prepare the public to understand that the range of possibilities may change as the models improve and as we get new data about the Earth system. This will not mean that scientists are changing their minds or contradicting themselves; it is part of the normal process of science. The very real uncertainties in the projections of the future should not be allowed to obscure the fact that climate change is occurring and cannot be ignored any longer.

APPENDIX A

Program of the Scientific Workshop

Thursday, April 12, 2007

Background and Impact

Inez Fung (*University of California at Berkeley*)	"Issues in Climate Change"
Cecilia Bitz (*University of Washington*)	"Sea ice cover in a changing climate"

Uncertainty, Risks, and Decisions

Lisa Goldberg (*MSCI Barra, Inc.*)	"Forecasting the Risk of Extreme Events"
Max Auffhammer (*University of California at Berkeley*)	"Impact of Aerosols on Rice Production in India Through Local Climate"
Lenny Smith (*London School of Economics*)	"Seeing Through Climate Models"

Identifying Climate Change

Ben Santer (*Lawrence Livermore National Laboratory*)	"Detection and Attribution of Climate Change"

Statistical Issues and Incorporating Data

Claudia Tebaldi (*National Center for Atmospheric Research*)	"Future Climate Projections from Multi-Model Ensembles: Motivation, Challenges, Approaches and Ways Forward"
Jeff Anderson (*National Center for Atmospheric Research*)	"Using Observations to Estimate Climate Model Parameters"

Friday, April 13, 2007

Computational Issues

Phil Colella (*Lawrence Berkeley National Laboratory*),	"Algorithms for the Study of Climate Change"
Kathy Yelick (*University of California at Berkeley*)	"Architectural Trends and Programming Model Strategies for Large-Scale Machines"

New Modeling Challenges

David Neelin (*University of California at Los Angeles*)	"Precipitation Change and the Challenges of Modeling"
Cecile Penland (*National Oceanic and Aeronautic Administration*)	"When We Can't Keep Track Of Everything: On Diffusion Processes and Levy Flights in Climate Modeling"
Jim McWilliams (*University of California at Los Angeles*)	"Irreducible Imprecision in Atmospheric and Oceanic Simulation"

Future Directions

Bill Collins (*National Center for Atmospheric Research*)	"Where do we go from here?"

Videotapes of these lectures may be found at the MSRI website, www.msri.org.

Attendees of the Scientific Workshop "Climate Change: From Global Models to Local Action"

Mathematical Sciences Research Institute, April 12-13, 2007

Rafael Abramov, *University of Illinois*
Malcolm Adams, *University of Georgia*
Jeff Anderson, *National Center for Atmospheric Research*
Ari Ariyawansa, *Washington State University*
Max Auffhammer, *University of California at Berkeley*
George Avalos, *University of Nebraska*
Nathaniel Berkowitz, *no affiliation given*
Bjorn Birnir, *University of California at Santa Barbara*
Cecilia Bitz, *University of Washington*
Jonathan Block, *University of Pennsylvania*
Robert Bryant, *Duke University*
Alin Carsteanu, *Cinvestav*
Bem Cayco, *San Jose State University*
Fatih Celiker, *Wayne State University*
Bill Collins, *National Center for Atmospheric Research*
Louis Crane, *Kansas State University*
Forrest DeGroff, *no affiliation given*
Maarten v. de Hoop, *Purdue University*
Eric DeWeaver, *University of Wisconsin*
Frank Drost, *University of New South Wales*
Philip Duffy, *no affiliation given*
Bahman Engheta, *University of California at Riverside*
Greg Eyink, *Johns Hopkins University*
Yue Fang, *University of California at Berkeley*
Inez Fung, *University of California at Berkeley*
Jimmy Fung, *Hong Kong University of Science and Technology*
Ashis Gangopadhyay, *Boston University*
Daryl Neil Geller, *Stony Brook University*
Serge Guillas, *Georgia Institute of Technology*
Dan Gunter, *Lawrence Berkeley National Laboratory*
A. G. Helmick, *North Carolina State University*
Robert Higdon, *Oregon State University*
Matthew Hoffman, *University of Maryland*
Masaki Iino, *University of Utah*
Larens Imanyuel, *Technominiaturization Project*
John Kahl, *University of Missouri at Columbia*
Hans Kaper, *National Science Foundation*
Allan Kaufman, *Lawrence Berkeley National Laboratory*
Boualem Khouider, *University of Victoria*
Eric Kostelich, *Arizona State University*
Charlie Koven, *University of California at Berkeley*
Hugo Lambert, *University of California at Berkeley*
Chloe Lewis, *University of California at Berkeley*
Wing Suet Li, *Georgia Institute of Technology*
Yi Li, *University of Iowa*
Regina Y. Liu, *Rutgers University*
Douglas Lind, *University of Washington*
Frank Ling, *University of California at Berkeley*
David Lobell, *Lawrence Livermore National Laboratory*
John MacDonald, *University of British Columbia*
Brian Maurizi, *Washington University of St. Louis*
Richard McGehee, *University of Minnesota*
Jim McWilliams, *University of California at Los Angeles*
Robert Megginson, *University of Michigan*
Juan Meza, *Lawrence Berkeley National Laboratory*
Norman Miller, *Lawrence Berkeley National Laboratory*
John Moussouris, *MicroUnity*
David Neelin, *University of California at Los Angeles*
Douglas Nychka, *Institute for Mathematics Applied to Geosciences*
Myunghyun Oh, *University of Kansas*
Sergei Ovchinnikov, *San Francisco State University*
Hyo-Seok Park, *University of California at Berkeley*
Cecile Penland, *National Oceanic and Atmospheric Administration*
Alexandra Piryatinska, *San Francisco State University*
Dimitros Politis, *University of California at San Diego*
Serge Preston, *Portland State University*
Renny Rueda, *University of Externado (Colombia)*
Ben Santer, *Lawrence Livermore National Laboratory*
Fadil Santosa, *University of Minnesota*
Eric Schechter, *Vanderbilt University*
Brad Shelton, *University of Oregon*
Emily Shuckburgh, *University of Cambridge*
Lenny Smith, *London School of Economics*
Hartland Snyder, *no affiliation given*
A. Spehr, *no affiliation given*
Dave Stainforth, *University of Oxford*
Alexander Stine, *University of California at Berkeley*
Andrea Taschetto, *University of New South Wales*
Claudia Tebaldi, *Athene Software, Inc.*
Bruce Turkington, *University of Massachusetts at Amherst*
Chunming Wang, *University of Southern California*
Shouhong Wang, *Indiana University*
Michael Wehner, *National Energy Research Scientific Computing Center*
Chris Wiggins, *Columbia University*
Peter Wolenski, *Louisiana State University*
Carol S. Wood, *Wesleyan University*
Katherine Yelick, *University of California at Berkeley*
Mary Lou Zeeman, *Bowdoin College*

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